

Forecasting Price Changes Under TFP Based Incentive Regulation

ONDŘEJ NOWAK, ONDŘEJ MACHEK

Department of Business Economics

University of Economics, Prague

W. Churchill Sq. 4, 130 67 Prague 3

CZECH REPUBLIC

ondrej.nowak@vse.cz <http://kpe.fph.vse.cz>

Abstract: - In this paper, a model based on Monte Carlo simulation is proposed in order to estimate the future price cap set by the regulator under incentive regulation based on productivity benchmarking. The forecast of future price changes is demonstrated on an empirical example based on a study on the Australian National Railways carried out by the Australian Industry Commission. The resulting forecast is represented by a probability distribution of a random variable with its parameters. Factors with the highest influence on the price changes are also identified using the sensitivity analysis.

Key-Words: Price Change Forecast; Total Factor Productivity; Uncertainty; Monte Carlo Simulation

1 Introduction

Network industries (energy and natural gas, water and sewerage, communications and transportation industries) are often considered to be affected with public interest and the protection and stability of these industries are of high importance for the government and its environmental, social and economic policy. Therefore, in most countries in the world, these industries are regulated by the government. The regulatory process devotes considerable attention to the price level.

The firms operating in price-regulated industries do not face competitive pressures, but regulatory constraints. Their profits depend on the choice of the regulatory method and its parameters. Total factor productivity (TFP) has recently become an important tool of regulation. Under this regulatory regime, the maximum price of services is set according to the relative performance of the firms.

For a regulated firm, forecasting future prices is an essential task, since the regulated prices directly influence its future cash flows, performance and growth. However, future prices are also determined by factors which may not be directly affected by the regulated firms, especially measurement errors, but also by fluctuation of price level and demand. Therefore, some degree of uncertainty in the calculations is practically inevitable.

Monte Carlo simulation has become a popular tool for decision-making under uncertainty for its simplicity and flexibility. In this paper, we propose a model based on the Monte Carlo simulation which can be used in the estimation of future price cap set

by the regulator under incentive regulation based on productivity benchmarking. We demonstrate the forecast of future price changes on an empirical example based on a study on the Australian National Railways carried out by the Australian Industry Commission.

1.1 Price Regulation and X-factor

All methods of economic regulation are based on the idea that a company should recover its costs and earn a commensurable return on the investments. The revenue requirements (RR) can be calculated as

Equation 1

$$RR = O \& M + D + T + RB \times RoR$$

where $O \& M$ denotes operating and maintenance costs, D denotes depreciation, T denotes taxes, RB is the regulatory asset base and RoR is the rate of return.

The traditional cost-of-service regulation is based on summing allowed expenses and calculating a required rate of return. This approach has several disadvantages – for example, information asymmetries between regulator and regulated companies, incentive to overinvest (A-J-W effect, [1]) or to invest imprudently and the tariff level has to be reviewed frequently.

The purpose of incentive regulation is to reduce the impact of negative aspects of cost-of-service regulation and motivate the company to behave efficiently. In general, we distinguish two basic

alternatives of incentive regulation: price-cap and revenue-cap.

The price-cap method is based on setting maximum tariffs for services provided, whereas the revenue-cap method is based on capping total revenues. The formula is

Equation 2

$$P(t) = (1 + RPI - X) \times P(t - 1)$$

where $P(t)$ is the price level in period t and $P(t-1)$ denotes the price level in period $t-1$. Since the tariffs are capped according to the inflation rate (RPI-factor) and efficiency (X-factor), this method of regulation is also referred to as RPI-X regulation.

The principle that the revenue requirements should not be based only on the regulated firm's costs is also called regulatory benchmarking [10]. This approach, if properly applied, strengthens the incentives for the regulated firms to behave efficiently. The regulated firm's productivity growth is compared with the productivity growth of the economy. When the firm succeeds to improve its productivity more than other firms, it is rewarded by greater profits. In order to apply incentive regulation, it is necessary to determine the above-mentioned X-factor, which can be calculated using total factor productivity (TFP) change. It is possible to derive the regulated price change as [7]

Equation 3

$$\Delta p = \Delta p_n - (\Delta TFP - \Delta TFP_n - (\Delta w - \Delta w_n))$$

where Δp_n is the inflation rate of economy outputs, Δw_n is the inflation rate of economy inputs, Δw is the inflation rate of the regulated industry inputs, ΔTFP is the change of productivity of the regulated industry and ΔTFP_n is the change of productivity of the economy. The X-factor of efficiency can be rewritten as

Equation 4

$$X = (\Delta TFP - \Delta TFP_n) - (\Delta w - \Delta w_n)$$

This expression is based on the assumption that all terms are deterministic. However, differences in productivity growth can also be caused by factors which may not be affected by the firms, especially measurement errors and random external influences. In the next section, we will introduce the TFP measurement and discuss possible random variables in the calculations.

1.2 Total Factor Productivity Measurement

Traditionally, productivity is defined as the ratio of output over input. The total factor productivity (TFP) approach takes into account all possible inputs and outputs of the firm. In this case, it is necessary to aggregate the set of outputs and inputs to obtain scalar values in the numerator and denominator.

In economic practice, TFP is measured by productivity indexes. Indexes are a common tool to measure price or quantity changes between two periods. Since in TFP calculations, we deal with the ratio of output and input quantities, we employ quantity indexes.

Indexes can be based on distance function or on price aggregation (for detailed discussion, see e.g. [3]). Among measures based on distance function, we can cite the Malmquist productivity index [2]. These measures require optimization problem solving (data envelopment analysis) or regression methods which measure the distance from a real, but unknown frontier.

Other TFP measures are based on price aggregation, such as Törnqvist productivity index [11] or Fisher productivity index [5]. These measures require data about input and output prices, but can be derived directly from empirical data. In this paper, we will use the Törnqvist index to measure productivity growth.

Törnqvist index is an example of the so-called superlative indexes [4]. The calculation is based on observed or estimated prices of inputs and outputs.

Let $\mathbf{x} = (x_1, x_2, \dots, x_n)$ denote the vector of input quantities, let $\mathbf{y} = (y_1, y_2, \dots, y_m)$ denote the vector of output quantities, $\mathbf{w} = (w_1, w_2, \dots, w_n)$ is the vector of input prices and $\mathbf{p} = (p_1, p_2, \dots, p_m)$ is the vector of output prices. The Törnqvist index of productivity is defined as a ratio of output quantity index Y_T and input quantity index X_T . Usually, the two quantity indexes are specified in their logarithmic form as

Equation 5

$$\ln Y_T(\mathbf{y}, \mathbf{p}) = \frac{1}{2} \left(\sum_m \left[\frac{p_{m,t} y_{m,t}}{\sum_m p_{m,t} y_{m,t}} + \frac{p_{m,t+1} y_{m,t+1}}{\sum_m p_{m,t+1} y_{m,t+1}} \right] \ln \frac{y_{m,t+1}}{y_{m,t}} \right)$$

$$\ln X_T(\mathbf{x}, \mathbf{w}) = \frac{1}{2} \left(\sum_n \left[\frac{w_{n,t} x_{n,t}}{\sum_n w_{n,t} x_{n,t}} + \frac{w_{n,t+1} x_{n,t+1}}{\sum_n w_{n,t+1} x_{n,t+1}} \right] \ln \frac{x_{n,t+1}}{x_{n,t}} \right)$$

And finally, the Törnqvist index of productivity can be specified as

Equation 6

$$\Pi_T(\mathbf{x}, \mathbf{y}, \mathbf{w}, \mathbf{p}) = \frac{Y_T(\mathbf{y}, \mathbf{p})}{X_T(\mathbf{x}, \mathbf{w})} = e^{\ln Y_T(\mathbf{y}, \mathbf{p}) - \ln X_T(\mathbf{x}, \mathbf{w})}$$

The Törnqvist index depends on four vectors, out of which all can be represented by random variables; both input and output price fluctuations and input and output quantities which are dependent on the demand. The source data for the productivity calculations will most probably differ in years as the calculation for each year will be based on unique set of data.

However, for a regulated company, the estimation of the price or revenue cap in the following regulatory period is a challenging but important part of planning, since the above-defined revenue requirements (RR) directly influence future cash flows and future performance and growth. The inherent uncertainty in the estimation of future revenues can be investigated quantitatively by the Monte Carlo simulation techniques.

1.3 Uncertainty and the Monte Carlo Simulation

Generally all the estimates are uncertain numbers with the occurrence rather on an interval with varying density of expectations than on a few discrete data points. Deterministic models working with a single point estimate are usually using only one number out of such an interval thus ignoring important and valuable information about the uncertainty [6]. Limiting ourselves only to single points however makes us lose information concerning not only the variance (degree of uncertainty or risk), but also the shape of the probability distribution, which might not be symmetrical or unimodal [9].

Furthermore, distinguishing the expected and most likely value is an integral part of business modeling, because the two values can differ significantly which can result in wrong model input. This is usually a case of asymmetric distributions where mean, modus and likeliest value do not overlap [6].

There are several methods for incorporating the uncertainty into the financial model, but due to its simplicity and flexibility, the Monte Carlo simulation is the most popular. This approach calculates numerous scenarios of a model by repeatedly drawing values from a user-predefined probability distributions and inserting them in the model. The output of the simulation is a distribution of a monitored variable (e.g. total factor productivity

or price change), which however should not be understood as a probability distribution, but rather as a distribution of our ignorance (or uncertainty) concerning the model output.

The structure of a simulation model is very similar to a deterministic model, with all functions and operations that link variables together, except that each variable is represented by a probability distribution instead of a single value. The objective is to calculate the combined impact of the variability in the model's parameters in order to determine a probability distribution of the possible model outcomes.

Replacing uncertain numbers with distributions allows for an integration of a wide range of improvements into financial modeling. Separately analyzing each uncertain variable, approximating its potential occurrence with a corresponding shape of the distribution and observing their joint influence on model outputs provides useful insight into what is beyond the average scenario.

There are essentially two sources of information used to quantify the variables within a risk analysis model: available data and expert opinions [12]. In the model, which will be introduced in the following section of this paper, historical data will be the primary source for the uncertainty quantification.

2 Proposed Model

In this section, we propose a model which can be used in the estimation of future price cap set by the regulator which employs the above-described RPI-X incentive regulation based on productivity benchmarking. We use the notion of random vectors whose components are scalar-valued random variables on the same probability space.

Let $\mathbf{X} = (x_1, x_2, \dots, x_n)$ denote a random vector of input quantities, let $\mathbf{Y} = (y_1, y_2, \dots, y_m)$ denote a random vector of output quantities, $\mathbf{W} = (w_1, w_2, \dots, w_n)$ is a random vector of input prices and $\mathbf{P} = (p_1, p_2, \dots, p_m)$ is a random vector of output prices, with each component having a proper marginal probability distribution. We can use the Törnqvist index to estimate the productivity growth as in the previous sections, but in this case using random variables as the function arguments, i.e.

Equation 7

$$\Pi_T(\mathbf{X}, \mathbf{Y}, \mathbf{W}, \mathbf{P}) = \frac{Y_T(\mathbf{Y}, \mathbf{P})}{X_T(\mathbf{X}, \mathbf{W})}$$

The productivity growth ΔTFP will be represented by a random variable with a specific

probability distribution and its parameters, as well as the future price cap in the following regulatory period, which can be estimated as

Equation 8

$$\Delta p = \Delta p_n - (\Pi_T(\mathbf{X}, \mathbf{Y}, \mathbf{W}, \mathbf{P}) - \Delta TFP_n - (\Delta w - \Delta w_n))$$

In order to apply this model in practice, the analyst has to estimate the probability distributions of the function arguments \mathbf{X} , \mathbf{Y} , \mathbf{W} and \mathbf{P} . This can be achieved either by using expert estimates or by employing numerical methods.

2.1 Empirical Example

In this article, we will demonstrate the estimation of an uncertain price level change in the year 1991 using real data from years 1979-1990 based on a study on the Australian National Railways carried out by the Australian Industry Commission[3].

Three categories of output are considered:

- y_1 : mainland freight, measured in net-tonne-kilometers;
- y_2 : tasrail freight (net-tonne-kilometers);
- y_3 : passenger services(passenger-train-kilometers).

Further, six categories of input are considered:

- x_1 : land, building and perway(1.000\$);
- x_2 : plant and equipment (1.000\$);
- x_3 : rolling stock (1.000\$);
- x_4 : labor (full-time staff);
- x_5 : fuel (1.000 liters);
- x_6 : other inputs (1.000\$).

The more complex the relations between input variables are, the more difficult the analytical solution of the estimation of the price level change becomes. The estimation is the result of an aggregation of many random variables that are often mutually dependent and whose probability distributions are not usually known in advance and therefore must be estimated with their interdependencies included. At this point the employment of Monte Carlo simulation is convenient.

2.2 Employment of Simulation Techniques

The model offers historical dataset from years 1979-1990, which can be subjected to a regression analysis for each variable in order to fit the regression curve. The data, however, never fit the

regression curve perfectly (i.e.usually, there is no functional relationship), but rather they are scattered along.

The regression curve enables to obtain the expected value of Y (i.e. dependent variable) for the year 1991. To make the process more realistic, a factor of an estimation error shall be added further to the expected value of Y . The estimation error should theoretically follow normal distribution with zero mean and standard deviation equal to the standard error of the regression(also known as the root mean square error or RMSE).

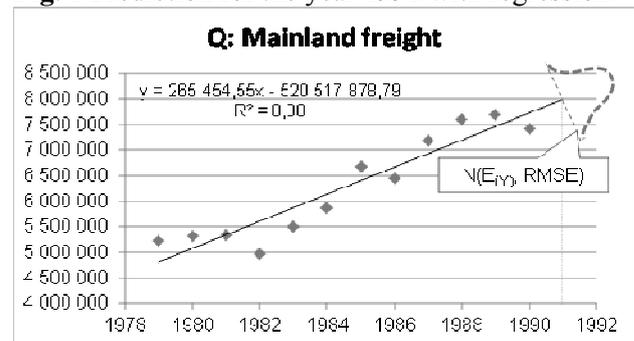
An example of this approach is shown at Fig.1, which provides the historical quantity of mainland freight with its prediction for the year 1991. The historical data fit the regression line quite well ($R^2 = 88\%$), with the standard error of regression equal to 371 091. After putting values into the regression equation, the calculation proceeds as

Equation 9

$$Y = 265\,454 \cdot 1991 - 520\,517\,878 = 8\,002\,121$$

Parameters of the normal distribution are therefore the estimation of Y (8 002 121) as the mean and RMSE (371 091) as the standard deviation. With this procedure it is possible to combine regression analysis with Monte Carlo simulation in order to generate random predictions for all the variables within the model.

Fig. 1 Prediction for the year 1991 with regression



Source: authors' analysis

In business modeling it is also necessary to deal with the question of to what extent the behavior of one variable determines the expected occurrence of others. In this matter working with linear dependencies expressed by correlations is usually the easiest method. However, it is important to keep in mind that their application is correct only if there is a presumption that the dependence is "approximately" linear.

Two types of dependencies can be distinguished in this model. First, each variable is following a certain trend (i.e. some are increasing other are decreasing in time). This trend is being accounted for by the slope of the regression curve.

Second type of a possible dependency is the deviation from this trend between variables (i.e. correlations between regression residuals of variables). This dependency reflects situations when deviations from trend tend to occur simultaneously for some variables. To cover this phenomenon, the correlation matrix of regression residuals is used as a proxy for capturing dependencies between uncertain variables within the model. This correlation matrix defines correlations between the normal distributions introduced earlier.

To determine the price level change in the year 1991, we would also have to estimate the inflation rate of economy outputs $v\Delta p_n$, the inflation rate of economy inputs Δw_n , the inflation rate of the regulated industry inputs Δw and the total factor productivity change of the economy ΔTFP_n .

In the context of forecasting, these four parameters are uncertain and should also be the subjected to risk analysis, which will most likely result in their quantification by a probability distribution. This is however beyond the scope of this paper. Hence, this step will be excluded from the following analysis and these variables will be arbitrarily chosen as follows:

- $\Delta p_n = 1.0728$ (the inflation rate in Australia between 1990/1991);
- $\Delta w - \Delta w_n = 0$ (we do not suppose that input price inflation will exceed the overall input price inflation, following [8]);
- $\Delta TFP_n = 0.7$ (the TFP growth between 1987-1992 calculated by OECD).

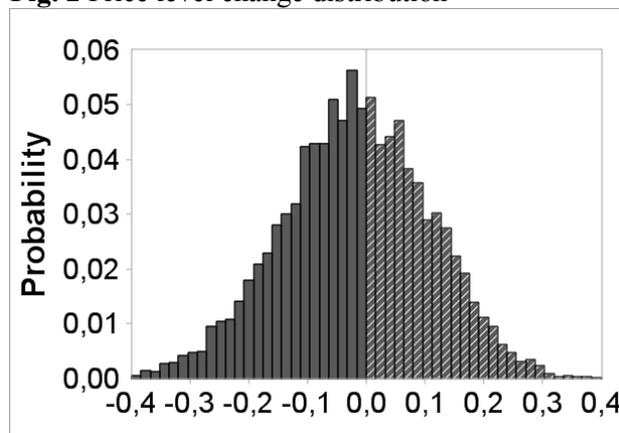
All the inputs for Monte Carlo simulation were entered to the spreadsheet using ModelRisk simulation software which is designed as an add-in for MS Excel. Its outputs are discussed in the following section of this paper.

2.3 Results

The distribution of the price level change in the year 1991, which is the output of Monte Carlo simulation, is shown at Fig. 2. The non-shaded area on the left side of the chart reflects the 55.3% probability of the decrease in the price level, which is higher than the 44.7% probability of its increase reflected by the shaded area on the right. Expected value of the change in the price level in

the year 1991 is -1.8% with the standard deviation of 12.6%.

Fig. 2 Price level change distribution

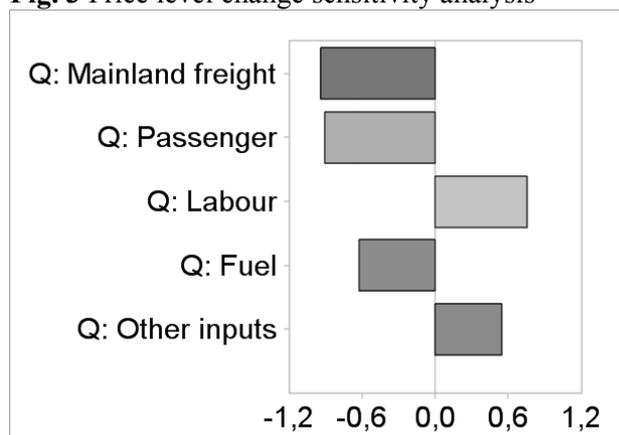


Source: authors' analysis, ModelRisk software

Part of the interpretation of the result is often the identification of significant intervals. For example, the lower and upper fifth percentile can be cut off thus obtaining observed value at the 90% confidence level. In this case the price change is expected to be between -22.9% and +18.3% on the 90% level of confidence.

Another output of the simulation is the sensitivity analysis displayed at Fig. 3, which shows six variables (Q denotes quantity) with the highest impact on the expected price change in the year 1991.

Fig. 3 Price level change sensitivity analysis



Source: authors' analysis, ModelRisk software

Different models varying in structure and inputs shall have different sets of the most influential factors. These factors should be the primary concern of the management as they drive the forecasted value.

3 Conclusion

The uncertainty in the estimation of future revenues under government regulation, which is caused by measurement errors, but also by random fluctuations of price level and demand, can be investigated quantitatively by simulation techniques.

In this paper, we proposed a simple model based on Monte Carlo simulation which can be used in the estimation of future price cap set by the regulator under incentive regulation based on total factor productivity benchmarking. We demonstrated the forecast of future price changes on an empirical example based on a study on the Australian National Railways carried out by the Australian Industry Commission.

The proposed model builds on a bivariate regression analysis, by which the time trend for each variable is estimated. The estimation error is added as a potential deviation from this trend in the forecasted period. The estimation error should follow a normal distribution with zero mean and standard deviation equal to RMSE of the regression analysis.

A possible interdependency between variables in deviations from this trend in partial years was also included in the model. To cover this phenomenon, the correlation matrix of regression residuals was used in order to define correlations between the above defined normal distributions.

Possible outputs of Monte Carlo simulation include probability distribution of the forecasted variable with relevant statistics. Sensitivity analysis further identifies variables with the highest influence on the forecasted variable, which can be represented for example by a tornado chart.

Our model can be used in dealing with uncertainty under TFP-based incentive regulation regime. Under different regimes (such as efficiency benchmarking or data envelopment analysis), the principle would be similar – to avoid using single-point estimates, but rather to introduce random variables represented by probability distributions and to understand their interdependencies. In a dynamic and constantly changing environment, which is the case of network industries, all regulatory methods are affected by some degree of uncertainty and measurement errors, which have to be treated with discretion.

Acknowledgment

The authors would like to express their gratitude for the financial support of the University of Economics, Prague - the paper is one of the outcomes of the research project VŠE IP300040 The crucial aspects of the competitiveness of enterprises and national economies in the global economic system.

References:

- [1] H. Averch H and L.L. Johnson, Behavior of the Firm Under Regulatory Constraint, *American Economic Review*, Vol.52, No.5, 1962, pp. 1052-69.
- [2] D.W. Caves, L.R. Christensen and W.E. Diewert, Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers, *Economic Journal*, Vol.92, 1982, pp. 73-86.
- [3] T.J. Coelli, D.S. PrasadaRao, C.J. O'Donnell, G.E. Battese, *An introduction to Efficiency and Productivity Analysis*, New York: Springer, 2005.
- [4] W.E. Diewert, Exact and Superlative Index Numbers, *Journal of Econometrics*, Vol.4, No.2, 1976, pp. 115-145.
- [5] I. Fisher, *The Making of Index Numbers*, Boston: Houghton-Mifflin, 1922.
- [6] D. Hertz, Risk Analysis in Capital Investment. *Harvard Business Review*, Vol.42, 1964, pp. 95-106.
- [7] J. Makhholm, Elusive Efficiency and the X-Factor in Incentive Regulation: the Törnqvist v. DEA/Malmquist Dispute, *NERA Publications*, 2007.
- [8] Pacific Economics Group, *X factor recommendations for New Zealand electricity distribution price controls*, Pacific Economics Group, 2009.
- [9] S.L. Savage, and J. Danziger, *The Flaw of Averages: Why We Underestimate Risk in the Face of Uncertainty*. New York: John Wiley & Sons, 2009.
- [10] A. Schleifer, A theory of yardstick competition, *Rand Journal of Economics*, Vol.16, No.3, 1985, pp. 319-28.
- [11] L. Törnqvist, The Bank of Finland's Consumption Price Index, *Bank of Finland Monthly Bulletin*, Vol.10, 1936, pp. 1-8.
- [12] D. Vose, *Risk Analysis. A Quantitative Guide*, Third Edition. New York: John Wiley & Sons, 2008.