Analysis of Dimensionless Numbers for Convective Heat Transfer Models Used in the Study of Spark Ignition Engines

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Abstract: Convective heat transfer models derived from equations for turbulent heat transfer in pipes are still used when undertaking thermodynamic calculations with limited data available for engine geometry characteristics. While computational fluid dynamics codes can offer an improved accuracy, they require increased computational power, extensive knowledge of fluid dynamics and numerical modeling. Simple equations derived from turbulent flow in tubes models require empirical correction factors that often need to be adjusted for a specific engine and working conditions. This study proposes an analysis of the dimensionless numbers employed in such convective heat transfer models in order to identify equations that are best suited for premixed charge spark ignition engines. For this purpose, a simple zero–dimensional model was employed to reconstruct the in–cylinder pressure and temperature traces and these values were then used for calculating heat transfer parameters. Ensemble averaged mean fluid velocity was identified as one of the main parameters that influences heat losses to the combustion chamber walls, and therefore, its prediction as accurately as possible is considered to be paramount.

Key-Words: spark ignition engines, premixed charge, convective heat transfer models, dimensionless numbers

1 Introduction

Heat losses during combustion and expansion are of great influence to engine efficiency, exhaust emissions and component thermal loading [1]. When considering premixed charge spark ignition (SI) engines, convective heat transfer is the main mechanism, as radiation from gases and the flame are not that significant. Models for predicting heat losses to the combustion chamber walls are usually based on equations for turbulent flow in tubes, as they provide an acceptable level of accuracy with several correction factors needed to fit engine experimental data [2]. Complex models that employ computational fluid dynamics (CFD) are more accurate without the need for extensive corrections, but require a detailed description of engine geometry and increased computational resources [3, 4].

In–cylinder pressure analysis is one of the most widely used tools in the field of internal combustion engines. For this reason, a simple zero–dimensional thermodynamic model was used to reconstruct the in–cylinder pressure trace with limited input data available for engine geometry and operational parameters. This reconstructed trace was compared with experimental values, with good agreement between the two sets of data. In–cylinder temperature and heat transfer rates throughout the closed valve part of the cycle were then calculated.

A discussion with reference to the dimensionless numbers and required corrections is presented. One of the main conclusions of this study is that an accurate description of the fluid flow inside the cylinder is the determining factor when considering the precision of convective heat transfer models.

2 Thermodynamic Model

As combustion is the most complex process, emphasis was directed towards this part of the working cycle. Only the period when the valves are closed was investigated, with imposed values for intake (p_i) and exhaust (p_e) pressure. Intake was modeled so that a volumetric efficiency of 50% was obtained, in order to match experimental data [5].

Compression and expansion can be modeled with acceptable accuracy using simple equations such as (1) and (2)

$$p = p_i \cdot \left(\frac{V_d + V_c}{V} \right)^{m_c}, \qquad (1)$$

$$p = p_b \cdot \left(\frac{V_b}{V}\right)^{m_e}, \qquad (2)$$

where p is the momentary pressure, with subscript i for intake and b for end of combustion, all measured in Pa, V_d displacement, V_c combustion chamber volume at top dead center (TDC) and V momentary cylinder volume, all measured in m^3, m constant exponent, with c for compression and e for expansion.

Heat release was modeled using a Wiebe function for defining burn mass fraction (x_b), with the end of combustion determined from the pressure trace analysis, and defining the location of maximum heat release rate, combined with solving the equation $d^2x_b(\theta) = 0$ for calculating parameter m. With a set as 7 for x_b = 0.999 at combustion completion, all four Wiebe parameters were defined, and equation (3) could be solved

$$x_b = 1 - e^{-a\cdot\left(\frac{\theta-\theta_i}{\theta_b-\theta_i}\right)^{m+1}}, \qquad (3)$$

where x_b is the burned mass fraction, θ momentary crank angle measured in rad, with subscript i for ignition, and parameter m obtained as 3.477.

Once the burned mass fraction was available, the energy equation could be written

$$dx_b \cdot Q_{av} = \frac{1}{\gamma - 1} \cdot V \cdot dp + \frac{\gamma}{\gamma - 1} \cdot p \cdot dV, \qquad (4)$$

where Q_{av} is the available heat calculated using the equation $Q_{av} = \eta_c \cdot m_f \cdot LHV - Q_w$, measured in J, with combustion efficiency η_c taken as 0.95, m_f fuel mass per cycle, in kg, LHV lower heating value in J/kg, Q_w heat loss to the combustion chamber walls, in J, and γ ratio of specific heats, calculated as an average value for the burned and unburned regions.

An iterative calculation was employed, with choosing m_c, rate Q_w / Q_f, with $Q_f = \eta_c \cdot m_f \cdot LHV$, m_e and then calculating the in–cylinder pressure and temperature traces.

Table 1 Model constants and relative error

Parameter	m_c [-]	m_e [-]	Q_{av} / Q_f [-]	η_i [-]	Error [%]
Model	1.356	1.326	0.847	0.358	6.87
Experiment [5]	-	-	-	0.335	-

With the gas temperature values, heat transfer rates were then calculated and resulting heat losses during compression, combustion and expansion were compared to originally imposed values by choosing m_c, Q_w and m_e. Table 1 presents the resulting values for these three parameters for methane combustion at an engine speed of 1500 rev/min, with the resulting pressure trace shown in figure 1, by calculating heat flux with the correlation defined by Gnielinski [6], after convergence was attained. The reason for choosing this equation will be discussed later in the article. Other relevant engine geometry characteristics are given as 85 mm bore, 88 mm stroke, 8.5 compression ratio and 149 mm connecting rod length. As for other operational parameters, ignition advance was set at 27 deg before TDC, and combustion end at 400 deg crank angle.

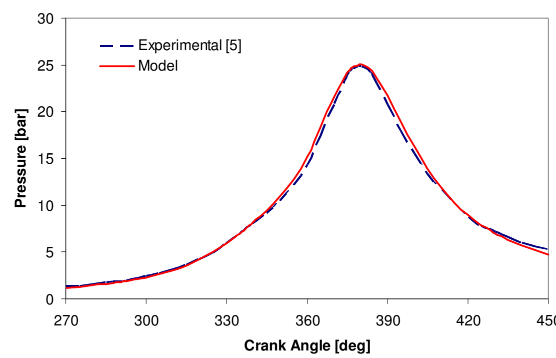

Fig.1 Experimental and reconstructed in–cylinder pressure trace

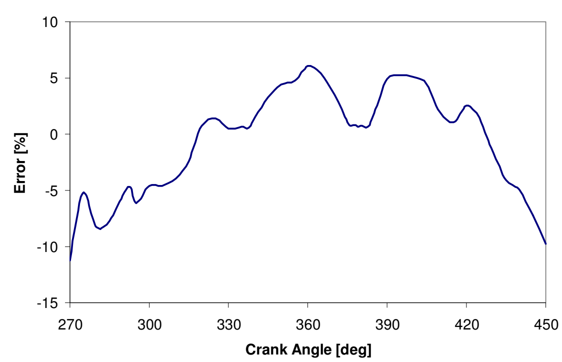

Fig.2 Relative error for the reconstructed in–cylinder pressure trace

A good agreement was obtained for the calculated and measured pressure traces, with relative errors slightly over 5% during combustion and below 12% during compression and expansion. One possible reason for the increased error in the lower pressure regions of the diagram is that

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pressure sensors usually feature increased accuracy of ±1–2% only over a certain threshold, usually over 5 bar [7]. The overall error level of the model can be better evaluated by comparing calculated indicated efficiency (η_i) with the measured values (table 1). Given that the model is a relatively simple one, an error below 7% can be considered as satisfactory.

3 Convective Heat Transfer Models

One of the most widely used correlations for convective heat transfer calculations in the study of SI engines is the equation proposed by Woschni, developed from a simple $Nu = c\,Re^m$ power law

$$h = 3.26 \cdot B^{-0.2} \cdot p^{0.8} \cdot T^{-0.55} \cdot w^{0.8}, \qquad (5)$$

where h is the average convective heat transfer coefficient, in W/m^{2}K, B cylinder bore, in m, with pressure p measured in kPa, T gas temperature, in K and speed w defined by equation (6)

$$w = C_1 \cdot S_p + C_2 \cdot V_d \cdot T_r / (p_r \cdot V_r) \cdot (p - p_m), \qquad (6)$$

where w is measured in m/s, S_p mean piston speed in m/s, V cylinder volume, in m^3, T_r, p_r and V_r temperature, pressure and volume at a reference point, usually at the start of combustion, p_m motored cylinder pressure at the same crank angle as p, with C_1 equals 6.18 for gas exchange and 2.28 for compression, combustion and expansion, C_2 equals 0 for the gas exchange and compression period, and $3.24 \cdot 10^{-3}$ for combustion and expansion [2].

An alternative to this calculation is to develop the cylinder-pipe flow analogy [8]

$$h = 86.39 \cdot \frac{p^{0.8} \cdot w^{0.8}}{T^{0.546} \cdot L^{0.2}} \cdot C_1 \cdot \left(\frac{L_{BDC}}{L}\right)^{C_2}, \qquad (7)$$

where C_1 equals 1.9, C_2 can be calibrated with a value of 0.4, and L is a specific length for disc shaped combustion chambers (with subscript BDC meaning Bottom Dead Centre), defined by equation (8)

$$L = 2 \cdot h_c / (1 + h_c / B), \qquad (8)$$

where h_c is the clearance height, measured in m.

Equation (9) is used to define speed w as

$$w = r_i \cdot w_r, \qquad (9)$$

where w_r is a mean average speed of the working fluid, at a reference state, usually mean intake flow speed, and r a kinetic energy dissipation factor, defined by equation (10)

$$r_i = \left(1 + 18 \cdot \frac{\theta_i - \theta_r}{6 \cdot N} \cdot \frac{\upsilon^{0.2}}{L^{1.2}} \cdot (r_{i-1} \cdot w_r)^{0.8}\right)^{-0.5}, \qquad (10)$$

where θ_i is the momentary crank angle, with θ_r at the reference state, both measured in deg, N engine speed, in rev/min and υ kinematic viscosity, in m^2/s.

Both of these correlations assume a constant value of 1 for the Prandtl (Pr) number. The major difference between equations (5) and (7) is how speed w is defined, the choice of characteristic length and the correction developed by Grünwald [8]. A more detailed description of the flow inside the cylinder can be obtained by employing a simplified k–ε turbulence model [9].

Figure 3 shows the average mean value of the in–cylinder velocity and figure 4 presents a comparison of different speed definitions, all normalized to the mean piston speed.

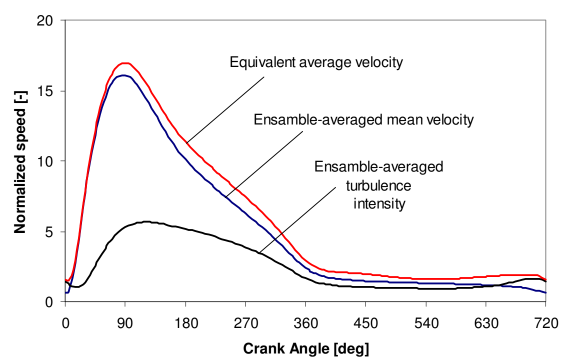

Fig.3 Velocity profiles calculated using the simplified k–ε turbulence model

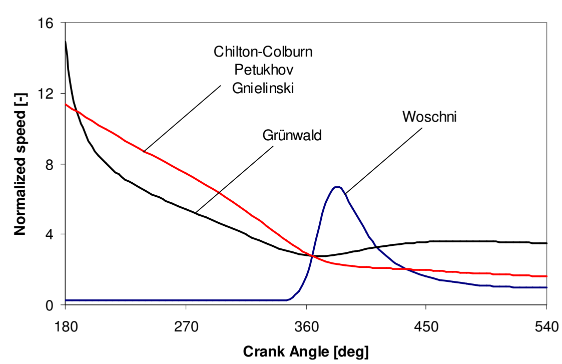

Fig.4 Comparison of different in–cylinder velocity profiles used for convective heat transfer equations

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It should be noted that mean average speed values obtained by using equation (9), combined with the definition of factor r in equation (10), as well as the results of the simplified k–ε model are comparable to measured velocity profiles, except for the exhaust stroke.

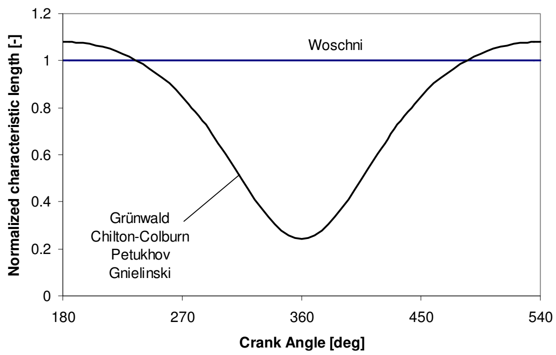

Fig.5 Characteristic length values used for convective heat transfer equations

Characteristic length values normalized to the cylinder bore used in this study are shown in figure 5, though other formulations can be used.

Other heat transfer equations for the Nusselt (Nu) number investigated in this study include the Chilton–Colburn analogy [10]

$$Nu = 0.125 \cdot f \cdot Re \cdot Pr^{1/3}, \qquad (11)$$

where the friction factor $f = (0.79 \ln Re - 1.64)^{-2}$ and Nu has a relative error level of up to 25%.

An equation that features an improved accuracy of 10% was developed by Petukhov [11]

$$Nu = \frac{(f/8) \cdot Re \cdot Pr}{1.07 + 12.7 \cdot (f/8)^{0.5} \cdot \left(Pr^{2/3} - 1\right)}, \qquad (12)$$

and for Reynolds (Re) numbers below the 10^4 threshold, equation (13) is recommended [6]

$$Nu = \frac{(f/8) \cdot (Re - 1000) \cdot Pr}{1 + 12.7 \cdot (f/8)^{0.5} \cdot \left(Pr^{2/3} - 1\right)}, \qquad (13)$$

given that at engine speed values of 800 rev/min, Re can be as low as 4000 during expansion.

4 Results and Discussion

Equations (5), (7), (11), (12) and (13) were used for calculating convective heat transfer rates throughout the closed valve part of the engine's working cycle.

Three distinct cases can be identified for Re (figure 6), as for equations (11), (12) and (13) the same speed and characteristic length were used. Calculated Re values are comparable for the correlations developed by Grünwald, Chilton–Colburn, Petukhov and Gnielinski, even if the speed calculated using equation (9) is different than the one obtained by employing the simplified turbulence model. A significant difference can be observed for Re using velocity values given by equation (6). It should be noted that the three distinct cases for Re are not directly comparable, as for the Woshni correlation a different characteristic length was used compared to the other two cases.

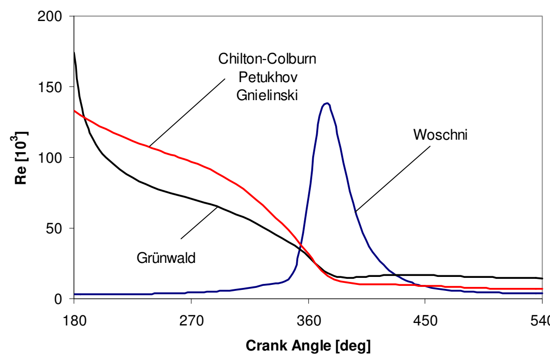

Fig.6 Calculated Reynolds number for different convective heat transfer models

As mentioned before, the equations developed by Woschni and Grünwald, both assume a constant Pr value of 1. This was mainly done to reduce computational effort, but it artificially increases the Nusselt number by 8 to 10%, depending on engine operational parameters. Calculated Pr values for the investigated case of methane combustion, 1500 rev/min and 50% volumetric efficiency are shown in figure 7.

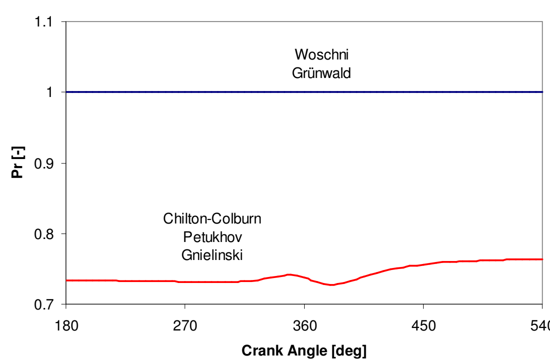

Fig.7 Calculated Prandtl number for different convective heat transfer models

Only a slight variation of *Pr* is observed, with higher values during combustion and expansion.

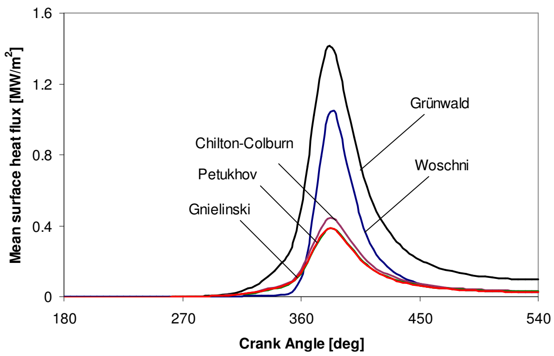

Fig.8 Uncorrected mean surface heat flux

Figure 8 presents calculated mean surface heat flux values without any corrections applied to any of the five equations. The correlation developed by Grünwald yields the highest values, comparable to data obtained by measurements available in the literature [1]. This is mainly because of the correction factor $C_1 \cdot (L_{BDC} / L)^{C_2}$. The author of equation (7) developed this correction by considering the large variation in cylinder geometry as the piston moves from BDC to TDC. He defined the correction factor as a way to account for the instability of cylinder geometry compared to the fixed diameter and length of a pipe. As the correlations for turbulent heat transfer were developed for pipes, this correction is required, much in the same way that *Nu* needs to be corrected for the entry region. Indeed, when comparing the Grünwald correction with the equations developed for turbulent flow pipe entry region, applied in a similar manner, very close results are obtained (figure 9). The correction for equation (5) is based on experimental data for methane combustion [12].

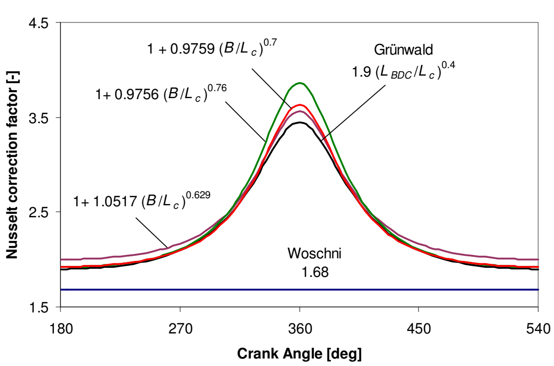

Fig.9 Correction factors

Another correction that was employed is the one required to account for the large difference in temperature from wall to the bulk gas. Only the friction factor in equations (11), (12) and (13) was corrected with a ratio of $(T_b / T_w)^{0.1}$. Values as high as 0.38 are suggested in some studies [11], but other authors found that this value is too high for gas cooling [13], especially at high bulk–wall temperature ratios. This correction increases the friction coefficient *f* by up to 17% during expansion. A more detailed study into the use of such correction factors is required for their application to SI engines, based on more fundamental aspects of fluid motion inside the cylinder rather than empirical equations that fit experimental data.

Values for corrected *Nu* using equations (7), (11), (12) and (13) are close, while the correlation developed by Woschni shows very different results, especially during compression and combustion (figure 10). This is mainly due to the difference in the equations used to define in–cylinder velocity profiles. Given that the characteristic length is also different, *Nu* cannot be directly compared for these different correlations, much the same as for *Re*.

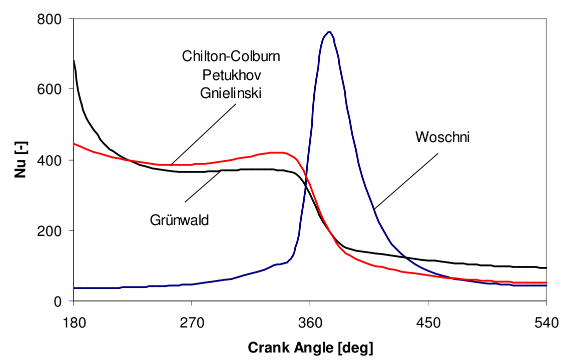

Fig.10 Corrected Nusselt numbers

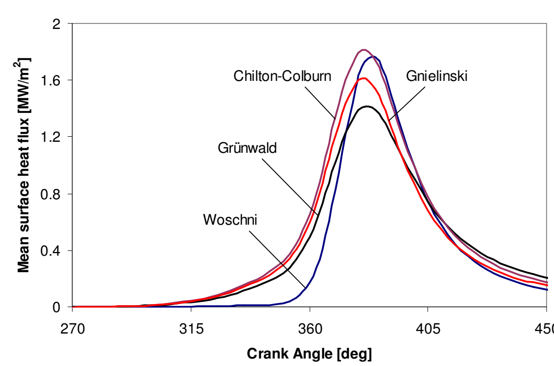

Fig.11 Corrected mean surface heat flux

While *Re* and *Nu* numbers cannot be directly compared due to the different velocity profiles and characteristic length used, corrected mean surface heat flux values are comparable for all correlations used (figure 11). Peak values are somewhat lower compared to experimental data. This could explain why the thermodynamic model underpredicts heat losses during combustion and the indicated efficiency is higher than the experimental value. An interesting observation is the difference between Woschni's equation and the rest of the correlations, given that equation (5) predicts very high transfer rates during combustion, due to the increased velocity profile. As this velocity during combustion is based on correlations of experimental data, a likely explanation is that the models do not capture specific phenomena, such as fluid motion and turbulence increase induced by the combustion process. Therefore, more refinement is required in order to ensure the application of such models with little, if any, empirical correction factors that depend on specific engine geometry and operational parameters.

5 Conclusion

A simple thermodynamic model validated with experimental data on methane combustion in a premixed charge SI engine was used for the study of convective heat transfer characteristics during the closed valve part of the working cycle.

Given that the correlations use different in–cylinder velocity profiles and characteristic length values, *Re* and *Nu* numbers are not directly comparable. All equations investigated require empirical correction coefficients in order to fit the experimental data and underpredict transfer rates, especially during combustion. As a result, a more detalied investigation is required to identify proper in–cylinder velocity profiles that would allow the removal of empirical correction coefficients.

6 Acknowledgement

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