Modeling and optimization of ATM cash replenishment

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Abstract: - Deriving the optimal cash deployment strategy for a network of ATMs involves the analysis of logistics costs, inventory policies as well as of the routing of replenishment vehicles. The optimal cash deployment strategy focuses on the reduction of cash-related expenses provided that ATMs do not run out of cash. In the paper, this rich vehicle routing problem is solved using genetic algorithm. The periodic vehicle routing problem can be seen as a problem of generating a group of routes for each day of a planning horizon so that the constraints involved are satisfied and the global costs are minimized. The proposed model furthermore enables to specify cases with requirements of two replenishments of some of the customers (ATMs) daily, while other customers may need only one replenishing in two days.

Key-Words: - ATM cash replenishment, vehicle routing problem, multi-depot, periodic, time-windows, genetic algorithm, optimization

1 Introduction
Deriving the optimal cash deployment strategy for a network of ATMs involves the analysis of logistics costs, inventory policies as well as of the routing of replenishment vehicles. The problem is thus twofold requiring first a conceptual framework to derive the optimal cash deployment strategy for a network of ATMs and second an assessment of potential benefits of sophisticated cash management software. Given the state of the ATM industry the optimization objective is clearly to minimize costs.

Consequently, the optimal cash deployment and replenishing strategy focuses on the reduction of cash-related expenses provided that ATMs do not run out of cash. The Vehicle Routing Problem (VRP) is a generic name given to a whole class of problems in which a set of routes for a fleet of vehicles based at one or several depots must be minimized for a number of geographically dispersed cities or customers [8]. The VRP arises naturally as a central problem in the fields of transportation, distribution and logistics.

Combined VRP and Inventory Allocation Problem is Inventory Routing Problem (IRP). IRP objective is to minimize the overall inventory cost (holding and transportation) given that customers (i.e. ATMs) do not run out of stock at any given time. IRP has two sub-problems:
- Inventory allocation problem (determines the replenishment quantity and frequency)
- Routing problem (determines the delivery route).

Authors of [10] address a scheduling and routing problem faced by a third-party logistics provider in planning its day-of-week delivery schedule and routes for a set of existing and/or prospective customers that need to make shipments to their customers. The goal is to minimize the total cost of transportation and inventory while satisfying a customer service requirement that stipulates a minimum number of visits to each customer each week and satisfaction of time-varying demand at the end-customers. Explicit constraints on the minimum number of visits to each customer each week give rise to interdependencies that result in a dimension of problem difficulty not commonly found in models in the literature.

Single-period models are well suited for instances with stochastic demand where forecasting accuracy is low. We are going to use multi-period models with forecasting based on historical data.

The Periodic VRP (PVRP) is a multi-period vehicle routing problem in which the decisions are the service days for each customer and the vehicle routes for a service provider on each day. The emphasis is on minimizing routing costs and the number of required vehicles.

The proposed model combines a multi-periodic vehicle routing problem with assignment of time windows to customers. The number of used vehicles
is not constrained, but it should be as low as possible. We suppose that each vehicle makes one route per day. Each route starts in one of the depots; while several routes can start in the same depot, there also can be a depot where no route starts. ATMs that require two visits per day are assigned TWs in 2 hours slots, the other ATMs are assigned wide TWs equal to a vehicles working shift.

2 Solution Techniques for VRP
Optimization plays a central role in science and engineering. In optimization, the systems under consideration can often (but not always) be expressed in terms of a mathematical function, and the goal is then to find the minimum or maximum of this function. So, many problems can be formulated as the task of minimizing (or maximizing) a mathematical function, called the objective function. In most practical applications of optimization, there are constraints, i.e. limits on the allowed range of the variables. Unfortunately, classical optimization algorithms are not suitable for all optimization problems and one has to use alternative, stochastic algorithms, mostly inspired by biological phenomena.

Recent advances in evolutionary algorithms (EAs) focus on how to solve practical optimization problems. EAs are stochastic algorithms whose search strategies model the natural evolutionary phenomena. Usually it is necessary to design a problem-oriented algorithm for the different types of optimization problems according to the characteristics of the problem to be treated. Generally, EAs involve the following meta-heuristic optimization algorithms: genetic algorithm (GA), evolutionary programming (EP), evolution strategy (ES), genetic programming (GP), learning classifier systems (LCS), and swarm intelligence (comprising ant colony optimization ACO and particle swarm optimization PSO). Among them, genetic algorithms are the most widely known type of evolutionary algorithms today [6].

The most commonly used techniques for solving VRP are modifications of Travelling Salesman Problem. Nearly all of them are heuristic and meta-heuristic because no exact algorithm can be guaranteed to find optimal routes within reasonable computing time when the number of cities is large. This is due to NP-hardness of the problem. We can find a classification of the solution techniques:

**Exact Approaches:** this approach proposes to compute every possible solution until one of the bests is reached.

**Heuristics:** Heuristic methods perform a relatively limited exploration of the search space and typically produce good quality solutions within modest computing times.
- Constructive Methods
- 2-Phase Algorithm

**Meta-Heuristics:** Metaheuristics are applied to “I know it when I see it” problems [7]. These algorithms are used to find answers to problems when there is very little knowledge about the character of the optimal solution, very little heuristic information to build on and brute-force search is out of the question because the space is too large. But if there is a candidate solution to the problem, it can be tested and assessed how good it is. In metaheuristics, the emphasis is on performing a deep exploration of the most promising regions of the solution space. The quality of solutions produced by these methods is much higher than that obtained by classical heuristics. Some examples: Ant Algorithms, Constraint Programming, Simulated Annealing, Deterministic Annealing, Genetic Algorithms, Tabu Search.

3 Rich Vehicle Routing Problems

**Capacitated VRP - CVRP:** Every vehicle has a limited capacity.

**VRP with time windows - VRPTW:** Every customer has to be supplied within a certain time window [2]. Each customer provides a time frame within which a particular service or task must be completed, such as loading or unloading a vehicle. A vehicle may arrive early, but it must wait until the start of service time is possible. Some VRPTW models (soft time window models) allow for early or late window service, but with some form of penalty, others have focused on the hard time window models.

**Multiple Depot VRP - MDVRP:** The vendor uses many depots to supply the customers. Each vehicle originates from one depot, services the customers assigned to that depot, and returns to the same depot. The objective of the problem is to service all customers while minimizing the number of vehicles and travel distance. A solution is feasible if each route satisfies the standard VRP constraints and begins and ends at the same depot.
VRP with **Pick-Up and Delivering**: Customers may return some goods to the depot.

**Split Delivery** VRP: The customers may be served by different vehicles.

**Stochastic** VRP: Some values (like number of customers, their demands, serve time or travel time) are random.

**Periodic** VRP - PVRP: The deliveries should be done repeatedly in previously specified days, i.e. one has to plan over several days (planning horizon of T days). Each customer has frequency of visit requirements (e.g., k out of T days). Visits to customers must occur on allowed k-day combinations. For each client, acceptable combinations of visits called patterns are created. The goals are: Each customer must be assigned to a single depot and a single pattern; Routes must be constructed for each depot and day; The total cost of all routes should be minimized. Rolling-horizon procedure is described in [5].

In MDPVPRPTW context, a tour for each couple (day, depot) needs to be optimized. The proposed model furthermore enables to specify cases with requirements of two replenishments of some of the customers (ATMs) daily, while other customers may need only one replenishing in two days.

It is important to highlight the complications introduced by the minimum days-of-service constraints. The problem exhibits strong links across multiple periods, not only because of the inventory-related costs induced by the day-of-week delivery pattern for an individual customer, but because the customer’s days-of-service constraint depends upon the delivery patterns of the other customers.

Actually no efficient exact methods are known for PVRP, different heuristics are used. For example, hybrid genetic algorithms exist for VRP, VRPTW, MDVRP; few on periodic problems.

The **Inventory Routing Problem** (IRP) involves the repeated distribution of a single product from a single facility to n customers over T days. Customers consume the product on a daily basis and maintain a small, local inventory. The objective is to minimize the sum of transportation and inventory-related costs (stock-outs can be costly). This is a very rich multi-period problem. One must first assign customers to patterns (certain days of the period) and then find routes on each day servicing the customers scheduled on that day. One seeks to minimize total distance traveled throughout the period. As an example, a waste management company has to assign customers to certain days of the week and then create daily routes.

The proposed model specifically accounts for effects of different delivery patterns. In contrast to many IRP models, the model directly addresses a multi-period problem with time-varying demand that may need to be satisfied by more than one shipment during the horizon.

### 4 Vehicle Routing Problem’s Formulation

The VRP is a combinatorial problem whose ground set is the edges of a graph \(G(V,E)\). The notation: \(G = (V,E)\), where \(V = \{v_0, v_1, \ldots, v_n\}\) is a vertex set; \(v_0\) denotes a depot

\(E = \{(i,j) : \ i, j \in V, \ i \neq j\}\) is an edge set

\(C\) … matrix of non-negative costs or distances (travel time) \(c_{ij}\) associated with every edge \((i,j) \in E\)

\(q\) … vector of customers demands

\(R_k\) is the route for vehicle \(k\)

\(m\) is the number of vehicles

\(s_i \geq 0\) is a service time (duration) required by a vehicle to unload the quantity \(q_i\) at place \(v_i\)

A feasible solution is composed of:

- Routes \(R_1, \ldots, R_m\) which are partitions of vertex set \(V\)
- Permutations \(P_i\) of \(R_i\) specifying the order of customers on route \(i\)

The cost of a given route \(R_i\) is given by:

\[
\text{Cost}(R_i) = \sum_{j=0}^{m} c_{i,i+1} + \sum_{l=1}^{m} s_l
\]

A route \(R_i\) is feasible if the vehicle stop exactly once in each customer and the total duration of the route does not exceed a specified bound \(D_k\), \(k = 1, \ldots, m\).

In VRPs, typically the planning period is a single day. In the case of the PVRP, the classical VRP is generalized by extending the planning period to T days.

Each vertex \(i \in V\) has a demand \(q_i \geq 0\) on each day of the planning horizon of T days, and requires a fixed number of visits \(f_i\) to be performed according to one of the allowable visit-day patterns in the list \(Z_i\) (pattern represents the days the associated customer receives a visit).

The objective is to minimize the vehicle fleet and the sum of travel time needed to supply all customers. A solution is feasible if all constraints of VRP are satisfied. Over the T-day period, each customer must be visited at least once.

In the model of PVRP, the daily demand of a customer is always fixed. In proposed model of ATM cash replenishment, the daily demands of
 customers (ATMs) are varying. Forecasting is based on historical data.

The PVRP can be seen as a problem of generating a group of routes for each day so that the constraints involved are satisfied and the global costs are minimized.

PVRP can also be seen as a multi-level combinatorial optimization problem: In the first level, the objective is to generate a group of feasible alternatives (combinations) for each customer. For example, if the planning period has \( T = 3 \) days \( \{ d_1, d_2, d_3 \} \), then the possible combinations are: \( 0 \rightarrow 000, 1 \rightarrow 010, 3 \rightarrow 011, 4 \rightarrow 100, 5 \rightarrow 101, 6 \rightarrow 110, 7 \rightarrow 111 \). If a customer requests two visits, then this customer has the following visiting alternatives: \( \{ d_1, d_2 \} \), \( \{ d_1, d_3 \} \), and \( \{ d_2, d_3 \} \). In the second level, one of the alternatives for each customer has to be selected. In the third level, the vehicle routing problem is solved for each day.

In VRPTW a time window is associated with each customer defining an interval wherein the customer has to be supplied. The objective is to minimize the vehicle fleet and the sum of travel time and waiting time needed to supply all customers in their required hours. Each vertex \( i \in V \) has a time window \([ e_i, l_i] \), where \( e_i \) is the earliest time service may begin and \( l_i \) is the latest time.

PVRPTW can be seen as the problem of generating at most \( m \) vehicle routes for each day of the planning horizon, to minimize the total cost over the entire planning horizon, such as (1) each vertex \( i \) is visited the required number of times, corresponding to a single pattern of visit-days chosen from visit day alternatives \( Z_i \), and is serviced within its time window, (2) each route starts from the depot, visits the vertices selected for that day and returns to the depot after a duration (travel time) not exceeding \( D \).

Decision variables:
- route selection:
  \[
  r_{ijk}^t = \begin{cases} 
  1 & \text{if vehicle } k \text{ traverses edge } (i,j) \text{ on day } t \\
  0 & \text{otherwise} 
  \end{cases}
  \]
- pattern selection:
  \[
  u_{iz} = \begin{cases} 
  1 & \text{if pattern } z \in Z_i \text{ is assigned to ATM } i \in V \\
  0 & \text{otherwise} 
  \end{cases}
  \]
- pattern assignment:
  \[
  a_{zt} = \begin{cases} 
  1 & \text{if day } t \in T \text{ belongs to pattern } z \\
  0 & \text{otherwise} 
  \end{cases}
  \]
- further requirement: service starting time at the next ATM on the given route needs to be greater (or at least equal) than service starting time plus service duration time at the previous ATM plus travel time.

The PVRPTW can be formulated as minimization of
\[
\sum_{t \in T} \sum_{(i,j) \in E} \sum_{k \in K} c_{ij} r_{ijk}^t 
\]
s.t.
\[
\sum_{z \in Z_i} u_{iz} = 1, \forall i \in V 
\]
\[
\sum_{k \in K} \sum_{j \in E} r_{ijk}^t = \sum_{z \in Z_i} u_{iz} a_{zt}, \forall i \in V, t \in T 
\]
\[
\sum_{k \in K} \sum_{j \in E} \tau_{ijk}^t \leq m, \forall t \in T 
\]

The objective function (1) minimizes the total travel cost. The constraint (2) specifies that a pattern is assigned to each ATM. The constraint (3) specifies that each ATM is visited on the days given by the pattern. The constraint (4) guarantees that the number of used vehicles is at most \( m \). To complete the given formulation, one has to specify constraints on time continuation based on service starting times, service duration times, and travelling distances.

5 The Proposed Meta-Heuristic
In the described model, we have 3 depots and 120 ATMs. The locations of ATMs are gained from Google maps of town Bratislava with its surroundings. The number of used vehicles is not constrained, but the aim is to minimize the number of used vehicles. We suppose that each vehicle runs only one route per day. Any route can start in one of the 3 depots (while several routes can start in the same depot, there also can be a depot where no route starts). An individual for the genetic algorithm we propose corresponds to a feasible or infeasible solution to the MDPVRPTW, which specifies: the pattern assigned to each customer, the number of routes, the delivery order within each route.

We presuppose that each vehicle works for one shift (8 hours) per day. Some of the vehicles may work in morning shift from 6 a.m. till 2 p.m., and some of them work in afternoon shift from 14 p.m. till 10 p.m.

Patterns in Fig. 1 represent the days (divided into 8 hour long time intervals) in which the associated customer receives a visit. For example, the pattern number 1 specifies that the ATM to which the pattern will be assigned requires cash replenishment only the third day in the afternoon. The pattern number 63 specifies that the ATM requires replenishing each day two times. We suppose that the third 8 hour long time interval (night) will not be used for replenishing.

5.1 Individual Representation
Each individual is represented by two chromosomes: the first addressing the pattern-to-
customer assignments, the second corresponding to the routes performed on each day of the planning horizon.

**Pattern chromosome** is associated with n customers (see Fig. 2a). Each entry i of this chromosome is a positive integer b that describes the pattern assigned to customer i. The binary representation of b stands for the days (more precisely parts of the days) the associated customer receives a visit. For each day in the planning horizon, a group of routes services customers on that day.

**Route chromosome** corresponds to the combination of the set of vectors, each representing the ordered sequence of customers for one route of a day. Each of the routes starts and ends in the same depot (see Fig. 2b).

### 5.2 Clustering and Routing

In the proposed model, cluster first - route second scheme is used (a model using route first - cluster second scheme would also be possible):

- Customers (ATMs) are sorted in increasing order of their angle distance with the depots.
- Sequence of n customers is divided into clusters of ATMs surrounding each of the depots.
- Some of the customers the positions of which are near the clusters boundary are allowed to be serviced by vehicles starting from both relating depots.

Routing is performed iteratively for each cluster. Another possibility is to use multi-criterion optimization.

### 5.3 Evaluation

Given a solution \(sol\), denote the total travel cost of its routes \(ttc(sol)\), \(d(sol)\) ... total violation of route duration (computed on a route basis with respect to \(D\)), \(w(sol)\) ... total violation of time window restrictions, \(deficit(sol)\) ... total deficit of cash, \(surplus(sol)\) ... total cost of acquired surplus money in ATMs.

Solutions are evaluated according to the fitness function:

\[
f(sol) = ttc(sol) + \alpha d(sol) + \beta w(sol) + \gamma deficit(sol) + \delta surplus(sol)
\]

where \(\alpha, \beta, \gamma, \delta\) are penalty parameters. Parameters \(\alpha\) and \(\beta\) are repeatedly adjusted so as to positively influence the total travel cost of the routes. Parameters \(\gamma\) and \(\delta\) are constants. Parameter \(\alpha\) amplifying the violation penalty of route duration needs to be set in accordance with cost of vehicle services per day (if this cost is high, a small violation of the route duration may be profitable with respect to fitness of the individual). Parameter \(\beta\) amplifying the violation penalty of time window restrictions needs to be set in accordance with distance which can be passed through in a time equaling to total violation of TW (based on the average speed of the vehicles in the area), as shorter routes with high TW violations should not be profitable.

We use random creation of the initial population and evaluate fitness of each individual in the population for the current generation, using parallel computation. Selection, crossover and mutation operators are used in the proposed algorithm. Control parameters used for the run:

- population size = 5000
- number of generations = 2000
- probability of crossover = 0.6

The proposed genetic algorithm is implemented in Matlab [1], [3], [4], [9]. Output of the simulation:

| Number of depots: 3 (labeled as 1, 2, 3) |
| Number of ATMs: 120 (labeled as 4, 5, ..., 123) |
| Number of routes: 6 |

**Blue Route (vehicle no 1)** = 1, 28, 119, 45, 1
**Green Route (vehicle no 2)** = 3, 78, 73, 113, 104, 102, 101, 103, 99, 115, 100, 116, 71, 72, 75, 79, 66, 82, 74, 92, 83, 98, 12, 61, 3
**Red Route (vehicle no 3)** = 2, 89, 85, 4, 76, 50, 67, 69, 77, 70, 18, 10, 2
Yellow Route (vehicle no 4) = 1, 31, 36, 30, 120, 117, 122, 121, 112, 106, 111, 55, 47, 29, 34, 22, 14, 32, 123, 64, 54, 42, 40, 39, 35, 44, 53, 1

Magenta Route (vehicle no 5) = 2, 13, 19, 24, 6, 41, 5, 27, 37, 16, 65, 59, 51, 60, 62, 25, 57, 63, 46, 11, 114, 108, 109, 110, 107, 118, 105, 33, 48, 36, 84, 95, 93, 87, 2

Cyan Route (vehicle no 6) = 3, 68, 81, 80, 8, 26, 9, 17, 23, 91, 90, 97, 86, 96, 94, 88, 43, 7, 58, 52, 56, 15, 20, 21, 3

Total distance traveled is 460 km. For illustration, sub-optimal routes for one day are given in Fig. 3.

6 Conclusion

Deriving the optimal cash deployment strategy for a network of ATMs involves the analysis of logistics costs, inventory policies as well as of the routing of replenishment vehicles. The optimal cash deployment strategy focuses on the reduction of cash-related expenses provided that ATMs do not run out of cash. The periodic vehicle routing problem can be seen as a problem of generating a group of routes for each day of a planning horizon so that the constraints involved are satisfied and global costs are minimized. Furthermore, the proposed model enables to specify cases with requirements of two replenishments for some of the customers (ATMs) daily, while other customers may need only one replenishing in two days.

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