Probability Density Functions of SSC Combiner Output Signal Derivatives at Two Time Instants in Weibull Fading Channel

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Abstract: - In this paper, the probability density functions (PDFs) of dual branch Switch and Stay Combiner (SSC) output signals time derivatives at two time instants in the presence of Weibull fading are determined. The second order characteristics as level crossing rate and average fade duration, for complex combiners which decide based on sampling at two time instants, can be calculated by using the form of single integral for probability density functions, obtained in this paper.

Key-Words: - Probability Density Function; Weibull Fading; Switch and Stay Combining; Time Derivative, Two Time Istants

1 Introduction

One of the main causes of signal degradation in wireless telecommunication systems is fading [1]. Multipath fading, or short term fading, has been modelled as Rayleigh, Rice, Nakagami–m and Weibull. Empirical studies have shown that Weibull distribution model is an effective model in both indoor [2] and outdoor [3] environments.

One of the most efficient method to reduce fading influence and improve system's quality of service, without enlarging channel capacity or signal power, is to use diversity reception [4] with different combining techniques [5] in order to acquire as larger as possible signal to noise ratio at the output. They are diversity systems with different complexities and amounts of channel information necessary at the receiver.

For example, maximal ratio combining (MRC) [6] and equal gain combining (EGC) [7] require all or some of the channel state information of received signal. Because separate receiver chain is needed for each branch of diversity system, the system complexity and price are increased. Less complicated and expensive combining technique is selection combining (SC) [8] because SC combiner processes only one of diversity branches. Because SC receiver processes only one of the diversity branches, it is simpler for practical realization. There is type of selection combining that chooses the branch with highest signal and noise sum [4] and in fading environments where the level of the cochannel interference is sufficiently high comparing with the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [8].

Finally, SSC is the simplest diversity method but with loss in performances. SSC receiver selects one antenna until its quality falls below a predetermined threshold. After this, receiver switches to second antenna whereas the signal from this antenna is below or above the threshold [9].

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The expressions for probability density functions and joint probability density functions for SSC combiner output signals at two time instants in the presence of different fading distributions are determined and then, these expressions are used for design of systems with better performances, such as the bit error rate and the outage probability. Performance analysis of SSC/SC combiner in the presence of Rayleigh and log-normal fading are given in [10] and [11], respectively.

The level crossing rate and the average fade duration are also very often used in designing of wireless communication systems as measures for their quality. Channel capacity and second order statistics in Weibull fading are derived in [12]. To obtain second order system characteristics we need the expressions for signal derivatives [13].

Because of this, the probability density functions of derivatives in two time instants for SSC combiner in Rician, Rayleigh and Nakagami-m fading channels are determined in [14]-[16], and the probability density functions (PDFs) of the signal derivative of dual switch and stay combining (SSC) combiner output signals at two time instants in the presence of Weibull fading will be derived in this paper.

2 System Model and Problem Formulation

The system model of dual brunch SSC combiner at two time moments is shown in Fig.1. At the first time moment the input signals are r_{11} and r_{21}, but r_{12} and r_{22} at the second time moment. The output signals are r_1 and r_2. The derivatives are $\dot{r}_{11}$ and $\dot{r}_{21}$ at the first time instant, and $\dot{r}_{12}$ and $\dot{r}_{22}$ at the second time instant. The derivatives at the SSC combiner outputs are $\dot{r}_1$ and $\dot{r}_2$.

The first index represents the branch ordinal number and the other time moment observed. At output signal the indices correspond to the time moments considered.

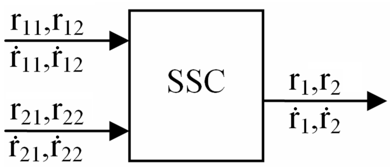

Fig.1. Dual SSC combiner at two time moments

The probabilities that combiner examines first the signal from the first, i.e. second branch are P_1, i.e. P_2. The values of P_1 and P_2 for SSC combiner are obtained in [1].

The four different cases depending on the size of the input signal with respect to the threshold are discussed here:

I $r_1<r_T,\ r_2<r_T$

In the first case all signals are less then threshold r_T, i.e.: $r_{11}<r_T,\ r_{12}<r_T\ \ r_{21}<r_T,$ and $r_{22}<r_T$. Let combiner considers first the signal r_{11}. Because of $r_{11}<r_T$, then $\dot{r}_1=\dot{r}_{21}$, and because of $r_{22}<r_T$, then $\dot{r}_2=\dot{r}_{12}$. The probability of this advent is P_1. If combiner examines first the signal r_{21}, then $r_{21}<r_T$, $\dot{r}_1=\dot{r}_{11}$, also $r_{21}<r_T$, $\dot{r}_2=\dot{r}_{22}$. The probability of this advent is P_2.

The possible combinations for this case are presented in :

$r_{11}\geq r_T,\ r_{12}<r_T, r_{22}<r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_1$

$r_{11}<r_T, r_{21}\geq r_T,\ r_{22}<r_T, r_{12}<r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_1$

$r_{21}\geq r_T,\ r_{22}<r_T, r_{12}<r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_2$

$r_{21}<r_T, r_{11}\geq r_T,\ r_{12}<r_T, r_{22}<r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_2$

III $r_1<r_T,\ r_2\geq r_T$

Now, the possible combinations are:

$r_{11}<r_T, r_{21}<r_T,\ r_{22}\geq r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_1$

$r_{11}<r_T, r_{21}<r_T,\ r_{22}<r_T, r_{12}\geq r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_1$

$r_{21}<r_T, r_{11}<r_T,\ r_{12}\geq r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_2$

$r_{21}<r_T, r_{11}<r_T,\ r_{12}<r_T, r_{22}\geq r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_2$

IV $r_1\geq r_T,\ r_2\geq r_T$

In the last case the possible combinations are:

$r_{11}\geq r_T,\ r_{12}\geq r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_1$

$r_{11}\geq r_T,\ r_{12}<r_T, r_{22}\geq r_T\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_1$

$r_{11}<r_T, r_{21}\geq r_T, r_{22}\geq r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_1$

$r_{11}<r_T, r_{21}\geq r_T, r_{22}<r_T, r_{12}<r_T\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_1$

$r_{21}\geq r_T, r_{22}\geq r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_2$

$r_{21}\geq r_T, r_{22}<r_T, r_{12}\geq r_T,\qquad \dot{r}_1=\dot{r}_{21}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_2$

$r_{21}<r_T, r_{11}\geq r_T, r_{12}\geq r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{12}\ \ P_2$

$r_{21}<r_T, r_{11}\geq r_T,\ r_{12}<r_T, r_{22}\geq r_T,\qquad \dot{r}_1=\dot{r}_{11}\ \ \dot{r}_2=\dot{r}_{22}\ \ P_2$

3 Probability Density Functions of Derivatives

The joint probability density functions of signals and derivatives are:

$r_1<r_T,\ r_2<r_T$

$$p_{r_1 r_2 \dot r_1 \dot r_2}(r_1,r_2,\dot r_1,\dot r_2)=P_1\int_0^{r_T} dr_{11}\int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{11},r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{21},r_{12},r_1,r_2,\dot r_1,\dot r_2) \tag{1}$$

$r_1\geq r_T,\ r_2<r_T$

$$p_{r_1 r_2 \dot r_1 \dot r_2}(r_1,r_2,\dot r_1,\dot r_2)=P_1\int_0^{r_T} dr_{12}\, p_{r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{12},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_1\int_0^{r_T} dr_{11}\int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{11},r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{22}\, p_{r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{21},r_{12},r_1,r,\dot r_1,\dot r_2) \tag{2}$$

$r_1<r_T,\ r_2\geq r_T$

$$p_{r_1 r_2 \dot r_1 \dot r_2}(r_1,r_2,\dot r_1,\dot r_2)=P_1\int_0^{r_T} dr_{11}\, p_{r_{11}r_{21}r_{22}\dot r_{21}\dot r_{22}}(r_{11},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_1\int_0^{r_T} dr_{11}\int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{11},r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\, p_{r_{21}r_{11}r_{12}\dot r_{11}\dot r_{12}}(r_{21},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{21},r_{12},r_1,r,\dot r_1,\dot r_2) \tag{3}$$

$r_1\geq r_T,\ r_2\geq r_T$

$$p_{r_1 r_2 \dot r_1 \dot r_2}(r_1,r_2,\dot r_1,\dot r_2)=P_1 p_{r_{11}r_{12}\dot r_{11}\dot r_{12}}(r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_1\int_0^{r_T} dr_{12}\, p_{r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{12},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_1\int_0^{r_T} dr_{11}\, p_{r_{11}r_{21}r_{22}\dot r_{21}\dot r_{22}}(r_{11},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_1\int_0^{r_T} dr_{11}\int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{11},r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2 p_{r_{21}r_{22}\dot r_{21}\dot r_{22}}(r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{22}\, p_{r_{22}r_{21}r_{12}\dot r_{21}\dot r_{12}}(r_{22},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\, p_{r_{21}r_{11}r_{12}\dot r_{11}\dot r_{12}}(r_{21},r_1,r_2,\dot r_1,\dot r_2)+$$
$$+P_2\int_0^{r_T} dr_{21}\int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}\dot r_{11}\dot r_{22}}(r_{21},r_{12},r_1,r,\dot r_1,\dot r_2) \tag{4}$$

The probability density functions (PDFs) of the signals at the combiner input channels in the presence of Weibull fading, $r_{i,j}$, are [17]:

$$p_{r_i}(r_{i,j})=\frac{\beta_i}{\Omega_i}r_{i,j}^{\ \beta_i-1}e^{-\frac{r_{i,j}^{\ \beta_i}}{\Omega_i}}\quad,\quad r_{i,j}\geq 0 \tag{5}$$

where $i=1,2$, $j=1,2$, β is the Weibull fading parameter where ($\beta\geq 0$) which expresses the severity of fading and $\Omega=E(r^\beta)$. With increasing the value of β, the severity of the fading decreases, while for the special case of $\beta = 2$, Weibull distribution reduces to the Rayleigh.

The probability density functions of signal derivatives at the combiner input in the presence of Weibull fading have a normal distribution with zero mean value and are conditioned on signals [18]:

$$p_{\dot r_i}(\dot r_{i,j})=\frac{1}{\sqrt{2\pi}\dot\sigma_i}e^{-\frac{\dot r_{i,j}^{\ 2}}{2\dot\sigma_i^2}},\quad -\infty<\dot r_{i,j}<\infty \tag{6}$$

with $\dot\sigma_i^{\ 2}=2\Omega\pi^2 f_m^{\ 2}\beta^{-2}r^{2-\beta}$ as variance.

The joint probability densities of the combiner input signals $r_{i,j}$ and their derivatives $\dot r_{i,j}$ in the presence of Weibull fading, are obtained using (5) and (6) [18,19]:

$$p_{r_i\dot r_i}(r_{i,j},\dot r_{i,j})=\frac{\beta_i^2 r_{i,j}^{\ 3\beta_i/2-2}}{f_m(2\pi\Omega_i)^{3/2}}e^{-\frac{\beta_i^2 r_{i,j}^{\ \beta_i-2}\dot r_{i,j}^2}{8\pi^2 f_m^{\ 2}\Omega_i}-\frac{r_{i,j}^{\ \beta_i}}{\Omega_i}}\quad, r_i\geq 0 \tag{7}$$

where f_m is the maximum Doppler frequency shift.

The joint PDF of derivatives at the output can be obtained after integrating of the whole range of signal values and some mathematical manipulations assuming that input signals from different branches in the same time moment and from different time moments in one branch are independent.

The signal derivative's PDFs can be found from joint PDF based on [20]:

$$p_{\dot{r}_1}(\dot{r}_1) = \int_{-\infty}^{\infty} p_{\dot{r}_1\dot{r}_2}(\dot{r}_1, \dot{r}_2) d\dot{r}_2 \quad (8)$$

$$p_{\dot{r}_2}(\dot{r}_2) = \int_{-\infty}^{\infty} p_{\dot{r}_1\dot{r}_2}(\dot{r}_1, \dot{r}_2) d\dot{r}_1 \quad (9)$$

PDF of the signal derivatives are:

$$\begin{aligned} p_{\dot{r}_1}(\dot{r}_1) &= P_1 A_1(\dot{r}_1) + P_2 A_2(\dot{r}_1) + \\ &+ P_1 F_{r_{11}}(r_T) A_2(\dot{r}_1) - P_1 F_{r_{11}}(r_T) A_1(\dot{r}_1) + \\ &+ P_2 F_{r_{21}}(r_T) F_{r_{12}}(r_T) A_1(\dot{r}_1) - P_2 F_{r_{21}}(r_T) F_{r_{12}}(r_T) A_1(\dot{r}_1) \end{aligned} \quad (10)$$

$$\begin{aligned} p_{\dot{r}_2}(\dot{r}_2) &= P_1 A_1(\dot{r}_2) + P_2 A_2(\dot{r}_2) + \\ &+ P_1 F_{r_{11}}(r_T) A_2(\dot{r}_2) - P_1 F_{r_{11}}(r_T) A_2(\dot{r}_2) + \\ &+ P_2 F_{r_{21}}(r_T) F_{r_{12}}(r_T) A_1(\dot{r}_2) - P_2 F_{r_{21}}(r_T) F_{r_{12}}(r_T) A_1(\dot{r}_2) \end{aligned} \quad (11)$$

where $F_{r_{ij}}(r_T)$ are signal's cumulative distribution functions (CDFs) and $F_{r_i}(r_T) = F_{r_{i1}}(r_T) = F_{r_{i2}}(r_T)$, while $A_i(\dot{r}_{i,j})$ is:

$$A_i(\dot{r}_{i,j}) = \int_0^{\infty} \frac{\beta_i^{\,2} r_{i,j}^{\;3\beta_i/2-2}}{f_m (2\pi\Omega_i)^{3/2}} e^{-\frac{\beta_i^{\,2} r_{i,j}^{\;\beta_i - 2} \dot{r}_{i,j}^2}{8\pi^2 f_m^{\,2} \Omega_i} - \frac{r_{i,j}^{\;\beta_i}}{\Omega_i}} dr_{i,j} \quad (12)$$

$p_{\dot{r}_1}(\dot{r}_1)$ and $p_{\dot{r}_2}(\dot{r}_2)$ are obtained in the form of single integral and are suitable for calculating of the level crossing rate and average fade duration.

The signal's CDF can be replaced with [17]:

$$F_{r_i}(r_T) = 1 - e^{-\frac{r_T^{\;\beta_i}}{\Omega_i}} \quad (13)$$

where $i=1, 2$.

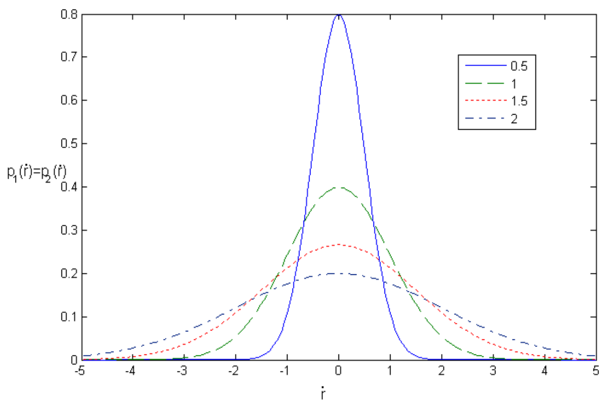

Fig.2. The probability density functions of derivatives at the SSC combiner output at two time instants

The PDFs of signal derivatives are presented in Fig. 2 for different values of parameter $\dot{\sigma}_i$,where $\dot{\sigma}_i^{\,2} = 2\Omega\pi^2 f_m^{\,2} \beta^{-2} r^{2-\beta}$, in the case of channels with identical distributions.

4 Conclusion

The expressions for probability density functions of the signal time derivatives in two time instants are introduced in this paper for dual branch SSC combiner output signals and for the presence of Weibull fading at the input branches. The expressions obtained in this paper can be used for obtaining the second order characteristics as the average level crossing rate and the average fade duration for complex SSC/SC and SSC/MRC combiners at two time instants.

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References:

[1] M. K. Simon, M. S. Alouni, *Digital Communication over Fading Channels*, Second Edition, Wiley-Interscience, A John Wiley&Sons, Inc., Publications, New Jersey, 2005.

[2] H. Hashemi, The indoor radio propagation channel, *Proceedings IEEE*, vol. 81 (7), 1993, pp. 943–968.

[3] N. S. Adawi, et al. Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range, *IEEE Transactions on Vehicular Technology*, vol. 37, (1), 1988, pp. 3–72.

[4] D. Tse, and P. Viswanath, *Fundamentals of Wireless Communication,* Cambrige University Press, July 2005.

[5] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[6] Z. Popovic, S. Panic, J. Anastasov, M. Stefanovic, and P. Spalevic, Cooperative MRC diversity over Hoyt fading channels, *Electrical Review,* vol. 87 no. 12, Dec. 2011, pp. 150-152.

[7] G. T. Djordjevic, D. N. Milic, A. M. Cvetkovic, and M. C. Stefanovic, Influence of Imperfect Cophasing on Performance of EGC Receiver of BPSK and QPSK Signals Transmitted over

Weibull Fading Channel, accepted for publication in *European Transactions on Telecommunications*, published online at wileyonlinelibrary.com, DOI: 10.1002/ett.1475.

[8] D. Milovic, M. Stefanovic, and D. Pokrajac, Stochastic approach for output SINR computation at SC diversity systems with correlated Nakagami-m fading, *European Transactions on Telecommunications*, vol. 20, no. 5, 2009, pp. 482-486.

[9] Đ. V. Banđur, M. Stefanović, and M. V. Banđur, "Performance analysis of SSC diversity receiver over correlated Ricean fading channels in the presence of co-channel interference", *Electronics Letters*, vol. 44, no. 9, 2008, pp. 587-588.

[10] P. Nikolić, D. Krstić, M. Milić, and M. Stefanović, Performance Analysis of SSC/SC Combiner at Two Time Instants in The Presence of Rayleigh Fading, *Frequenz.*, vol. 65, Issue 11-12, ISSN (Online) 2191-6349, ISSN (Print) 0016-1136, November/2011, pp. 319–325.

[11] M. Stefanović, P. Nikolić, D. Krstić, and V. Doljak, Outage probability of the SSC/SC combiner at two time instants in the presence of lognormal fading, *Przeglad Elektrotechniczny (Electrical Review)*, ISSN 0033-2097, R. 88 NR 3a/2012, March 2012, pp.237-240.

[12] A. Mitic, and M. Jakovljevic, Second-Order Statistics in Weibull-Lognormal Fading Channels, Proc. of *TEKSIKS'07,* Serbia, Nis, Sept. 26-28, 2007.

[13] L. Yang, and M. S. Alouini, Average Level Crossing Rate and Average Outage Duration of Switched Diversity Systems, Proc. of *Global Telecommunications Conference, GLOBECOM '02. IEEE*, vol. 2, Print ISBN: 0-7803-7632-3, 2002, pp. 1420-1424.

[14] D. Krstić, P. Nikolić, and G. Stamenović, Probability Density Functions of Derivatives in Two Time Instants for SSC Combiner in Rician Fading Channel, The Eighth International Conference on Wireless and Mobile Communications, *ICWMC 2012,* June 24-29, 2012, Venice, Italy

[15] P. B. Nikolić, D. S. Krstić, G. Lj. Stamenović, and Z. J. Popović, PDFs of Time Derivatives for SSC Combiner at Two Time Instants in Rayleigh Fading Channels, 8th IEEE, IET Int. Symposium on Communication Systems, Networks and Digital Signal Processing, *CSNDSP 2012*, 18-20 July, Poznan, Poland

[16] D. Krstić, P. Nikolić, G. Stamenović, PDFs of Time Derivatives at Two Time Moments in the Presence of Nakagami Fading, 16th WSEAS International Conference on Communications (part of CSCC '12), July 14-17, 2012, Kos Island, Greece

[17] N. C. Sagias, G. K. Karagiannidis, D. A. Zogas, P. T. Mathiopoulos, G. S. Tombras, Performance analysis of dual selection diversity over correlated Weibull fading channels*, IEEE Transactions on Communications*, vol. 52, no. 7, pp. 1063-1067, July 2004.

[18] N. C. Sagias, D. A. Zogas., G. K. Karagiannidis., G. S. Tombras, Channel capacity and second order statistics in Weibull fading, *IEEE Communication Letters*, Vol. 8, No. 6, June 2004, pp. 377–379.

[19] G. Fraidenraich, M. D. Yacoub, J. C. Filho, Second-Order Statistics of Maximal-Ratio and Equal-Gain Combining in Weibull Fading, *IEEE Communications letters*, Vol. 9, No. 6, pp. 499–501, June 2005.

[20] A. Goldsmith, Wireless Communications, Cambridge University Press, September 2005.