Modeling Road Traffic Congestion by Quasi-Dynamic Traffic Assignment

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Abstract – The paper deals with simulation of congested road networks through dynamic traffic assignment models and presents a new quasi-dynamic traffic assignment model that improves realism and effectiveness of both usual static traffic assignment models and other quasi-dynamic models recently introduced in the literature, as it simulates flow progression onto the network by moving link flows according to the link speed at every time step. The new model has been applied to the real large-scale road network of the town of Rome and has provided a comparable fit of real data with that obtained in a different application by a much more sophisticated simulation-based dynamic traffic assignment model, although it requires much lower calibration effort and smaller computation time.

Key-words – Road traffic simulation models, Dynamic Traffic Assignment, Quasi-Dynamic Traffic Assignment

1. Introduction

Reducing traffic congestion of urban road networks is one of the toughest challenges in most Countries worldwide. To tackle this problem effectively, traffic plans and control techniques need to apply simulation models. The core problem of traffic network modeling consists of assigning a specified demand matrix of vehicle movements between points of entry and exit in a modeled road network. Traffic network models of increasing complexity and realism were developed in the last 50 years. Earliest models were based on simplistic assumptions on drivers’ behavior and link performances [1]. Behavioral assumptions are: a) drivers have perfect information on topology and actual travel times of the road network; b) all drivers choose their most convenient route to reach their destination. Under these assumptions, for each O-D pair, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path. This property is known as Deterministic User Equilibrium, or more briefly, DUE [2].

Moreover, vehicle interactions are simplified by assuming that: a) vehicular traffic constitutes a homogeneous flow; b) origin-destination flow is constant during the time interval of simulation; c) travel time on each link is a continuously differentiable function of the flow on the same link. It has been demonstrated that, if link travel time functions are continuous for all values of link flows, a solution of the assignment problem exists. The solution is unique if the Jacobian of the travel time functions and that of its derivatives are positive definite [3].

In the following years, more general and more realistic models were introduced. Drivers have been assumed having different perceptions of route performances and complex choice mechanism. Drivers’ choices were represented through probabilistic models among discrete alternatives [4]-[6]; existence and uniqueness conditions were derived for traffic network models with multiple classes of vehicles [7]; more realistic queuing models were introduced to reproduce capacity constraints and delays at bottlenecks [8]-[10], which on the other hand implied missing uniqueness property.

The greatest inadequacy of these models lies in the assumption of time invariant demand, which implies steady-state traffic condition. It is clear that such a model cannot reproduce dynamics of traffic congestion on the road network properly. Two kinds of traffic dynamics that cannot be simulated through equilibrium models are individuated. The first concerns transients due to modifications of demand and/or supply and is addressed by day-to-day dynamic models [11]. The second relates to within-day dynamics and allows describing temporary oversaturated conditions, when demand exceeds capacity and queues raise on some elements of the road network. To achieve a more realistic representation of within-day traffic dynamics, different
formulations of Dynamic Traffic Assignment (DTA) models were introduced in the last years [12]. While many of them generalize traditional static formulations, the most advanced and realistic models use a traffic simulator to replicate the complex traffic flow dynamics. Several sophisticated software tools for DTA have been developed; the most noticeable are mentioned here: DynaMIT [13], DYNASMART [14], Dynameq [15], AIMSUN [16]. Implementation of such complex models is a rather cumbersome task, which involves the construction of detailed graphs of thousands of links and requires a long process for calibrating hundreds or thousands of coefficients. In a recent paper, Ben-Akiva et al. [17] describe the huge calibration efforts in implementing the mesoscopic dynamic traffic assignment model DynaMIT to a highly congested subnetwork of the city of Beijing, China.

In order to reduce both calibration and computational efforts required by simulation-based DTA models, several authors in the last years proposed a quasi-dynamic (or semi-dynamic) approach, which exploits some useful properties of simpler steady-state assignment models. Quasi-dynamic approach to modeling traffic resulting from time-varying demand is to divide the modeled period into time slices in each of which demand is steady and a corresponding steady-state assignment is modeled, and where demand exceeds capacity some links are overloaded [18]. The quasi-dynamic approach to traffic model is not new. It was firstly introduced by Van Vliet [19] and then abandoned for more complex dynamic simulation models. All quasi-dynamic models developed so far assume that most travelers reach their destination within the period in which they depart and represent residual flows propagation by modifying the O-D demand in the current and next periods [20]-[24]. Specifically, Ujii et al. add the remained flow in each period to the O-D flow of the next time period [20]. Nakayama et al. assign half of the remaining flow to the current period and the other half to the next period [21]. Chen et al. [22] apply a probabilistic logit model to reproduce drivers’ route choice behavior and implement a cell transmission model [25] to load traffic to the network. They avoid so steady-state assumptions in each time period. However, they do not consider the queues at intersections and assume that links can accommodate all the vehicles assigned by the route choice model. Our model does not require hypothesizing short trips to ensure steady-state conditions, but assumes that no great change in traffic conditions occurs from one time period to the next, so that the steady-state link performance functions can be applied to model congestion. Unlike other models in literature, it provides a realistic representation of flow progression onto the network as it moves forward each link flow according to the related speed at every time step.

2. Quasi-dynamic assignment model

The quasi-dynamic approach carries out a traffic network simulation in different time intervals, starting from empty network. At each time interval, the demand flow starting from each origin to each destination is loaded on the network, supposing that in that time period steady state conditions are valid. Obviously, demand variation involves a variation of link flows that depend also on the progression in time of demand flows already on the network. In fact, for each simulation time step, i.e. 15 minutes, the corresponding O-D demand portion is assigned to the network links that can be reached in upper bound of the time interval. In the next time interval the procedure is repeated for another slice of O-D demand, while the flows on the links advance on the network up to reach the links achievable in the second time interval. Travel times that depend on assigned demand because of link congestion and O-D routes depending on the new travel times are computed at every time step. Supposing that users don’t modify their route choice during the travel, only paths for the new users that enter in the network are computed.

As the network is represented by a series of steady-state conditions, the same road cost functions of the static approach can be used. This is a great advantage, because it allows the analysts exploiting traditional link-cost functions largely used and calibrated in practical applications. However, because of the time-dependent interaction of flows on the network, O-D flows starting at a generic time interval may be affected by flows starting at a successive time interval from a different origin that overlap their route and alter so their travel time. Thus, there is no guarantee to find the user equilibrium solution and a heuristic solution procedure has to be applied for network loading.

3. Mathematical Formulation

Mathematical formulation of Quasi-Dynamic Traffic Assignment (QDTA) generalizes the steady-state probabilistic user equilibrium assignment problem. Unknowns of the problem are the flows $h'_{ik}$ on the generic route $k$ departed in time interval $t$ from origin $i$ to destination $j$, both belonging to a given set of traffic zones $Z$. Flows $h'_{ik}$ are updated at every time interval $t$ and are assumed to be independent of the solutions found in the successive time intervals. This assumption corresponds to a demand composed by usual travelers (e.g., commuters) that choose their routes according to their statistical knowledge of usual network congestion.

On each time interval $t$ the flow conservation law holds for each O-D pair $(i,j)$, whose flow $d$ is distributed on the set of $K_{ij}$ feasible routes:

$$\sum_{k \in K_{ij}} h'_{ik} = d_{ij} \quad \forall i,j \in Z$$

$$h'_{ik} \geq 0 \quad \forall k \in K'_{ij} \quad \forall i,j \in Z$$

(1)
Link flows $x^a_\omega$ at the generic time interval $\omega \geq t$ are determined by applying link-route incidence condition, which in the dynamic case is a function of the time interval $\omega$ in which the O-D flow $d^i_j$ that started at the time period $t$ reaches link $a$:

$$x^a_\omega = \sum_{i=1}^\omega \sum_{k \in K^i_j} \delta^i_{a,k} (\tau^i_{a,k}) h^i_k \quad \forall a \in Z$$  \hspace{1cm} (2)

For each link $a$, time interval $\omega$ is determined by computing travel time $\tau$ on the shortest path $k$ for flows departed at time $t$.

$$\delta^i_{a,k} = 1 \iff a \in k, k \in K^i_j, i,j \in Z, \omega - \Delta t < \tau^i_{a,k} \leq \omega$$

$$\delta^i_{a,k} = 0 \quad \text{otherwise}$$  \hspace{1cm} (3)

If steady-state conditions are assumed during every time period $\omega$ of length $\Delta t$, it is possible to apply the traditional link cost-flow functions

$$c^a(x^a_\omega) \forall a \in A$$

and the related path cost-flow functions

$$C_k^p = \sum_{a \in k} c^a(x^a_\omega)$$  \hspace{1cm} (4)

The route choice model is based on the assumption that a rational driver chooses the route that maximizes the utility related to his or her choice. Utility of each alternative is modeled as a function of both the observed attributes of the alternative and the observed characteristics of the decision maker. To incorporate the effects of unobserved attributes and characteristics, the utility of a route $k$ is expressed as a random variable consisting of a deterministic component $V_k$ and an additive random term $\varepsilon_k$:

$$U_k = V_k + \varepsilon_k$$  \hspace{1cm} (5)

If the random terms of each utility function are independently and identically distributed Gumbel variables with zero mean and parameter $\theta$, the choice model is a multinominal logit function. In this case, the probability $p$ that a generic driver departing during time slice $t$ chooses route $k$ is:

$$p_k = \frac{e^{\frac{V_k}{\theta}}}{\sum_{h \in K(i,j)} e^{\frac{V_h}{\theta}}}$$  \hspace{1cm} (6)

where the deterministic utility $V^i_k$ at the decision time period $t$ is defined as the linear combination with coefficients $\beta_i$ of the attributes $Y^i_{ak}$ of alternative $k$:

$$V^i_k = \sum_{i=1}^n \beta_i Y_{ai}^i$$  \hspace{1cm} (7)

In congested urban networks it is usual to assume the attributes coincide with travel costs (that is, with travel times and tolls, if any), which, in dynamic case, are time-dependent.

However, we assume that drivers choose their route at their departure on the basis of their statistical knowledge of the network and do not modify their choice during their trip. The model allows considering different classes of users having different levels of knowledge of usual road traffic conditions, that is, different coefficients $\theta$. According to these assumptions, the expected flow $h^i_k$ on route $k$ is:

$$h^i_k = d^i_j p^i_k \quad k \in K(i,j)$$  \hspace{1cm} (9)

where $d^i_j$ is the flow started in the time period $t$ from origin $i$ to destination $j$ and $p^i_k$ is the probability of choosing the route $k$.

It is easy to verify that, since $p^i_k$ is a probability, route flows comply the conservation conditions (1). It is also worth noting that $p^i_k$ is time-dependent, as link and route costs vary with the time.

4. Solution procedure

The solution procedure consists of:

- computing, in every time interval, the K optimal routes for each O-D pair (in our case, K=3);
- calculating the fraction of O-D demand on each route;
- moving, in every time interval, the flow assigned on each route of the distance travelled, depending on the traffic speed on these routes.

Step 0: Computation of initial solution (empty network)

Let $\omega = c_s(0), \forall a \in A$;

Let current time period $\omega = 1$.

Step 1: Minimum paths computation

Let $t = \omega$ and $\forall$ origin $i \in R$ with $d^i_j > 0$ and each destination $j \in R$, compute K shortest paths trees, their relative costs $\{C_{ij}, C_{ij2}, \ldots, C_{ijk}\}$ and time instants $\tau^i_{a,k}$ in which OD flow $d^i_j$ started at period $t = \omega$ reaches each link belonging to route $K$.

Step 2: Computation of path flows departed at time interval $t = \omega$

In this step the route choice model (7) is applied using the route costs computed at step 1. O-D demand departed at time interval $t = \omega$ is assigned to the different routes by applying equation (9).

Step 3: Traffic flow simulation on the network at generic time interval $\omega$

It consists of simulation of flows progression on the network by applying equations (2) and (3); the results of this step are link flows in every time interval.

Step 4: Updating of link travel times at generic time interval $\omega$

Link costs corresponding to link flows computed at step 3 are calculated through (4). Costs are assumed to be equivalent to travel times, so that they are inverse functions of link speeds.

Step 5: Stopping criterion

If $\omega = T$, the algorithm stops; otherwise, let $t = \omega + 1$ and come back to step 1.
5. Validation of Quasi Dynamic Traffic Assignment Model on the road network of Rome

QDTA model has been applied to a large-scale road network, namely, Rome network. It is modeled by a road graph composed by 15,000 directed links and 6,000 nodes. O-D flows are represented by a matrix of 850 x 850 items. As only O-D trips in the rush hour were known from statistical surveys, this static demand was processed to derive a time-dependent demand in 15 minutes interval from traffic counts detected over the 24 hours.

After a preliminary comparison between model’s output and real flows and speeds detected on the network, a quick calibration phase has been carried out, adjusting parameter’s values of link cost-flow functions. The comparison with real data has been performed because a large database of floating cars was available. In the study area the equipped fleet counts 80,000 vehicles, which travelled 9 millions trips and provided 104 millions of records, containing their positions and speeds during one month. The same floating car data have been used to test the goodness of fit of the model after the calibration phase.

A further comparison has been executed by applying a traditional Deterministic User Equilibrium model (DUE) to the same network. The results of this comparison are summarized in the table below, where mean speed and mean travel time are shown. The analysis of results show that QDTA model provides a quite satisfactory approximation of mean speed and mean travel time revealed by floating cars. The difference in terms of mean speed between QDTA and floating car data (FCD) is about 5,2%, while the difference in terms of mean travel time is about 4,3%.

The mean speed computed by applying DUE model is higher than that calculated by QDTA (16,5%), while the mean travel time is lower than that calculated by QDTA model (-18,4%). Similar results are obtained comparing the DUE model with FCD (speed is higher as 13,6% and travel time lower as -11,9%).

Table 1. Difference between observed (FCD) and simulated road network performances by QDTA and static DUE models

<table>
<thead>
<tr>
<th></th>
<th>Mean Speed (Km/h)</th>
<th>Mean Travel Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floating Car Data</td>
<td>27,1</td>
<td>53,3</td>
</tr>
<tr>
<td>Quasi-Dynamic Traffic Assignment</td>
<td>25,7</td>
<td>55,7</td>
</tr>
<tr>
<td>QDTA vs FCD</td>
<td>-5,2%</td>
<td>4,3%</td>
</tr>
<tr>
<td>Static DUE Traffic Assignment</td>
<td>30,8</td>
<td>47</td>
</tr>
<tr>
<td>DUE vs FCD</td>
<td>13,6%</td>
<td>-11,9%</td>
</tr>
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</tr>
</tbody>
</table>

In a second step the time period of analysis has been extended to a whole day. In Fig.1 and Fig.2 the fit-to-link flows and fit-to-link speeds on a sample of 12 relevant links of the network during the whole day are depicted respectively to evaluate the goodness of model calibration. In both figures, the x-axis relates to observed data and y-axis to simulated data. The 45° line indicates a perfect match between the simulated and observed values. Although the very brief calibration process, error statistics (Root Mean Square Error and Root Mean Square Normalized error reported in the figures), are close to those obtained in one of the most advanced applications of a dynamic simulation traffic assignment model [17], where a root mean square normalized error on link travel time as 0.436 has been obtained after having applied a very advanced calibration procedure to a sub-network in Beijing having comparable size and characteristics of that experienced here.

Figure 1. Comparison between observed and simulated flows on a sample of main road links

Figure 2. Comparison between observed and simulated speed on a sample of main road links
Link flows are the main output of the assignment model, which takes O-D flows as inputs and determines how they distribute on different links of the network. Link speeds, on the other hand, are a very relevant result of the model because they are the most direct estimate of road network congestion. A more detailed validation of the model has been carried out on specific links, where speeds computed by QDTA are compared with corresponding values detected by floating cars, for each hour of day. It’s possible to note in Fig.3 that in some hours of the day the trend and the values of speeds computed by QDTA are quite coincident to detected data (e.g.: Viale Marconi in time interval 05:00 a.m.-10:00 a.m; Viale Emo in time interval 04:00 a.m.-09:00 a.m). Observing these results it is evident that a better calibration has been achieved for the peak period in the morning, even if a rather good trend of speeds can be appreciated also in the peak hour of the evening.

On this regard, it is worth mentioning that a statistical estimate of traffic demand was available for the peak hour of the morning, while a very simple method has been applied to estimate travel demand in the remaining hours of day. Such differences should be reduced in a further calibration phase.

6. Conclusions

The paper has presented a quasi-dynamic traffic assignment model that deals with time-dependent traffic demand and simulates time evolution of traffic congestion on an urban traffic network. Results obtained by applying the model to the road network of Rome, Italy, show that the quasi-dynamic traffic assignment (QDTA) model is a good compromise between sophisticated simulation-based dynamic traffic assignment models and traditional static user equilibrium traffic assignment models. In fact, the aggregated representation of traffic dynamics provided by QDTA allows simulating several hours of traffic on a large road network in few minutes by using a standard personal computer.

Use of traditional link-cost functions facilitates the calibration process, which is however very cumbersome in simulation-based dynamic traffic assignment models.

On the other hand, the dynamic network loading procedure introduced in this model has shown a capability of reproducing traffic phenomenon which is fairly comparable to that obtained by much more complex simulation-based dynamic traffic models. Further research will concern exploitation of floating car data to improve reliability of K-shortest path algorithm and the implementation of a general optimization technique for global calibration of the quasi-dynamic traffic assignment model.

7. References


