Smith Predictor Based Autotuners for Time-delay Systems

ROMAN PROKOP, JIŘÍ KORBEL, RADEK MATUŠŮ
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nám. TGM 5555, 760 01 Zlín
CZECH REPUBLIC
prokop@fai.utb.cz    http://www.fai.utb.cz

Abstract: This paper presents a set of autotuners for single input-output systems with time delay. Autotuners represent a combination of relay feedback identification and some control design method. In this contribution, models with up to three parameters are estimated by means of a single asymmetrical relay experiment. Then a stable low order transfer function with a time delay term is identified. Controller parameters are analytically derived from a general solution of Diophantine equations in the ring of proper and stable rational functions $R_{PS}$. This approach covers a generalization of PID controllers and it enables to define a scalar positive parameter for further tuning of the control performance. The Smith predictor scheme is applied for systems with a time delay term. The simulations are performed in the Matlab environment and a toolbox for automatic design and simulation was developed.

Key-Words: Algebraic control design, Relay experiment, Autotuning, Pole-placement problem, Smith predictor

1 Introduction

The development of various autotuning principles was started by a simple symmetrical relay feedback experiment proposed by Åström and Hägglund in [1] in 1984. The ultimate gain and ultimate frequency are then used for adjusting of parameters by original Ziegler-Nichols rules. From that time, many studies have been reported to extend and improve autotuners principles; see e.g. [2] - [4], [9], [10]. Over time, the direct estimation of transfer function parameters instead of critical values began to appear. The extension in relay utilization was performed in e.g. [8] - [11] by an asymmetry and hysteresis of a relay. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

In this paper, a novel combination for autotuning method of PI and PID like controllers is proposed and developed. The basic autotuning principle combines an asymmetrical relay identification experiment and a control design performed in the ring of proper and stable rational functions $R_{PS}$. The factorization approach proposed in [12] was generalized to a wide spectrum of control problems in [13], [15] - [17]. The pole placement problem in $R_{PS}$ ring is formulated through a Diophantine equation and the pole is analytically tuned according to the required response of the closed loop. Naturally, there exist many principles of control design syntheses which can be used for autotuning principles, e.g. [2], [8], [9], [18]. This contribution deals with two simplest SISO linear dynamic systems with a delay term. The first model of the first order (stable) plus dead time (FOPDT) is supposed in the form:

$$G(s) = \frac{K}{Ts + 1}e^{-\theta s}$$  \hspace{1cm} (1)

Similarly, the second order model plus dead time (SOPDT) is assumed in the form:

$$G(s) = \frac{K}{(Ts + 1)^2}e^{-\theta s}$$  \hspace{1cm} (2)

The contribution is organized as follows. Section 2 outlines a background of algebraic control design, see [15] - [19] for details. In section 3 the principle of the Smith predictor is introduced. Section 4 presents some facts about relay identification for autotuning principles. The developed Matlab program environment for design and simulations is described in Section 5. Finally, section 6 presents simulation results in two examples of SISO systems.

2 Algebraic Control Design

The control design is based on the fractional approach; see [12], [13], [15]. Any transfer function $G(s)$ of a (continuous-time) linear system is expressed as a ratio of two elements of $R_{PS}$. The set $R_{PS}$ means the ring of (Hurwitz) stable and proper rational functions. Traditional transfer functions as a ratio of two polynomials can be easily transformed...
into the fractional form simply by dividing, both the polynomial denominator and numerator by the same stable polynomial of the appropriate order.

Then all transfer functions can be expressed by the ratio:

\[ G(s) = \frac{b(s)}{a(s)} = \frac{B(s)}{(s + m)^n} \]

\[ n = \max(\deg(a), \deg(b)), \quad m > 0 \]  

The similar conclusion is valid also for the load disturbance \( d = G_d / F_d \). The load disturbance attenuation is then achieved by divisibility of \( P \) by \( F_d \). More precisely, for tracking and attenuation in the closed loop according to Fig. 2 the multiple of \( AP \) must be divisible by the least common multiple of denominators of all input signals. The divisibility in \( \mathbb{R}_\mathbb{P} \) is defined through unstable zeros and it can be achieved by a suitable choice of a rational function \( Z \) in (6), see [10], [15] for details. The derivation of controller parameters can be found in [17], [19] also with aperiodic tuning, similar adjusting is solved in [14]. Time delay systems are studied in [18], [20].

3 Smith Predictors

The Smith predictor was designed in the late 1950s for systems with time delay, see [6], deep insight into time delay systems can be found in [5]. The basic classical interpretation of the Smith predictor is depicted in Fig. 3. The time delay term \( e^{-\theta s} \) has a negative influence to feedback stability which follows from the frequency analysis. The feedback signal for the main controller \( C(s) \) in Fig. 3 is a predicted value of the output. It means that the signal \( y(t) \) inputs into the control error instead of the delayed \( y(t-\theta) \), it explains the name predictor. The Smith predictor launched the high development of Internal model controllers (IMC), where the plant model is present in the feedback loop, see [7]. When the transfer function \( G(s) \) is stable then the feedback system in Fig. 3 is equivalent to the IMC version depicted in Fig. 4.

The main advantage of the Smith predictor is that the controller \( C(s) \) can be designed according to delay-free part \( G(s) \) of the plant. However, there are two main weak points in this sophisticated scheme. The first one is that the signal \( v(t) \) is zero only in the case when the transfer function \( G(s) \) is the same in the outer and inner loops in Fig. 3. The second weakness is that the transfer function must be stable. In the case of autotuning, the approximated transfer
function of the plant can always be incorporated into the feedback.

![Smith predictor – IMC version](image)

**4 Relay Feedback Estimation**

The estimation of the process or ultimate parameters is a crucial point in all autotuning principles. The relay feedback test can utilize various types of relay for the parameter estimation procedure. The classical relay feedback test [1] was proposed for stable processes by symmetrical relay without hysteresis and the scheme is depicted in Fig. 5. Following sustained oscillation are then used for determining the critical (ultimate) values. The control parameters (PI or PID) are then generated in standard manner.

![Block diagram of an autotuning principle](image)

Asymmetrical relays with or without hysteresis bring further progress [2], [8], [9], [11]. After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such experiment is depicted in Fig. 6. The relay asymmetry is required for the process gain estimation (10) while a symmetrical relay would cause the zero division in the appropriate formula.

In this paper, an asymmetrical relay with hysteresis is used. This relay test enables to estimate transfer function parameters as well as a time delay term. For the purpose of the aperiodic tuning the time delay is not exploited.

The process gain can be computed by the relation (see [22]):

\[
K = \frac{\int_{0}^{T_y} \frac{y(t)}{dt}}{\int_{0}^{T_y} u(t)dt}
\]

(10)

where \(T_y\) is a chosen suitable time for at least ten oscillations in the relay experiment.

For the first order model (1), the time constant and time delay term are given by [11]:

\[
T = \frac{T_y}{2\pi} \sqrt{\frac{16 \cdot k^2 \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1}
\]

\[
\Theta = \frac{T_y}{2\pi} \left[ \pi - \arctg \frac{2\pi T}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right]
\]

(11)

where \(a_y\) and \(T_y\) are depicted in Fig. 6 and \(\varepsilon\) is the hysteresis.

In the case of second order model (2), the gain is given by (10), the time constant and time delay term can be estimated according to [11] by the relation:

\[
T = \frac{T_y}{2\pi} \sqrt{\frac{4 \cdot k \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1}
\]

\[
\Theta = \frac{T_y}{2\pi} \left[ \pi - 2 \arctg \frac{2\pi T}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right]
\]

(12)

**5 Simulation and Program System**

A Matlab program system was developed for engineering applications of auto-tuning principles. The estimated model is of a first or second order transfer function with time delay while the controlled system is of arbitrary order. The user can choose three cases for the time delay term. In the first case the time term is neglected, in the second one the term is approximated by the Padé expansion and the third case utilizes the Smith predictor control structure. The Main menu window of the program system can be seen in Fig. 7.

In the first phase of the program routine, the controlled transfer function is defined and parameters for the relay experiment can be adjusted. Then, the relay experiment is performed and an
The estimated transfer function in the form of (1) or (2) is identified. Then controller parameters are generated after pushing of the appropriate button. The second phase begins with the “Design controller parameters” button and the chosen control design is performed and the controller is derived and displayed. The control scheme depends on the choice for the 1DOF or 2DOF structure and on the choice of the treatment with the time delay term.

The first and second order estimation results in the following transfer functions:

\[
\tilde{G}_1(s) = \frac{2.99}{5.88s + 1} \cdot e^{-10.35s}
\]

\[
\tilde{G}_2(s) = \frac{2.99}{11.19s^2 + 6.69s + 1} \cdot e^{-8.49s}
\]  \hspace{1cm} (14)

Then controllers were designed for the identified models (14) with neglected time delay terms. The PI controller was derived for the value of \(m = 0.13\) and the PID one was derived for \(m = 0.22\). Both controllers in the 1DOF structure have the transfer functions:

\[
C_1(s) = \frac{0.17s + 0.03}{s}
\]

\[
C_2(s) = \frac{0.42s^2 + 0.23s + 0.03}{3.35s^3 + s}
\]  \hspace{1cm} (15)

The control responses for the first order approximation and design are depicted in Fig. 9. In this case the difference of responses between neglected time delay term and with the use of the Smith predictor is remarkably strong. While the standard feedback control response is quite poor and oscillating then the response with Smith predictor in the loop is smooth and aperiodic. However, the Padé approximation gives also acceptable behaviour.

### 6 Examples and Simulations

**Example 1:** The example represents a fifth order system with a time delay term with transfer function

\[
G(s) = \frac{3}{(2s + 1)^3} \cdot e^{-5s}
\]  \hspace{1cm} (13)

![Simulink Control System](image)

**Fig. 7: Main Menu**

During the third phase, after pushing the “Start simulation” button, the simulation routine is performed and required outputs are displayed. Various simulation parameters can be specified in the Simulink environment. In all simulation a change of the step reference is performed in the second third of the simulation horizon and a step change in the load is injected in the last third. A typical control loop of the case with the Smith predictor in Simulink is depicted in Fig. 8.

![Simulink PID Controller](image)

**Fig. 8: Control loop in Simulink (Smith predictor)**

![Control responses](image)

**Fig. 9: Control responses 1DOF - first order**

Almost the same situation is showed in Fig. 10 where the second order approximation and synthesis were utilized. However, comparison of Fig. 9 and Fig. 10 shows that the first order synthesis is sufficient and the second order is redundant.
Example 2: This example represents a case of higher order system without delay approximated by a lower order system with a time delay term. A higher order system ($8^{th}$ order) with transfer function $G(s)$ is supposed:

$$G(s) = \frac{3}{(s+1)^3} \quad (16)$$

Again, after the relay experiment, a first order and second estimation gives the following transfer functions:

$$\hat{G}(s) = \frac{2.96}{4.22s + 1} e^{-4.96t}$$

$$\tilde{G}(s) = \frac{2.96}{4.83s^2 + 4.40s + 1} e^{-4t} \quad (17)$$

The step responses of systems (16) and (17) are compared in Fig. 11.

Naturally, both step responses of the estimated systems are quite different from the original system $G(s)$. PI controllers are generated from (5) and the tuning parameter $m>0$ can influence the control behaviour. Since the difference of controlled and estimated systems is considerable, it can be expected that not all values of and some of $m>0$ represent acceptable behaviour.

With respect to aperiodic tuning in [19], three responses are shown in Fig. 12. Generally, larger values of $m>0$ implicate larger overshoots and oscillations. As a consequence, for inaccurate relay identifications, lower values of $m>0$ can be recommended. The PI controller for $m = 0.18$ gives the transfer function:

$$C(s) = \frac{0.17s + 0.05}{s} \quad (18)$$

The control responses for (16) and (18) with and without the Smith predictor are shown in Fig. 12.

The second order identification and synthesis of example 3 for $m = 0.34$ gives the PID controller:

$$C(s) = \frac{0.28s^2 + 0.23s + 0.05}{2.20s^2 + s} \quad (19)$$

The control responses of the combination (16), (19) are depicted in Fig. 13.
7 Conclusion
This contribution gives some rules for autotuning principles with a combination of relay feedback identification and a control design method.

The estimation of a low order transfer function parameters is performed from asymmetric limit cycle data. The control synthesis is carried out through the solution of a linear Diophantine equation according to [12], [15], [17], [19]. This approach brings a scalar tuning parameter which can be adjusted by various strategies. A first order estimated model generates PI-like controllers while a second order model generates a class of PID ones. In both cases also the Smith predictor influence was compared with neglecting of time delay terms and/or the Padé approximation. The methodology supported by developed Matlab program system is illustrated by two examples. The results of all simulations prove that the Smith predictor structure brings a significant improvement of the aperiodic responses. The price for the improvement is a more complex structure of the feedback control system.

Acknowledgements
This work was supported by the European Regional Development Fund under the project CEBIA-Tech No. CZ.11.05/2.1.00/03.0089.

References: