# Design of an extended Luenberger observer for sensorless vector control of induction machines using virtual instrument

NICOLAE PATRASCOIU ADRIAN TOMUS Automatics, Computers, Electrical and energetics Engineering University of Petrosani Universitatii 20, 332006 Petrosani ROMANIA patrascoiu@upet.ro http://ime.upet.ro/caiac/cadre.html

*Abstract:* - This paper presents the simulation of the flux and of an induction motor rotor's speed estimator using the graphical programming language LabView. Also, in this paper it is realized a study of the Luenberger estimator's and extended Luenberger estimator's. For this, it's created a virtual instrument using LabView for the grapfical interface and Control Design and Simulation Module for LabView to implement the model of the induction motor and of the Luenberger estimator in order to realize the simulations.

Key-Words: - Luenberger estimator, ELO, virtual instrument, LabView, Control Design and Simulation.

#### **1** Introduction

Since the early papers, which concentrated on observers for purely deterministic continuous-time linear time-invariant system, observer theory has been extended by several researchers to include time-varying systems, discrete systems, and stochastic systems.

Sometimes all state space variables are not available for measurements, or it is not practical to measure all of them, or it is too expensive to measure all state space variables. In order to be able to apply the state feedback control to a system, all of its state space variables must be available at all times. Also, in some control system applications, one is interested in having information about system state space variables at any time instant. Thus, one is faced with the problem of estimating system state space variables. This can be done by constructing another dynamical system called the observer or estimator, connected to the system under consideration, whose role is to produce good estimates of the state space variables of the original system.

The theory of observers started with the work of Luenberger (1964, 1966, 1971) so that observers are very often called Luenberger observers. According to Luenberger, any system driven by the output of the given system can serve as an observer for that system.

Two main techniques are available for observer design.

The first one is used for the full-order observer design and produces an observer that has the same dimension as the original system.

The second technique exploits the knowledge of some state space variables available through the output algebraic equation (system measurements) so that a reduced-order observer is constructed only for estimating state space variables that are not directly obtainable from the system measurements [1], [4].

#### **2** Problem Formulation

The Extended Luenberger Observer (ELO) is based on the use of an adaptive mechanism, in which the reference model is the induction motor and the adjustable model is a Luenberger type linear state estimator.

The equations of the Luenberger linear estimator are:

 $\hat{\mathbf{x}}_{k+1} = \mathbf{F}_{d} \cdot \hat{\mathbf{x}}_{k} + \mathbf{H}_{d} \cdot \mathbf{u} + \mathbf{L}_{d} \cdot \left(\mathbf{y}_{k} - \mathbf{C} \cdot \hat{\mathbf{x}}_{k}\right)$ (1)

where:  $F_k$ ,  $H_k$  and  $L_k$  are matrixes obtained through complete discretization based on the following formulas:

$$F_{k} = I + A_{k} \cdot T + A_{k}^{2} \cdot \frac{T^{2}}{2}; H_{k} = B \cdot T + A_{k} \cdot B \cdot \frac{T^{2}}{2};$$
(2)  
$$L_{k} = L \cdot T + A_{k} \cdot L \cdot \frac{T^{2}}{2}$$



Fig. 1 The Principle Schematic of the ELO Estimator

where T is the sampling time and A, B and L have the following structure:

$$A_{k} = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} \cdot \omega_{k} \\ 0 & a_{11} & a_{14} \cdot \omega_{k} & a_{13} \\ a_{31} & 0 & a_{33} & -\omega_{k} \\ 0 & a_{31} & \omega_{k} & a_{33} \end{bmatrix}$$
$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 \end{bmatrix}^{T}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} k_{11} & k_{12} & k_{21} & k_{22} \\ -k_{12} & k_{11} & -k_{22} & k_{21} \end{bmatrix}$$
(3)

The component elements of the  $A_k$ , B and L matrixes are:

$$a_{11} = -\left(\frac{1}{T_{s} \cdot \sigma} + \frac{1 - \sigma}{T_{r} \cdot \sigma}\right); a_{13} = \frac{L_{m}}{L_{s} \cdot L_{r} \cdot T_{r} \cdot \sigma}$$

$$a_{14} = \frac{L_{m}}{L_{s} \cdot L_{r} \cdot \sigma}; a_{31} = \frac{L_{m}}{T_{r}}; a_{33} = -\frac{1}{T_{r}};$$

$$b_{11} = \frac{1}{L_{s} \cdot \sigma}; T_{s} = \frac{L_{s}}{R_{s}}; T_{r} = \frac{L_{r}}{R_{r}}; \sigma = 1 - \frac{L_{m}^{2}}{L_{s} \cdot L_{r}} (4)$$

$$k_{11} = (a_{11} + a_{33}) \cdot (1 - k_{1}); k_{12} = \omega_{k} \cdot (1 - k_{1});$$

$$k_{21} = \left(a_{31} + \frac{a_{11}}{a_{14}}\right) \cdot (1 - k_{1}^{2}) - \frac{k_{11}}{a_{14}}; k_{22} = \frac{k_{12}}{a_{14}}$$

where: Ls, Lr, Lm are the stator, rotor and mutual inductances; Rs, Rr are the stator and rotor's resistances and  $\sigma$  is the mutual dispersion coefficient.

The equations of the Luenberger linear estimator were obtained considering the fact that the structure of the state vector is:

$$\hat{\mathbf{x}}(\mathbf{k}) = \begin{bmatrix} \hat{\mathbf{i}}_{ds}(\mathbf{k}) & \hat{\mathbf{i}}_{qs}(\mathbf{k}) & \hat{\boldsymbol{\psi}}_{dr}(\mathbf{k}) & \hat{\boldsymbol{\psi}}_{qr}(\mathbf{k}) \end{bmatrix}^{\mathrm{T}}$$
(5)

Formed by the stator's currents and rotor's fluxes in the dq axes system and, on the other hand, the input vector is:

$$\mathbf{u}(\mathbf{k}) = \begin{bmatrix} \mathbf{u}_{ds}(\mathbf{k}) & \mathbf{u}_{qs}(\mathbf{k}) \end{bmatrix}^{\mathrm{T}}$$
(6)

and the output vector is:

$$\mathbf{y}(\mathbf{k}) = \begin{bmatrix} \mathbf{i}_{ds}(\mathbf{k}) & \mathbf{i}_{qs}(\mathbf{k}) \end{bmatrix}^{\mathrm{T}}$$
(7)

where  $u_{ds}$  and  $u_{qs}$  are the stator's tensions in the dq axes system and  $i_{ds}$  and  $i_{qs}$  are the stator's currents in the dq axes system.

In case of Luenberger matrix's projection was used a formula that ensures the proportionality between the induction machines self-values and the estimator's based on a  $k_1$  proportionality coefficient.

The formulas are deduced in the continuous case obtaining the L matrix, from which, through discretisation it results the  $L_k$  matrix based on the discretisation, number 2 formula. Through assuring the proportionality, there results the estimator's stability no matter what the  $\omega_k$  speed is. Obviously, the proportionality will not maintain after discretisation, but the stability will.

The Luenberger estimator's error is given by:

$$\widetilde{\mathbf{x}}(\mathbf{k}) = \mathbf{x}(\mathbf{k}) - \hat{\mathbf{x}}(\mathbf{k}) \tag{8}$$

However, as one may notice from Figure 1, the  $\omega_k$  speed, used in the  $F_k$ ,  $H_k$  and  $L_k$  matrixes calculations is the estimated speed obtained at the output of the adapting mechanism.

The adapting mechanism is deducted through the Popov type hyper-stability conditions obtaining, after the calculations that:

$$\hat{\omega} = K_{i} \cdot \int \left[ e_{ds} \cdot \hat{\psi}_{qr} - e_{qs} \cdot \hat{\psi}_{dr} \right] dt \qquad (9)$$

where:  $K_i$  is a constant chosen so that we have a good dynamics and  $e_{ds} = \dot{i}_{ds} - \hat{i}_{ds}$ ;  $e_{qs} = \dot{i}_{qs} - \hat{i}_{qs}$ ;

In case of numeric implementation of the ELO algorithm, the (9) relation is calculated using one of the integral's numeric evaluation methods, like the rectangle method, the trapezium method.

In this paper in the implementation of the adapting mechanism was used a more complex formula, namely:

$$\hat{\omega} = K_{p} \cdot \left[ e_{ds} \cdot \hat{\psi}_{qr} - e_{qs} \cdot \hat{\psi}_{dr} \right] + K_{i} \cdot \int \left[ e_{ds} \cdot \hat{\psi}_{qr} - e_{qs} \cdot \hat{\psi}_{dr} \right] dt$$
(9)

where we also used a proportional component. The discretisation method of the integral is the trapezium method (Tustin) [4], [5].

### **3** Problem Solution

To achieve the Luenberger estimator analysis on use graphical programming language LabView and the Control Design and Simulation module.

A program developed in LabVIEW is called a virtual instrument (VI) and it has two components the block diagram that represent program itself and the front panel that is the interface with user.

The Control Design and Simulation module for LabVIEW can be used to simulate dynamic systems. To facilitate model definition, this module adds functions to the LabVIEW environment that resemble those found in SIMULINK. There is also the ability to use m-file syntax directly in LabVIEW through the new MathScript node.

The Control Design and Simulation Module also provide VIs that can be used to create and develop control design applications in LabVIEW. These VIs can be used to develop mathematical models of a dynamic system, analyze the models to learn about their dynamic characteristics, and create controllers to achieve specified dynamic characteristics.

The Control Design and Simulation Module also include numerous functions that extend the functionality of the LabVIEW MathScript window. These functions are used to design and analyze controller models in a text-based environment and the LabVIEW MathScript engine is used to execute scripts previously written using The MathWorks, MATLAB [8].

For simulate in real time, we consider a motor with the next parameters:

$$\begin{split} P_N &= 2.2 \text{ kW}; \ z_p = 1; \\ L_s &= 0.03257 \text{ H}; \ L_r = 0.03245 \text{ H}; \\ R_s &= 0.385 \ \Omega; \ R_r = 0.342 \ \Omega; \\ J &= 0.0088 \text{ N} \cdot \text{m}; \ M_r = 2 \text{ kg} \cdot \text{m}^2; \\ f &= 50 \text{Hz}. \end{split}$$



Fig. 2. Simulation scheme and the internal structure of induction motor

For a better simulation, we created a simulation scheme for induction motor based on state equations, called model voltage (fig.2). The simulation is performed using LabVIEW MathScript from the Control Design and Simulation Module.

Based on this model of the induction motor is implemented in the same way the Luenberger flux estimator. Simulation scheme and the internal structure of the Luenberger estimator is shown in fig.3.



Fig.3. Internal structure of the Luenberger estimator

With this two models we create the simulation model for Luenberger observer and extented Luenberger observer using the Control Design and Simulation module of LabView. In the simulation scheme of the Luenberger estimator is observed speed estimator; the controller used in the speed estimator is a discrete PI. Block diagrams corresponding to these two simulation models are presented in fig.4 and fig.5



Fig.4. Simulation scheme of the Luenberger estimator





The front panel of virtual instrument contanis two tabs that allow you to select one of two simulation models: Luenberger estimator or extented Luenberger Estimator. Each tab contains the graph type indicators to display speed, flow or current chart. The front panel for these two cases is presented in fig.6.

From the simulation results we see that Extended Luenberger Observer (ELO) estimator has very good dynamics and it can be successfully used in sensorless vector control systems type (without speed sensor). High dynamics and low volume calculations make this type of estimator to be the most widely used today [6], [7], [8].



Fig.6. Front Panel of the virtual instrument for Luenberger estimator and extended Luenberger estimator

## **4** Conclusion

The simulation mode can be presently considered the most important design strategy and implementation of an estimator so that its analysis through simulation without hardware, introduced in a loop and also through hardware introduced within the simulation.

After the simulation it is noticed that the ELO estimator is behaving very well in case of decreasing or increasing the rotor's resistance by 10%, problems arising for values of resistance passing over 50%.

Regarding the stability, the ELO estimator is internally stable for a wide band of revolutions up to approximately 10.000 rpm.

Regarding the sensitivity and dynamic performances point of view one may say that the ELO estimator is very performing and it can successfully be used within the sensorless vectored acting systems with induction motors.

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