Dengue Transmission Model with Age-Dependent Survival Rates in the Presence of Wolbachia Infection

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Abstract: In this paper we devise a mathematical model for the transmission of dengue disease with an assumption that there is an age-dependent function describing the survivorship of individuals for both human and mosquito populations. We also assume that there is a wolbachia bacterial infection into the mosquito population, reflected by the decreasing of mosquito’s life expectancy and biting rate. We derive the basic reproduction number and re-establish the well known rule of thumb of the minimum vaccination coverage to control a disease in the context of vector-borne disease with age-dependent survival rates. The effect of the presence of wolbachia infection in the basic reproduction number of dengue is also discussed, to explore the infection as a potential biological control to eliminate the disease. We compare the strategy of wolbachia introduction with the existing strategy such as vaccination.

Key–Words: Dengue disease, wolbachia infection, biological control.

1 Mathematical Model

Brauer in [1] discussed a model for an infectious disease in which he assumed that an infective does not recover. It is also assumed that the disease is transmitted directly from an infective individual to a susceptible individual. The model discussed here is the generalization of his model for vector-born transmitted disease to include recovery and indirect transmission of the disease, such as in dengue disease transmission. To formulate the model, we assume that the host population $N_H$ is divided into three compartments of infectives ($S_H$), susceptibles ($I_H$) and recovered individuals ($R_H$). We assume that recovered host individuals are not susceptible anymore. We also assume that vectors remain infective for life due to their short life period compared to the duration of the disease, and hence the vector population only comprises of two compartments, namely infectives ($S_V$) and susceptibles ($I_V$). Recruitment rates for the host and the vector are $B_H$ and $B_V$, with the death rates $\mu_H$ and $\mu_V = \zeta \mu_V$, respectively. The disease transmission probability from vector to host and from host to vector are given by $\beta_H = \xi \beta_V$ and $\beta_V = \xi \beta_V$, respectively. Here $\mu_V$ is the natural vector death rate while $\beta_V$ are the natural disease transmission rates in the absence of wolbachia infection. We assume that wolbachia infection reduces the life span, or equivalently increases the death rate of the mosquitoes, and hence in the presence of wolbachia infection $\zeta$ is a magnifier factor with $\zeta > 1$. We also assume that the presence of wolbachia infection reduces the biting rate of the mosquitoes with the reduction factor $\xi < 1$. To mimic the age structure effect, suppose that there exist non-negative monotonically non-increasing functions of age, $Q_H(a)$ and $Q_V(a)$, describing the fraction of host and vector population, respectively, who survives to the age of $a$ or more, such that $Q_H(0) = 1$ and $Q_V(0) = 1$. Furthermore, we assume that host life expectancy is finite,

$$\int_0^\infty Q_H(a)da = L_H,$$

so that

$$\int_0^\infty aQ_H(a)da < \infty.$$

The host population is assumed to be constant and given by

$$N_H(t) = \int_0^\infty B_H Q_H(a)da = B_H L_H.$$

For the next discussion we will use the notations $X(a) = B_H Q_H(a)$ and $Y(a) = B_V Q_V(a)$ to indicate the portion of offsprings survive to the age $a$ or more. Since the per capita rate of infection in host
population at time $t$ is $\beta_H I_V(t)$, the number of susceptibles at time $t$ is given by

$$S_H(t) = \tilde{S}_H(t) + \int_0^t X(a)E(a, t, I_V) da,$$

with $\tilde{S}_H(t) = S_H(0)Q_H(t)e^{-\int_0^t \beta_H I_V(s) ds}$ and $E(a, t, I_V) = e^{-\int_{t-a}^t \beta_H I_V(s) ds}$. If we further assume that the rate of recovery is $\gamma$ then the number of infective host at time $t$ is given by

$$I_H(t) = \tilde{I}_H(t) + \int_0^t X(a)[1 - E(a, t, I_V)]e^{-\gamma a} da,$$

with $\tilde{I}_H(t) = I_H(0)Q_H(t)e^{-\gamma t}$. Here $e^{-\int_{t-a}^t \beta_H I_V(s) ds}$ is the probability that a susceptible of the age $t - a$ will survive at least up to time $t$. Furthermore, since $R_H(t) = N_H(t) - S_H(t) - I_H(t)$ then we have

$$R_H(t) = \tilde{R}_H(t) + \int_0^t X(a)[E(a, t, I_V)] [1 - e^{-\gamma a}] da,$$

with $\tilde{R}_H(t) = \tilde{N}_H(t) - \tilde{S}_H(t) - \tilde{I}_H(t)$ and $E(a, t, I_V) = (1 - E(a, t, I_V)$). Equations (4) to (6) constitute transmission dynamics in the host population. Analogously, the transmission dynamics in the vector population is governed by the following equations

$$N_V(t) = \tilde{N}_V(t) + \int_0^t Y(a) da = B_V L_V,$$

with $L_V = \int_0^\infty Q_V(a) da$,

$$S_V(t) = \tilde{S}_V(t) + \int_0^t Y(a)F(a, t, I_H) da,$$

with $\tilde{S}_V(t) = S_V(0)Q_V(t)e^{-\int_0^t \gamma a I_H(s) ds}$, and $F(a, t, I_H) = e^{-\int_{t-a}^t \beta_V I_H(s) ds}$

$$I_V(t) = \tilde{I}_V(t) + \int_0^t Y(a)[1 - F(a, t, I_H)] da,$$

with $\tilde{I}_V(t) = I_V(0)Q_V(t)$. It is also clear that $\lim_{t \to \infty} \tilde{N}_V(t) = \lim_{t \to \infty} \tilde{S}_V(t) = \lim_{t \to \infty} \tilde{I}_V(t) = 0$. The full model for the host-vector model is given by (4) to (6) and (8) to (9). In the next section we analyze these equations and show that there is a threshold epidemic $R_0$ determining the occurrence of the non-trivial equilibrium point. In the subsequent section we also show that this threshold is actually determining the stability of the non-trivial equilibrium point.

### 2 Results

It is well known that in epidemic modeling the appearance of a non-trivial or endemic equilibrium point depends on a threshold number, which often referred to the basic reproduction number $[2, 3]$. We found that the basic reproduction number of dengue disease from the model given by

$$R_0 = R_0^H R_0^V > 1$$

with $R_0^H = B_H \beta_H \int_0^\infty a Q_H(a)e^{-\gamma a} da$ and $R_0^V = B_V \beta_V \int_0^\infty a Q_V(a) da$.

If we assume that a portion $p$ of the host population is vaccinated at birth assuming that vaccination is perfect then we have an effective reproduction number due to the application of vaccination, given by

$$(1 - p)R_0,$$

where $R_0$ is the basic reproduction number. From the last expression we conclude that the endemic equilibrium vanishes whenever

$$p > 1 - \frac{1}{R_0} = p_c.$$

The quantity $p_c$ is usually referred as the critical vaccination level which depends on the value of the basic reproduction number. It is well known in literatures that in a direct transmission disease model the threshold number $R_0$ can be estimated by the ratio of life expectancy and mean age at infection. In [4] it is shown that if the survival function is a negative exponential function then this estimation is still valid for the indirect transmission model discussed in the previous sections.

If we further assume that there is a decaying effect of vaccination with $e^{-\tau a}$ is the survival of an individual of age $a$ from losing immunity then the effective reproduction number is given by

$$R_\tau = R_0 - pR_1,$$

where $R_0$ is the basic reproduction number and $R_1 = \beta_H \beta_V \int_0^\infty a X(a) (\int_0^\infty a^2 Y(a) da) e^{-(\gamma + \tau) a} da$. From the last expression we can observe two extreme conditions. If the rate of losing immunity is relatively low, i.e. $\tau \to 0$, then we can ignore the presence of decaying immunity, since the minimum level of vaccination $p$ is remain the same as before, that is, $p > 1 - \frac{1}{R_0}$. If, however, the rate of losing immunity is sufficiently high, i.e. $\tau \to \infty$, then there might be no reduction in value of the basic reproduction number $R_0$ no matter large the vaccination rate $p$ is. Generally, the value of the rate of immunity waning is in between those two extreme values. In this case, to eliminate the disease,
we need $R_0 < 1$ or equivalently $p > \frac{R_0}{k_1}$, in which the inequality $p > \frac{R_0 - 1}{k_1} > \frac{R_0}{R_0 - 1}$ holds. This indicates that ignoring the presence of decaying immunity will underestimate the critical vaccination level $p$, which results in failing to eliminate the disease.

In regards to the presence and absence of wolbachia in mosquito population, we can write the basic reproduction number in the presence of wolbachia $R_0(\zeta, \xi)$ and in the absence of wolbachia $R_0(\zeta = 1, \xi = 1)$ as follows:

$$R_0(\zeta, \xi) = \frac{B_H \xi \beta_H^* c}{(\mu_H + \gamma)^2} \left( \frac{B_V \xi \beta_V^* c}{\xi \mu_V^* c^2} \right) R_0(\zeta = 1, \xi = 1)$$

since the magnification factor of the mosquito death rate $\zeta > 1$ and the reduction factor of mosquito biting rate $\xi < 1$. This means that the presence of wolbachia is truly effective in decreasing the basic reproduction number of dengue. The following example show how much reduction of the basic reproduction number could be gained if there is a certain reduction on mosquito biting rate and survival rate for exponential function of survivorship.

Let $Q_H(a) = e^{-\mu_H a}$ and $Q_V(a) = e^{-\mu_V a}$ be the survival function for humans and mosquitoes, respectively, then the basic reproduction number in the presence of wolbachia is

$$R_0(\zeta, \xi) = \left( \frac{B_H \xi \beta_H^* c}{(\mu_H + \gamma)^2} \right) \left( \frac{B_V \xi \beta_V^* c}{\xi \mu_V^* c^2} \right) R_0(\zeta = 1, \xi = 1)$$

Now suppose that the presence of wolbachia can suppress the mosquitoes so that its death rate increases by 50% and its biting rate decrease by 50%. In this case $\zeta = 3/2$ and $\xi = 1/2$, and hence the reduction on the basic reproduction number can be achieved to 1/9 of the reproduction number in the absence of wolbachia. This illustration supports the belief of the effectiveness of wolbachia introduction to control dengue disease. As an example, it is recorded that the natural basic reproduction number for dengue disease in Bandung Indonesia, during the period 2002-2007, is approximately eight [4]. If there is a wolbachia infection that could reduce the biting rate into half and increase the mosquito death rate by half its current level, then theoretically the disease will die out. Compare to effort if we use vaccination strategy, then from 12 we need to vaccinate at least 87.5% of the total population assuming that there is no decaying vaccination. If there is a waning immunity, then from 13 the vaccination coverage should be larger than 87.5%. Other scenario can be analyzed by varying the control parameters in the basic reproduction number, such as the effect of combination of vaccination and biological control of wolbachia infection.

### 3 Conclusion

We have developed a mathematical model of dengue transmission in the presence of wolbachia infection in the mosquitoes. In general it has been shown that the introduction of wolbachia is prospective in controlling dengue transmission. A simple formula regarding the effect of wolbachia into natural basic reproduction of dengue disease has been created, by which we can derived some insights. Although we found some insights regarding the effect of wolbachia into the basic reproduction number of dengue, the result here is only a rough estimate. A more realistic model should consider the transmission mechanism of wolbachia among the mosquitoes. The dynamics of wolbachia transmission might alter the result in this paper to some extent. This work is currently under investigation.

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**References:**


