Numerical Simulation of Blood Flow in the System of Human Coronary Arteries with and without Bypass Graft

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Abstract: In this paper, we aim to present a three-dimensional simulation of a pulsatile blood flow in the system of human coronary arteries. We investigate the behavior of blood flow through a stenosed right coronary arteries (RCA) with and without bypass graft. The computational domains, constructed and based on real domain, consist of the aorta, the left and the right coronary arteries, and a graft vessel. Blood is considered as an incompressible non-Newtonian fluid. The governing equations are the continuity equation and the Navier-Stokes equations together with pulsatile boundary conditions. The solution of governing equations is solved by using finite element method. The results show that higher degree of blockage of the RCA gives higher drop of blood pressure at the stenosis region, and bypass surgery can improves the blood pressure.

Key–Words: blood flow, bypass graft, coronary artery, incompressible non-Newtonian fluid, pulsatile, stenosis.

1 Introduction

In present time, almost one fourth of human death are caused by ischaemic heart disease (IHD) and stroke and other cerebrovascular disease, especially in the middle-income and high-income countries. IHD, or called coronary artery disease (CAD), is a major cause of human death in the world. In 2008, the World Health Organization (WHO) had reported the list of the top 10 causes of death in the world [1]. There are 7.25 million men who died because of IHD which is 12.8%, the highest percentage of deaths in the world.

To deal with this disease, many surgery procedures have been invented. A coronary artery bypass grafting (CABG) is a technique has been widely used for patients with severe coronary artery diseases. However, up to 25% of grafts fail within one year and up to 50% fail within ten years after surgery [3]. It is believed that this failure depends on the abnormal flow of blood. To overcome the problem of blood flow in the stenosed coronary arteries, many researchers have studied the blood flow behavior in either the right coronary artery (RCA) or the left coronary artery (LCA) by using experimental and numerical studies [4, 11, 12].

Due to morality and ethics, the experimental research on real human body is impracticable. Mathematical modeling and numerical simulation are thus used for understanding the phenomena involved in vascular disease. The problem of stenosis was received attention from many researchers. Blood flow was simulated in a single tube with stenotic region [8, 10]. To understand the behavior of blood in a vessel with bypass graft, the blood flow was simulated in a straight tube with graft [2, 6]. Wiwatanapataphee et al.(2006) investigated the behavior of blood flow in a stenosed RCA with a bypass graft [5]. The result shows that blood pressure drops dramatically in stenosis area and high wall shear stress occurs around the stenosis site. However, it has been recognized that the results obtained from unrealistic computational domain may not be use for treatment.

The three-dimensional realistic geometry which mimic human blood vessel was constructed and studied [7, 14]. Chuchard et al.(2011), studied blood flow through the system of coronary arteries with diseased left anterior descending [13]. Their computational domain consists of the base of the aorta, the RCA and the LCA. The result shows that the blood flow with high speed and the pressure rapidly drops in the area of stenosis.

In this work, we simulate a non-Newtonian pulsatile blood flow in the system of human coronary arteries with and without bypass graft. Figure 1 illustrates a computational domain used in this study. It consists of the ascending part of the aorta, the RCA with a bypass graft and the LCA. The effects of stenosis severity and bypass graft on the pressure distribution are investigated.
2 Governing equations and Boundary conditions

In this study, blood is assumed as an incompressible non-Newtonian fluid. The motion of blood flow is governed by the continuity equation and the Navier-Stokes equations, which can be expressed in vector notation as follows:

\[ \nabla \cdot \mathbf{u} = 0, \]
\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{\rho} \nabla \cdot \mathbf{\sigma}, \]

where \( \mathbf{u} \) is the blood velocity vector in the lumen region, \( \rho \) is blood density and \( \mathbf{\sigma} \) is the total stress tensor which is defined by

\[ \mathbf{\sigma} = -p \mathbf{I} + \eta(\dot{\gamma})[\nabla \mathbf{u} + (\nabla \mathbf{u})^T], \]

where \( p \) is the blood pressure. \( \eta \) and \( \dot{\gamma} \) denote the viscosity of blood and shear rate, respectively. In this work, the relation between \( \eta \) and \( \dot{\gamma} \) are described by Carreau’s shear-thinning model, namely,

\[ \eta = \eta_\infty + (\eta_0 - \eta_\infty) \left[ 1 + (\lambda \dot{\gamma})^2 \right]^{(n-1)/2}, \]

where \( \eta_\infty, \eta_0, \lambda, \) and \( n \) are parameters and the shear rate defined by

\[ \dot{\gamma} = \sqrt{2tr\left( \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right)^2}. \]

As a cyclic nature of heart pump, the pulsatile behavior of blood flow is considered. We assume that, in cardiac cycle, there has no difference of time variation. Therefore, the pulsatile pressure and flow rate can be expressed by the functions \( p(t) = p(t + nT) \) and \( Q(t) = Q(t + nT) \) for \( n = 0, 1, 2, 3, \ldots \) and \( T \) is the cardiac period [5]. Mathematically, the pulsatile pressure \( p(t) \) and flow rate \( Q(t) \) can be expressed by the truncated Fourier series:

\[ p(t) = \bar{p} + \sum_{k=1}^{4} \alpha^p_k \cos(k \omega t) + \beta^p_k \sin(k \omega t), \]
\[ Q(t) = \bar{Q} + \sum_{k=1}^{4} \alpha^Q_k \cos(k \omega t) + \beta^Q_k \sin(k \omega t), \]

where \( \omega = \frac{2\pi}{T} \) is the angular frequency with the period \( T \), \( \bar{p} \) and \( \bar{Q} \) are the mean pressure and the mean flow rate, respectively. The values of \( \bar{p}, \bar{Q}, \alpha^p_k, \beta^p_k, \alpha^Q_k, \) and \( \beta^Q_k \) are as shown in Table 1.

In Figure 2, on the inflow surface \( \Gamma_{\text{aorta}} \) of aorta, we let the pulsatile velocity condition is

\[ \mathbf{u}_n(t) = \frac{Q(t)}{A}, \]

where \( A \) is the surface area of inlet boundary.

On the outflow surfaces including \( \Gamma_{\text{aorta}}, \Gamma_{\text{RCA}}, \Gamma_{\text{RCA}}, \Gamma_{\text{LCA}}, \Gamma_{\text{LCA}} \), and \( \Gamma_{\text{LCA}} \), we impose the corresponding pulsatile pressure condition, i.e.,

\[ p(t) = p_0(t) - \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)^2. \]

No-slip condition is applied to the outer arterial wall.
Table 1: Values of the parameters $\alpha_n^Q$, $\beta_n^Q$, $\alpha_n^p$, and $\beta_n^p$

<table>
<thead>
<tr>
<th>Vessel</th>
<th>n</th>
<th>$\alpha_n^Q$</th>
<th>$\beta_n^Q$</th>
<th>$\alpha_n^p$</th>
<th>$\beta_n^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aorta</td>
<td>1</td>
<td>1.7048</td>
<td>-7.5836</td>
<td>8.1269</td>
<td>-12.4156</td>
</tr>
<tr>
<td>$\bar{Q} = 5.7222$</td>
<td>2</td>
<td>-6.7035</td>
<td>-2.1714</td>
<td>-6.1510</td>
<td>-1.1072</td>
</tr>
<tr>
<td>$\bar{p} = 97.2222$</td>
<td>3</td>
<td>-2.6389</td>
<td>2.6462</td>
<td>-1.333</td>
<td>-0.3849</td>
</tr>
<tr>
<td>$A = 6.862712$</td>
<td>4</td>
<td>0.7198</td>
<td>0.2687</td>
<td>-2.9473</td>
<td>1.1603</td>
</tr>
<tr>
<td>LCA</td>
<td>1</td>
<td>0.1007</td>
<td>0.0764</td>
<td>-3.3107</td>
<td>-2.2932</td>
</tr>
<tr>
<td>$\bar{Q} = 0.1589$</td>
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<td>-0.0034</td>
<td>-0.0092</td>
<td>-9.8639</td>
<td>8.0487</td>
</tr>
<tr>
<td>$\bar{p} = 65.9722$</td>
<td>3</td>
<td>0.0294</td>
<td>0.0337</td>
<td>3.0278</td>
<td>3.8009</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0195</td>
<td>-0.0129</td>
<td>2.2476</td>
<td>-3.2564</td>
</tr>
<tr>
<td>RCA</td>
<td>1</td>
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<td>0.0241</td>
<td>5.9369</td>
<td>3.6334</td>
</tr>
<tr>
<td>$\bar{Q} = 0.0896$</td>
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<td>-0.0360</td>
<td>0.0342</td>
<td>-11.199</td>
<td>2.1255</td>
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<tr>
<td>$\bar{p} = 65.3333$</td>
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<td>0.0026</td>
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<td>-3.7528</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.0035</td>
<td>-0.0041</td>
<td>2.7333</td>
<td>-0.6375</td>
</tr>
</tbody>
</table>

In summary, the blood flow in the system of human coronary arteries is governed by the following boundary value problem.

**BVP:** Find $u$ and $p$ such that equations (1)-(2) and all boundary conditions are satisfied.

### 3 Numerical Example

In this study, we have simulated the three-dimensional blood flow in the stenosed coronary artery with and without bypass graft. The example under investigation is the system of human coronary arteries including the ascending part of the aorta, the RCA in which the stenosis presents at the proximal part, and the LCA. Figure 3 shows two examples of domain meshes where the area of the aorta inlet surface, $A$, is about 6.86 cm$^2$.

To investigate the effect of the stenosis severity, we study the pressure distribution along the arterial axis of the stenosed RCA during a cardiac cycle for the cardiac period, $T$, is 0.8 s. Four different degrees of stenosis including 39.13%, 49.95%, 62.69% and 71.85% are chosen. Finite element method based on COMSOL multiphysics was used to solve the problem of blood flow. The results indicate that the stenosis has a significant effect on blood pressure. We can explain this effect by comparing the pressure profile along the RCA axis as shown in Figure 4. It is noted that the blood pressure drops more the higher the stenosis is. Remarkably, pressure drops a lot when the narrowing of the artery is more than 50 percent.

To study how the coronary artery bypass grafting can improve the blood flow, the pressure distribution in the system of coronary arteries having 71.85% stenosis at the proximal part of the RCA is considered. Systolic pressure and diastolic pressure obtained from the model without and with bypass graft are shown in Figure 6 and Figure 7, respectively. Under the severe condition of the stenosis, abnormal pattern of pressure occurs. A high pressure drop causes an decrease in blood pressure in the stenosed artery. The model with a bypass graft causes an increase in blood pressure to the normal pressure. Figure 5 provides a comparison between the pressure along the axis of normal RCA and the axis of the 71.85% stenosed RCA with bypass graft.

### 4 Conclusion

Three-dimensional blood flow in the system of human coronary arteries with and without bypass graft is simulated. The effects of stenosis severity and bypass grafting on the blood pressure in the stenosed artery are investigated. The results shows that stenosis and bypass grafting have significant effects on the blood pressure. Higer degree of stenosis severity generates larger drop of the pressure. This causes an decrease of pressure in the stenosed artery. Bypass grafting can improve the blood pressure.

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Figure 2: Inlet and outlet surfaces of the computational models: (a) a model without a bypass graft, (b) a model with a bypass graft.

Figure 3: Domain meshes of the computational models: (a) the model without a bypass graft, (b) the model with a bypass graft.

References:


Figure 4: Pressure profile along the stenosed RCA having different degree of stenosis including 0% (the normal RCA), 39.13%, 49.95%, 62.69% and 71.85% at two different times: (a) the peak of systole, (b) the peak of diastole.

Figure 5: Pressure profile along the RCA axis at the peak of systole (solid line) and the peak of diastole (dashed line), obtained from the model having 71.85% stenosed RCA with a bypass graft and the normal RCA.


Figure 6: Pressure profile obtained from the model having 71.85% stenosed RCA without a bypass graft: (a) Systolic pressure in the model, (b) Pressure profile along the RCA axis at the peak of systole (solid line) and the peak of diastole (dashed line).

Figure 7: Pressure profile obtained from the model having 71.85% stenosed RCA with a bypass graft: (a) Systolic pressure in the model, (b) Pressure profile along the RCA axis at the peak of systole (solid line) and the peak of diastole (dashed line).