Customization of Wavelet Function for Pupil Fluctuation Analysis to Evaluate Levels of Sleepiness

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Abstract: This paper proposes a method to customize a wavelet function for the analysis of pupil diameter fluctuation in the detection of drowsiness states under a driving simulation. The methodology relies on a genetic algorithm–based optimization and lifting schemes, which are a flexible and fast implementation of the discrete wavelet transform. To customize the wavelet function a clustering separability metric is employed as a fitness function so that the feature space created by the wavelet analysis exhibits the maximum class separability favorable for classification. Therefore, a completely new wavelet function is created, having unique characteristics customized to pupil diameter fluctuation analysis. It is demonstrated that the customized wavelet function own distinguished frequency and temporal responses suitable specifically for pupil diameter fluctuation analysis (namely, application–dependent), and in the classification they outperform classical wavelet families including Daubechies, Coiflet and Symlet, which are assumed to be application–independent. Thus the proposed method is useful for analysis of pupil fluctuation in evaluating sleepiness levels, as has been demonstrated in other applications.

Key Words: Wavelet function customization, evolutionary optimization, pupil diameter fluctuation, drowsiness detection.

1 Introduction

Wavelet analysis, owning to its capability for representation of non–stationary biological signals, has inspired several methodologies in applications such as recognition of the nuclei of the basal ganglia from their neural spike trains [1], identification of epileptic seizures and classification of drowsiness states out of EEG [2, 3], recognition of hand movements by using EMG activity [4] among others. More recently, particular attention has been given to customized wavelet analysis that encloses the design of new wavelet mother functions [1, 5, 6, 7, 8, 9, 10]. Here, taking advantage in recent advances in wavelet customization, a methodology for designing application–dependent wavelet functions via genetic algorithms (GAs) are applied for evaluation of pupil diameter (PD) time series aiming at drowsiness assessment [1, 5].

During time–frequency analysis with the wavelet transform, the input signal is projected into an orthogonal or bi–orthogonal space created by the scaling and translation of one single wave–like function, called wavelet mother. It is comprehensible that the successful representation of the signal mainly depends on the proper selection of a wavelet mother. Although there are plenty of wavelet prototypes in the literature, there is not an established rule that states which wavelet should be used for each application. Instead, it is an usual task for the researcher to test more or less arbitrarily different wavelet functions to find one suited for the data to be analyzed [2]. However, existing wavelets may not be adequate for every application. For this reason several methods have been proposed to create new wavelets with desired features by either deterministic or evolutionary approaches. On one hand, deterministic methods have been shown to be burdensome due to the complexity of mathematical conditions on orthogonality, symmetry, compactness, and smoothness [6]. On the other hand, evolutionary approaches, such as GAs, cultural algorithms (CAs), and ant systems (AS), have been presented to have news-worthy features for wavelet design. Therefore, in the present study, an evolutionary methodology for creation of a customized wavelet function is employed. The main motivation behind, additional to avoiding the use of a generic function, is to create a wavelet function that achieve maximum class separability. In other words, given a feature space $\mathbf{Z} \in \mathbb{R}^{n \times d}$, where $d$ is the number of features and $n$ the number of signals analyzed that are discriminated in $k$ classes or clusters, and generated by wavelet decomposition (1)-(7), the methodology attempts at generating a wavelet function that maximize the separability of classes un-
under a desired cost function $D$, so that the feature space yielded by the wavelet function presents characteristics bound towards classification such as intra-cluster compactness.

To evoke the benefits of the customized wavelet function, the application under consideration in this study is the analysis of fluctuation of PD. The pupil that determines eye’s optical properties including depth of focus, and spatial frequency of the retinal image is controlled by two kinds of antagonistic muscles, the dilator and the sphincter, which are each innervated by two kinds of the autonomic nervous system, the sympathetic and the parasympathetic. These two muscles interact with each other to secure the proper co-action of the two kinds of smooth muscles has been described as highly non-stationary as well as non-linear [11]. What is more, the autonomic nervous system whose activity is known to change in parallel with drowsiness states may be also evaluated in PD time series[12]. To the best knowledge of the authors, few works have approached the wavelet analysis of PD time series [13].

This paper is organized as follows: In Sec. 2 the theory basis of the methodology is introduced. Then in Sec. 3.2 a novel wavelet function is produced for PD fluctuation analysis. Sec. 3.3 evinces classification performance with the newly customized wavelet function and for the sake of comparison classical wavelet functions are also evaluated. Finally, conclusion and ending remarks are drawn in Sec. 4.

<table>
<thead>
<tr>
<th>Table 1: Equations involved in the proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{X} = {\mathbf{x}_i : i = 1, \ldots, N} \in \mathbb{R}^{n \times N}$</td>
</tr>
<tr>
<td>$\Lambda = {\lambda_i : i = 1, \ldots, N} \in \mathbb{Z}^{1 \times N}$</td>
</tr>
<tr>
<td>$\mathbf{x}<em>{(0)} = \mathbf{x}</em>{(2i)}; \mathbf{x}<em>{(1)} = \mathbf{x}</em>{(2i-1)}$</td>
</tr>
<tr>
<td>$\mathbf{x}<em>{(l)} = \mathbf{w}</em>{a,2i} \mathbf{x}_{(l-1)}$</td>
</tr>
<tr>
<td>$\mathbf{p} = {p_i : i = 1, \ldots, N_p}$, $\mathbf{u} = {u_i : i = 1, \ldots, N_u}$</td>
</tr>
<tr>
<td>$\mathbf{w}<em>{d,i} = \mathbf{x}</em>{(l-1)} - \sum_{r=-N_p/2+1}^{N_p/2-1} p_r \mathbf{x}_{(l-1)}$</td>
</tr>
<tr>
<td>$\mathbf{w}<em>{a,i} = \mathbf{x}</em>{(l-1)} - \sum_{j=-N_u/2}^{N_u/2-1} u_j \mathbf{w}_{d,i,j-1}$</td>
</tr>
<tr>
<td>$p_i = p(i-1), u_i = u(i-1+1)$, $i = 1, 2, \ldots, N_u/2+1$</td>
</tr>
<tr>
<td>$\sum_{i=1}^{N_p/2} p_i = 1/2$, $\sum_{i=1}^{N_u/2} u_i = 1/4$</td>
</tr>
<tr>
<td>$\mathbf{Z} = {\mathbf{z}_i : i = 1, \ldots, d} \in \mathbb{R}^{d \times n}$</td>
</tr>
<tr>
<td>$\mathbf{S}<em>j = \left(\frac{1}{n} \sum</em>{i=1}^{N} \lambda_i(z_i) \mathbf{z}_i \right)$</td>
</tr>
<tr>
<td>$d_{i,j} = |\mathbf{v}_i - \mathbf{v}_j|_2$, $i \neq j$</td>
</tr>
<tr>
<td>$D_{DB}(\Lambda, \mathbf{Z}) = \frac{1}{k} \sum_{i=1}^{k} \max_{j=1}^{N} \left{\frac{S_i + S_j}{d_{i,j}}\right}$</td>
</tr>
<tr>
<td>$\gamma_1 = \mathbb{E}{\mathbf{x}^2 - (\mathbf{x}<em>{(i-1)} \mathbf{x}</em>{(i+1)})}$</td>
</tr>
<tr>
<td>$\gamma_2 = \sqrt{\mathbb{E}{\mathbf{x}^2 - \mu^2}}$</td>
</tr>
<tr>
<td>$\gamma_3 = \sqrt{\mathbb{E}{\mathbf{x}^2}}$</td>
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</tbody>
</table>

2 Methods

2.1 Wavelet Customization

The precise procedure for optimization of wavelet functions has been reported elsewhere [1]. Therefore, only a summarized version is presented here referring to equations given in Table 1.

First, extract a subset of training signals $\mathbf{X}_e \in \mathbf{X}$. It has been shown that a 30% sample of the patterns available provides enough information for the methodology to converge [1]. Second, each pattern is decomposed with the wavelet transform up to level $l$ by using cascade lifting schemes (3 to 7). Lifting schemes are a fast and flexible implementation of the discrete wavelet transform comprised by a prediction.
filter \( p \) and an update filter \( u \) with order \( N_p \) and \( N_u \), respectively (5). Prediction filter is associated to the wavelet function in the classical wavelet scheme, and is responsible for extracting high frequency components in the signal by eliminating low order polynomials, while the update operator is related with the scaling function and the extraction of approximation coefficients.

Multiresolutional analysis up to level \( l \) is achievable by taking sequential decomposition on the approximation coefficients (4), similar as in the wavelet discrete transform case. Once the wavelet transform is computed, a matrix \( W \) of wavelet coefficients is yielded. The dimension of such matrix happens to be heavy load for classification. Indeed, for a signal with \( N \) samples, performing an \( l \) scale wavelet analysis produces a matrix \( W_{1 \times N} \) discriminated in \( l + 1 \) frequency bands. To reduce the dimension of the wavelet representation a quantitative vector (10) is proposed and constructed by a feature extraction transformation, i.e., \( \mathcal{O} : W \rightarrow Z \). The set of metrics that assemble \( \mathcal{O} = \{ \gamma_i : i = 1, \ldots, d \} \) is entirely application–dependent. Such metrics are to be introduced for the application under consideration in Sec. 3.2. At this point a major step of the methodology must be set, namely, the cost function in the optimization procedure. Owing to the focus of the present content, classification of drowsiness states, the cost function is a measure that allows the evaluation of classification potential of \( Z \) without depending upon the classifier itself, which could increase considerably the procedure time and by which the methodology may also decrease generalization among classifiers. Consequently, the Davies–Bouldin index \( D_{DB} \) is employed (10 to 14) as the cost function because the \( D_{DB} \) is a measure of classes separability that assesses not only the compactness of each class (12) but also the global dispersion between classes (13). Therefore, by employing the Davies–Bouldin index given a crisp partition of the data \( \Lambda \), i.e., \( D_{DB}(\Lambda, Z) \) the filters \( u \) and \( p \) are expected to be evolved to maximize the separability among classes in the feature space \( Z \).

The terminating step of the procedure is the evolutionary optimization of the lifting scheme filters (5) by using GAs under constraints (8, 9). The constrains secure the linear phase, symmetry, compact support, and normalization of lifting filters, in other words, the associated wavelet and scaling functions. Furthermore, due to (8, 9) the GAs only evolve \( N_p/2 + N_u/2 - 2 \) values.

### 2.2 Driving Simulation Experiment

In a custom made driving simulator, a subject sat comfortably on the driver’s seat equipped with a steering wheel, and brake and accelerator pedals (Logitech PRO-11000) in a dark room. The subject wore a goggle (NEWOPTO ET-60-L) with 2 CCD cameras each of which takes infrared images of each eye at the frame rate of 29.97 fps (NTSC). Brightness and contrast of the projector that presents driving simulation images were adjusted for each subject so that the initial PD became in its intermediate size (around 6 mm). The simulation image generated by a PC is projected onto the screen at 2.64 m away from the subject’s eye. The horizontal and vertical visual angles of the image are \( \pm 12.1 \), and \( \pm 10.7 \) deg, respectively. Subjects were instructed to follow the car in front, driving straight on the straight road at the maximum speed (monotonous driving situation). Only the white lane markings and texture of lawn on the shoulder of the road move radially backward depending on the speed of the car during the simulation. Every two minutes the subject was asked to inform his/her sleepiness level according to the following three scales 0: awake, 1: not sure if sleepy or not, 2: sleepy. Subject recruitment and experimental procedures for this study conformed to the Declaration of Helsinki and all are approved by the human research committee of Chubu University. All subjects had given their informed consent.

### 2.3 Database

Using custom made scripts, PD was calculated from the video recordings, and errors in the PD detection due to blinks and saccades were eliminated automatically [12]. The eliminated periods of data were interpolated by a cubic function. Then 2-minutes segments were extracted and categorized according to the self–sleepiness level.

![PD fluctuation signal during the monotonous driving situation simulation of a typical subject (top), and reported self–sleepiness level (bottom).](image)
3 Results

3.1 PD During Driving Simulation

A total of 31 healthy subjects (mean SD: 22.8 ± 7.1 years old) participated in the experiment. Fig. 1 illustrates the change in PD during the monotonous driving situation simulation of a typical subject with a relatively steady state until 10 min followed by gradual decrease in average PD until 13 min and large low frequency fluctuation after that. In total there are 150 segments belonging to state 0 labeled as class A, 98 of state 1 labeled as class B, and 131 of state 2 referred to as class C. The classification task under consideration is a two–class problem given by class A and C (k = 2 classes, N = 3596 samples, n = 281 segments). Class B has not been included in the present study because the boundaries of awake and sleepy are sometimes hard to decide subjectively.

3.2 New Wavelet Function for PD Analysis

In the customization of the new wavelet function for the PD data a major step is the GA–based optimization. Regarding the GA, the following five parameters must be selected: (i) the arithmetic operator, (ii) the mutation operator, (iii) the population scale, (iv) the number of generations, and (v) the bounds of iteration. Parameters (i) to (iv) can be set directly from literature [1]. Besides, the arithmetic crossover and no uniform mutation operators are employed as recommended in [1]. For the sake of simplicity, the population scale is set to be 30, whereas the number of generations is set equal to 20. The iteration parameters of the GA are selected to range within the interval [−0.5, 0.5], which are possible values for the predictor and update coefficients that meet the normalization constraints (9).

The number of parameters to be optimized during the GA procedure for $N_p = N_u = 6$ is $2(N/2 - 1) = 4$. The wavelet decomposition level $l$ is set to 6 so that the frequency band spanned by $w_d^6$, $w_d^6$, and $w_d^5$ (approximation coefficient at level six, detail coefficients of level six and five), reside within the interval [0, 1] Hz where discriminant information related with respiratory components has been identified for pupil diameter [14]. The set of metrics that composed $Z$ are three, and given that only three wavelet spaces are considered, i.e., $w_d^6$, $w_d^6$, and $w_d^5$ ($d = 3 \times 3 = 9$): the average non–linear energy, also known as Teager’s operator (15), the root mean square value (16), and the variability of absolute coefficients (17). Such metrics were selected among a set of statistical moments employed in previous works [1]. Only three metrics are employed to allow...
the construction of three–dimensional feature spaces and hence to allow its visual inspection. To avoid local minima in the GA convergence, above steps are repeated ten times with random initializations. Thus, ten new wavelet functions are generated. The best mother wavelet is selected as the one with the largest value (14), for the current application the best one corresponded to the operators $u^* = [-0.033, -0.039, 0.322, 0.322, -0.039, -0.033]$, and $p^* = [0.013, -0.027, 0.514, 0.514, -0.027, 0.013]$. Fig. 2 depicts the new wavelet function (a and b) that was calculated from $u$ and $p$, respectively by using the reverse decomposition method [15], along with two classical wavelets, Daubechies and Symlet of order four and six, respectively (c). As can be seen the main advantage of the proposed methodology is that the symmetry, and compact support of the customized wavelet can be secure by employing the constrained optimization in the GA. Additionally, it is important to notice that since the training set $X_e$ is composed of actual PD data, the temporal and frequency response of $u$ and $p$ must have been customized to extract features in PD fluctuation, whereas the classical wavelets are non-symmetrical and do not exhibit major specialization on temporal and frequency characteristics of PD fluctuation [5].

### 3.3 Classification Performance

To evaluate the performance of the new wavelet function, classification scores of sleepy and awake states, classes C and A respectively, are calculated. Since a classifier method such as support vector machines or neural networks may increase the discrimination aptitude of $Z$, here a basic linear Bayes classifier is used so that the classification rates will not be enhanced by the classifier. Four classical wavelet families with different supports are considered for comparison: Daubechies of order two, four and six (db2, db4, db6), Symlet of order two, four and six (sym2, sym4, sym6), Biorthogonal of order two and four (bior2, bior4), Coiflet of order one and four (coif1, coif4). The results yielded by the best customized wavelet function is also shown in Table 2. Outcomes are presented in terms of the Type I and Type II classification error. The former evaluates the number of sleepy states which are misclassified, while the later evaluates the number of awake states classified as sleepy states.

Classification results showed that the best customized wavelet outperformed the classical wavelets for the current application in PD fluctuation analysis. In particular type I error achieved with the new wavelet is lower than the classical ones. Indeed, the classifier is able to reach low type I error rate (20%), in other words, of 40 sleepy states samples fed to the classifier, 8 were erroneously labeled as awake states (lower than reported scores [13]). It is important to notice that the bior2 wavelet achieved similar type I rate, however the type II error is higher than the customized one. Regarding the customized wavelet function that yielded the lower fitness function, the classification error type I and type II were 21.5% and 34.2%, respectively, which are lower than some of the classical wavelets.

### 3.4 Generated Feature Space

Fig. 3 depicts the two–dimensional feature spaces constructed with a classical wavelet function (bior2) (right) and with the customized wavelet function $u^*$ and $p^*$ (left).

![Figure 3: Two–dimensional feature spaces constructed with a classical wavelet function (bior2) (right) and with the customized wavelet function $u^*$ and $p^*$ (left).](image)

<table>
<thead>
<tr>
<th>Error</th>
<th>Customized</th>
<th>db2</th>
<th>db3</th>
<th>db4</th>
<th>sym2</th>
<th>sym4</th>
<th>sym6</th>
<th>bior2</th>
<th>bior4</th>
<th>coif1</th>
<th>coif4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>0.204</td>
<td>0.266</td>
<td>0.235</td>
<td>0.240</td>
<td>0.266</td>
<td>0.235</td>
<td>0.240</td>
<td>0.206</td>
<td>0.228</td>
<td>0.215</td>
<td>0.237</td>
</tr>
<tr>
<td>Type II</td>
<td>0.320</td>
<td>0.357</td>
<td>0.350</td>
<td>0.350</td>
<td>0.357</td>
<td>0.307</td>
<td>0.325</td>
<td>0.342</td>
<td>0.342</td>
<td>0.305</td>
<td>0.395</td>
</tr>
</tbody>
</table>

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Table 2: Type I and Type II Classification Error with Classical and Customized Wavelet Functions

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In the present study a new wavelet function has been designed specifically for the analysis of pupil fluctuation.
ation signal aiming at identifying subjective sleepiness levels. The designed wavelet outperformed classical wavelets in terms of less misclassification of sleepiness states, suggesting that it acquired unique features suitable to classify the pupil fluctuation data in different sleepiness conditions. Although the new wavelet still misclassified 20.4% of sleepy data as awake, the error rate should be significantly improved by employing more sophisticated classifier such as support vector machines, instead of a basic linear Bayes classifier currently employed. Moreover, the self-sleepiness level is somewhat variable in different subjects and even within a subject [12], and does not always correlate with physiological indicators of drowsiness [16]. Thus in the next study, we will try to classify gradual myosis and large low frequency fluctuation of PD exemplified in Fig. 1, which allows us not only to detect driver’s sleepiness but to predict it before the driver perceives own sleepiness [12].

References:


