Frequency Characteristics of Dielectric Periodic Structures

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Abstract: In this paper, Wave diffraction characteristics of dielectric periodic structures are systematically investigated. Oblique incidence, reflection characteristics and arbitrary grating profiles are analyzed by a method which combines the multimode network theory with the rigorous mode matching procedure. The variations of the total reflection characteristics with the geometric dimensions of the dielectric periodic structures and effect of the different groove profiles, such as rectangle, trapezoid on the reflection characteristics frequency selective surface (FSS) are comparatively studied.

Key-Words: Dielectric Periodic Structure, Oblique Incidence, Reflection Characteristics, Arbitrary Grating Profiles

1 Introduction

(Frequency selective surfaces) FSS are widely used in the millimetre wave frequency band. With the further study of the frequency selective surfaces composed of periodic patches or apertures on a conductive screen, it is found that the frequency selective surface consisting of metal periodic structures are not much suitable for the millimetre wave frequency band because of the high loss. In 1989, Bertoni etc. presented the feasibility of using periodic dielectric structures as a frequency selective surface in the millimeter wave frequency band. Due to the advantage of low loss, little material requirement compared with metallic screens and flexible for modifying the frequency selective reflection characteristics by changing the structure parameters, dielectric periodic structures have been becoming a desired frequency selective surface for millimetre wave application.[1]

FSSs are widely used in the MMW frequency band. At present, dielectric periodic structures have been becoming a desired FSS for MMW applications due to the advantage of low loss, little material requirement compared with metallic patch screens and flexible for modifying frequency selective reflection characteristics by changing structure parameters. Among the structure parameters, the periodic groove profile has a remarkable effect of frequency selective characteristics of the dielectric periodic structures; therefore, study of frequency selective reflection characteristics of dielectric periodic structures with arbitrary groove profiles is of greatly practical significance.

2 Analysis

The dielectric periodic structure under consideration is showed in Fig.1. Periodic dielectric structure with arbitrary groove profile having the thickness \( t_g \) and the dielectric constant \( \varepsilon_d \) is sandwiched between two semi-infinite air regions. [2]

When a plane wave with a propagation factor \( K_0 \) is incident at a fixed angle \( \theta \) from the upper space, according to the Floquet Theorem, each periodic dielectric layer \( j = 1, 2, \ldots N \) supports an infinite set of Eigen modes, which are different in different layers. In each periodic layer, the Eigen mode propagates with different propagation factor \( K_m( m = 0, \pm 1, \pm 2 \ldots) \) along the \( z \) direction and the same fundamental longitudinal propagation factor \( K_{0z} = K_0 \sin \theta \) along the \( x \) direction due to the continuous condition of the tangential fields at the interfaces. The field of each mode consists of an infinite number of space harmonics; and the propagation factor along the \( x \) direction, \( K_{0x} \) for
each space harmonic satisfies \( K_{mn} = k_{0} + 2\pi n/d \) (n=0, ±1, ±2, ...) – The fields in the air regions are modified to satisfy the periodic boundary conditions, i.e., the propagation factor along the x direction, \( k_{xm} \), also satisfies \( K_{xm} = k_{0} + 2\pi n/d \) (n=0, ±1, ±2, ...) while the propagation factor along the z direction, \( K_{zn} \), satisfies \( K_{zn} = K_{0} + k_{d} \) (\( a \) denotes the two air regions). According to the theory of multimode network, the whole dielectric periodic structure can be analyzed by an equivalent multilayer dielectric periodic structure with multimode network representation shown as Fig 2, where the fields in the air regions should be represented by an infinite number of transmission lines; each line stands for one space harmonic [3]. The tangential components of electric and magnetic fields for the TE mode polarization in all the regions can be determined with following procedure. (Here, a time dependence \( \exp(i\omega t) \) is assumed and suppressed for simplicity)

\[
E_{x}(x, y) = \sum_{n} t_{n} \exp(-ik_{xn}x - ik_{zn}^{(a)}z) \\
-H_{x}(x, y) = \sum_{n} Y_{n}^{(a)}t_{n} \exp(-ik_{xn}x - ik_{zn}^{(a)}z)
\]

Where, \( t_{n} \) expresses the amplitude of the nth transmitted plane wave. In the region \( 0 \leq z \leq g \), the fields in the jth periodic layer (j = 1, 2... N) are the superpositions of all the space harmonics of the Eigen modes in the layer. [5]

\[
E_{j}(x, y) = \sum_{m} \left[ \sum_{n} \left[ \exp(-ik_{xn}x - ik_{zn}^{(a)}z) \right] \right] a_{mn}^{j} \exp(-ik_{xn}x - ik_{zn}^{(a)}z) \\
-H_{j}(x, y) = \sum_{m} \left[ \sum_{n} \left[ \exp(-ik_{xn}x - ik_{zn}^{(a)}z) \right] \right] b_{mn}^{j} \exp(-ik_{xn}x - ik_{zn}^{(a)}z)
\]

Where, \( a_{mn}^{j} \) and \( b_{mn}^{j} \) are respectively the amplitudes of the forward and backward waves of the mth mode in the jth layer. \( d_{mn}^{j} \) is the amplitude of the nth space harmonic of the mth modes in the jth layer, which can be obtained with the procedure given in[1] \( a_{mn}^{0} \) can be determined by the conventional modal condition

\[
\sum_{n} |a_{mn}^{j}|^2 = 1
\]

\[
\eta_{mn}^{j} = k_{zm} \mu_{0}
\]

\( K_{zm} \) is the solution of the following dispersion relation

\[
\cos k_{x0}d = \cos k_{x1}d_{1} \cos k_{x2}d_{2} \cos k_{x3}d_{3} \cos k_{x4}d_{4} \\
- \frac{1}{2} \left[ (Y_{1}' / Y_{2}') + (Y_{2}' / Y_{1}') \right] \sin k_{x3}d_{3} \cos k_{x4}d_{4} \cos k_{x5}d_{5} \\
+ \left( Y_{1}' / Y_{2}' + Y_{2}' / Y_{1}' \right) \cos k_{x5}d_{5} \sin k_{x4}d_{4} \cos k_{x6}d_{6} \\
+ \left( Y_{2}' / Y_{1}' + Y_{1}' / Y_{2}' \right) \cos k_{x6}d_{6} \cos k_{x5}d_{5} \sin k_{x4}d_{4} \sin k_{x5}d_{5}
\]

(3)

\[
K_{zm}^{j} = \left[ \left( k_{0}^2 e_{z}^{j} - k_{zm}^2 \right) \right]^{1/2} k_{zm} \sqrt{e_{z}^{j}} \geq |K_{zm}| \\
\left[ i \left( k_{zm}^2 - k_{0}^2 e_{z}^{j} \right) \right]^{1/2} k_{zm} \sqrt{e_{z}^{j}} \leq |K_{zm}|
\]

\[
Y_{i}' = k_{zi} / \omega \mu_{0}
\]
According to the boundary condition that the tangential components of the fields are continuous at the interfaces between every two periodic layers, and after some mathematics manipulations the following matrix equations are obtained:

$$R = P^{-1}(B^j - QF^j)$$

$$PU = F^j - QB^j$$

$$P = 2(I^1 + Y^a V^1)^{-1} Y^{(a)}$$

$$Q = (I^1 + Y^a V^1)^{-1}(I^1 - Y^{(a)} V^1)$$

where, $R$ is a reflection matrix, $U$ is a unit matrix with elements $\delta_{nm}$, $F'$ and $B'$ are matrices with elements $\gamma_m \delta_{nm}$, $b'_m \delta_{nm}$, the amplitudes of forward and backward waves of the mth modes in layer $j=1$, $Y^{(a)}$ is the elements of the diagonal matrix $Y^{(a)}$ are determined by (1), while the elements of the voltage and current matrices $V$, $\Gamma$ are determined as:

$$V^j = \left(a^j_{mm}\right)$$

$$I^j = \left(\eta^j_{mm} a^j_{mm}\right)$$

Where, $a^j_{mm}$, $\eta^j m$ is determined respectively by (2), (3). At the interface of $z = T_j$,

$$T = V^n E^n F^n + V^n \left(E^n\right)^{-1} B^n$$

$$B^n = E^n W E^n F^n$$

$$W = \left(I^n + Y^{(a)} V^n\right)^{-1} \left(I^n - Y^{(a)} V^n\right)$$

$$E^n = \left(\delta_{nn}, \exp(-ikj_{mn} t_j)\right)$$

Where, $T$ stands for transmission matrix $F^n$ and $B^n$ are diagonal matrices which are determined by the amplitudes of forward and backward waves $f_{nm}$, $b_{nm}$ in the Nth layer, while $V^n$ and $\Gamma$ are determined by the amplitudes of the space harmonics of the modes in the Nth layer [6]. In the region of $0 \leq z \leq l_j$, at each interface between layer $j$ and $j+1$

$$F^j = \left(\Gamma^j E^j\right)^{-1} \left(F^{j+1} + \Gamma^j B^{j+1}\right)$$

$$B^j = E^j \left(\Gamma^j F^{j+1} + B^{j+1}\right)$$

$$C^j = 2\left(\eta^j (V^j)^{-1} V^{j+1} + (V^j)^{-1} I^{j+1}\right)$$

$$\Gamma^j = \left(\eta^j (V^j)^{-1} V^{j+1} + (V^j)^{-1} I^{j+1}\right)$$

The elements of $\eta^j$ are determined by (3), the definitions of other matrices are similar to those of the matrices in the layer $j=1$, N. Therefore, in order to obtain matrices $R$ and $T$, the matrices $F^1$, $B^1$, $F^n$, $B^n$ must be determined first. According to the theory of network, the relation of $F^1$, $B^1$ ($j=1, \ldots, n$) can be expressed in terms of scattering matrix $S$, i.e:

$$\begin{pmatrix} F^1 \\ B^1 \end{pmatrix} = S \begin{pmatrix} F^{j-1} \\ B^{j-1} \end{pmatrix}$$

$$S^j = \begin{pmatrix} (C'E^j) \left(C'E^j\right)^{-1} \Gamma^j \\ E^j (C'^j)^{-1} \Gamma^j E^j (C'^j)^{-1} \end{pmatrix}$$

And the relation between $F^1$, $B^1$ and $F^n$, $B^n$ can be given by cascading $S$:

$$\begin{pmatrix} F^n \\ B^n \end{pmatrix} = S \begin{pmatrix} F^1 \\ B^1 \end{pmatrix} = \begin{pmatrix} S_{11} S_{21} & F^1 \\ S_{21} S_{22} & B^1 \end{pmatrix}$$

$$S = S^1 \ldots S^{j+1} S^j S^{j-1} \ldots S^1$$

From (4), (5), (6), (7) and (8), the reflection matrix $R$ and the transmission matrix $T$ can be solved.

### 3 CONCLUSION

In order to verify the effectiveness and accuracy of the present approach, the reflection characteristics of the sinusoidal groove profile are calculated. A comparison between the results obtained using the unimoment method and those obtained with present approach is shown in table I, where $P^2_1$, $P^2_1$, $P^2_0$ are respectively the transmitted powers of the space harmonics ($n=-2,-1,0$) for the $m=0$ mode, while $P_E$ is the total power [7].
As we know, at lower frequencies, only the lowest \( m=0 \) mode propagates along the \( z \) direction, while other modes are all below cut off; so the periodic layer acts approximately as if it was uniform layer with the uniform dielectric constant. When the frequency is rising to the millimetre wave frequency band, both \( m=0, m=-1 \) modes propagates along the \( z \) direction in the layer with the largest dielectric constant.

As examples, four different periodic groove profiles, namely rectangle, trapezoid, increasing cosine and triangle profiles, are analyze to show the effect of groove profiles on frequency selective reflection characteristics of dielectric periodic structure. The half periods of the four groove profiles are sketched in Fig. 3(a), (b), (c), (d), where the shallow regions denote dielectric regions with the dielectric constant \( \varepsilon_d \), the thickness \( t_g \) and period \( d \). The aspect ratio \( T \) is definite as

\[
T = \left( d - 2d_0 \right) / d.
\]

In the computation, the incident angle \( \theta = 45^\circ \) is fixed; the thickness \( t_g \) and the period \( d \) are all normalized as \( t_g / \lambda_0, d / \lambda_0 \) and \( t_g / \lambda_0 = 2.037 \) is taken for all calculations. Here, \( r_0 \) is the reflection coefficient of the \( m,n=0 \) mode.

**Table 1** Comparison between unimoment method and the present approach (U: unimoment method, M: approach in this paper)

<table>
<thead>
<tr>
<th>( d/\lambda_0 )</th>
<th>( P_{c2} )</th>
<th>( P_{c1} )</th>
<th>( P_{c0} )</th>
<th>( P_{s2} )</th>
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<tbody>
<tr>
<td>1.333</td>
<td>U 0.241</td>
<td>0.547</td>
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<tr>
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References:


