Prediction Problem’s Solution for the Finite Possibilistic Model of Expert Knowledge Streams

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Abstract: The solution of the prediction problem is presented for the finite possibilistic modelling [3,4,6,7]. A recurrent variant of finite possibilistic models is considered. In this variant, we define the regularization condition for constructing a quasi-optimal estimator of fuzzy transition operator (FTO). We construct the discrete recurrent extremal fuzzy process with possibilistic uncertainty, the source of which is an expert knowledge stream on the states of some evolutionary complex system. Expert knowledge stream is created on the basis of Dempster-Shafer temporalized belief structure. Based on the non-probabilistic utility theory the example is presented to illustrate the prediction problem solution for expert knowledge streams.

Key-Words: possibilistic modelling, fuzzy process, expert knowledge stream, Dempster-Shafer belief structure, prediction problem

1 Introduction

In recent years, both the dynamics of fuzzy system and its modelling problem have attracted significant attention. Dynamics is an obvious problem in optimization. Applications of the dynamics of fuzzy systems and of the modelling of dynamic systems by fuzzy systems range from physics to biology, economics, pattern recognition and time series prediction.

Evidence exists that fuzzy models can explain cooperative processes, such as in biology, chemistry, material sciences, or in economics. Relationships between dynamics of fuzzy systems and the performance of decision support systems were found, and chaotic processes in various classes of fuzzy systems were proved as a powerful tool in analyzing complex, weakly structurable systems, as abnormal and extremal processes.

In alternative classical approaches of modelling of the complex systems the main accent is placed on the assumption of fuzziness. As the complexity of systems increases, our ability to define their behaviour exactly drops to a certain level, below which such characteristics of information as imprecision and uncertainty become mutually excluding. In such situations an exact or stochastic quantitative analysis of real complex systems is apt not to be quite plausible. Hence the system approach to constructing models of complex systems with fuzzy uncertainty guarantees the creation of computer-aided systems forming the instrumental basis of the intelligent technology solutions of expert-analytic problems. It is obvious that the source of fuzzy-statistical samples is the population of fuzzy characteristics of expert knowledge.

Our research is concerned with quantitative-fundamental analysis of this uncertainty and its use for precision of information processes and decision modelling. Consequently one of main objects of our attention is the analysis of structures of expert knowledge. The most important of such analysis methods are the theory of the body of evidence [2,5]
which are the main instrument used in some decision making systems.

The objective of this research is to construct or modify existing heuristic methods of expert knowledge analysis and engineering based on some knowledge representations. Several methods can be chosen which show some credibility in practice for wide range of problems (medical diagnosis, business, marketing, management, information management, etc.). Based on the approach developed in this paper these methods will model more precise decisions for expert knowledge streams. The precision of decisions first of all means improvement of representation of decision making factors by the Dempster-Shafer Belief structure [2,5]. The novelty of our research is the technology for precision of the structure of body of evidence, which we call the temporalization of body of evidence. Temporalization means the construction of information precision relation on the bodies of evidence. This means the following: the data which is an input of considered methods will be represented using Dempster-Shafer structures and pessimistic-optimistic representations of knowledge. This will better exhibit the knowledge and intellectual activities of an expert.

After some heuristic method of decision making obtains temporal structure, its inputs become expert knowledge streams with the purpose of making more precise decisions about the states of system. The process of decision making, which condenses the imprecision and uncertainty of expert knowledge streams, can be considered as a fuzzy process [3,4,6,7], and used to continue the dynamics of the temporalization process using finite possibilistic model of expert knowledge streams. Based on [7], the fuzzy recurrent process, the input data of which is the temporalized process, is constructed. The dynamics of possibilistic modelling is described and the constructed model is converted to the finite model. A genetic algorithm approach is developed for identifying the transition operation of the EFDS finite model. This is why we use the prediction problem after identification problem. Thus we will obtain final and more precise decisions, where possible alternatives are sorted and ordered as the forecasts.


For illustration the use of temporalization process in the non-probability utility theory is discussed. An example on the optimal choice of the students project’s versions is presented.

## 2 Construction of a Finite Model

Let’s start describing objects of the finite possibilistic extremal model of expert knowledge streams [3,4,6,7]. Let

\[ D = \{ d_1, d_2, \ldots, d_n \}, \]

be the set of possible states (in our case possible decisions) of the evolutionary system, observed by the expert. The fuzzy trajectory of expert knowledge reflections on the possible decisions, with respect to some fuzzy term, given in certain time moments, will have the following form [6,7]:

\[
\{ \tilde{K}_t = \tilde{K}_s(d,t); \quad \tilde{K}_0^* = \tilde{K}_0^*(d,t) \},
\]

\[ t = 0,1, \ldots, s, \quad d \in D. \]  

(2)

Let’s introduce the following finite set of compatibility levels for some natural number N:

\[
F_N = \left\{ 0, \frac{1}{N}, \ldots, \frac{N}{N} \right\} \subset [0,1].
\]

(3)

In models of [7], we consider only those fuzzy transition operators \( \tilde{\rho}_N \), which belong to the following set:

\[
\Phi_N = \{ \tilde{\rho}_N \mid \tilde{\rho}_N : D^2 \rightarrow F_N \}. \]

(4)

This way, we have some kind of discretization of FTO. Thus, our goal is to construct the FTO from \( \Phi_N \) in which the following deviation achieves its minimum (regularization condition):

\[
\Delta = \sum_{d \in D} \sum_{t=0}^s \left[ | \tilde{K}_N(d,t) - \tilde{K}_0^*(d,t) | + \right.
\]

\[
\left. + | \tilde{K}_N(d,t) - \tilde{K}_s(d,t) | \right],
\]

(5)

where we use the finite possibilistic extremal model of expert knowledge stream (2) [6,7]:

\[
\tilde{K}_N(d,t) = \{ \tilde{\rho}_N(d',d) \land \tilde{K}_N(d',t-1) \} \circ \tilde{\rho}_N(d,t-1); \quad d \in D, \quad t = 1, 2, \ldots, s;
\]

(6)

with initial conditions:

\[
\tilde{K}_N(d',0) \equiv \tilde{K}_s(d',0); \quad \tilde{K}_N^*(d',0) = \tilde{K}_0^*(d',0);
\]

\[ d' \in D. \]
In (6) we use possibility and necessity measures [1] with their distribution, derived from the expert knowledge stream (2):

\[
\hat{P}_{os_{t-1}}(B) = \frac{\nabla \hat{K}_{N}(d,t-1)}{\nabla \hat{K}_{N}(d,t-1)}.
\]

\[
\hat{N}_{ec_{t-1}}(B) = 1 - \frac{\nabla \hat{K}^*(d,t-1)}{\nabla \hat{K}^*(d,t-1)}.
\]

Note, that the modelled extremal fuzzy process of expert knowledge stream (2) cannot be used directly as prediction model in many cases, because of, as already mentioned, the complexity of the process [3,4]. That is why we introduce a new additional fuzzy uncertainty which is the notion of prediction in the fuzzy time intervals [3,4,7].

If we follow the analysis of construction of extremal fuzzy processes in fuzzy time intervals from [3,4], we can use the fuzzy intervals and monotone measures of its uncertainty.

If in some prediction interval \([s, \tau]\), let \(\tilde{K}_{N}(d,t) \approx 0\), \(\tilde{K}_{N}(d,t') \equiv 1\), then \(\tau' \geq \tau\).

In this case we can define continuous prediction regularized extremal fuzzy process in the interval \(s \leq \tau_0 \leq \tau\) [3,7]:

\[
\tilde{Q}_{N}(d,\tau_0) = \tilde{Q}_{N}^*(d,\tau_0) \circ \tilde{g}_T(\cdot) = \int_{[s,\tau_0]} \tilde{K}_{N}^*(d,t') \circ \tilde{g}_T(\cdot) = \int_{[s,\tau_0]} \tilde{K}_{N}^*(d,t') \circ \tilde{g}_T(\cdot)
\]

where \(\tilde{K}_{N}(d,t)\) and \(\tilde{K}_{N}^*(d,t)\) are corresponding possibility and necessity measures, \(\forall B \subset D\).

\[
\tilde{K}_{N}(d,t) = \tilde{K}_{N}^*(d,t) = \tilde{K}_{N}(d,t) = \tilde{K}_{N}^*(d,t).
\]

Obviously, for each \(d \in D\) the trajectory of extremal fuzzy process of expert knowledge stream \(\tilde{K}_{N}(d,t) ; \tilde{K}_{N}^*(d,t)\) is monotone:

\[
[\tilde{K}_{N}(d,t); \tilde{K}_{N}(d,t)] \subseteq [\tilde{K}_{N}(d,t-1); \tilde{K}_{N}^*(d,t-1)],
\quad t = 1,2,\ldots, \tau (8)
\]

3 Prediction Problem

Note, that the modelled extremal fuzzy process of expert knowledge stream (2)

\[
[\tilde{K}_{N}(d,t); \tilde{K}_{N}^*(d,t)], \quad t = s + 1, s + 2, \ldots, \tau (9)
\]
possibilistic level of state \( d \in D \) is the interval \( \left[ Q^*_a \left(d, \tau_0 \right), \tilde{Q}^*_a \left(d, \tau_0 \right) \right] \) which has \( \varepsilon \) accuracy. Of course, \( \tau_0 \) depends on \( d \in D \) : \( \tau_0 = \tau^{d}_0 \). Let us introduce the notion of stopping of prediction time moment: \( \tau^{opt} = \max(\tau_0^n, \tau_0^n, \ldots, \tau_0^n) \) such that the intervals of possibility levels of \( d_1, d_2, \ldots, d_n \) states don’t intersect. If \( \tau^{opt} \leq \tau \), this means that stopping has been performed in prediction interval, and we can sort the possibility levels and the decision \( d \) with highest level will become the prediction at approximately \( \tau^{opt} \) moment. This problem is opt-stopping problem and the mechanism of choosing \( \varepsilon \) is simple. This means that in possibilistic distribution

\[
\begin{bmatrix}
[\tilde{Q}^*_a \left(x_1, \tau^{opt} \right), \tilde{Q}^*_a \left(x_1, \tau^{opt} \right)] & [\tilde{Q}^*_a \left(x_2, \tau^{opt} \right), \tilde{Q}^*_a \left(x_2, \tau^{opt} \right)] & \cdots & [\tilde{Q}^*_a \left(x_n, \tau^{opt} \right), \tilde{Q}^*_a \left(x_n, \tau^{opt} \right)]
\end{bmatrix}
\]

(12)

the possibilities are represented by intervals, the length of which are minimal and intervals don’t intersect. Thus we have the possibility to sort them according to their levels.

4 Construction of an Expert Knowledge Stream

In this section we construct an expert knowledge stream based on the temporalized procedure [5] of decision making person’s utilities. The temporalized procedure is a process of construction of more précised expert evaluations.

Let us consider a utility model in the case of non-existent information on initial step of temporalization total ignorance of the body of evidence can be taken as input, \( \Omega_{\tau=t} \) is some known operator and \( A_{j+1}^t \) are focal elements on the bodies of evidence at the temporalized step \( t+1 \) [2].

Note, that for each \( d \in D \), in extremal trajectory \([\hat{K}_a \left(d, t \right), \hat{K}^* \left(d, t \right)]\), \( t = 0, 1, \ldots, s \) the expert knowledge stream is condensed concerning the decisions, as from the point of view of imprecision as well as of uncertainty. Its dynamic model is represented by (6).

5 Example

Here we will state the problem, which needs the precision of decisions in dynamics using created hybrid (temporalization + fuzzy process modelling) method.

The authors of the paper have an experience of working with the students of graduate masters
module in “intelligent information systems”, in which students work on group projects, which involves the evolution, control, engineering and management of simulation models for studied complex systems. The students always create several versions of project and we face the complex problem of choosing the best version of project for implementation, because it is usually very hard to figure out the role of each student in the group and their utility. Also, we have to take into account the fact that each student is working in several groups. After studying the versions of projects we have the possibility to consider the levels of competency of each student concerning the implementation of the project and evaluate each student by utility levels for each given version of project.

In one of the cases we were dealing with the estimation of the financial state of some business organization. The estimation on linguistic variable is represented by several fuzzy terms, which represent the output of fuzzy control system. The input information was the objective-statistical data – linguistic variables, which influence the financial state of organization. After detalization of the problem, we found out that the number of input linguistic variables was 14. Their fuzzification was performed and the students elaborated three versions of project of construction of the system for the same input and output information: 1) the fuzzy logic rules, corresponding knowledge base and the decision support system must be built using MatLab Fuzzy-Logic ToolBox; 2) the fuzzy rules, knowledge base, architecture and interface all will be developed using the programming language C#; 3) the body of control system - the transaction between input and output variables will be developed using fuzzy relations and their compositions. Corresponding software also developed using C#.

Thus three versions of project have been created \( D = \{d_1, d_2, d_3\} \) in which 7 students were participating, say \( \Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\} \). All seven of them participated in development of all three versions, but in different sub-groups, as often happens in engineering and management of simulation modelling. They created four groups (in future called focal elements):

1. \( A_1 \) - the group for problem detalization, gathering of input data, its initial processing and construction of conceptual model;
2. \( A_2 \) - the group for conceptual model validation and software development;
3. \( A_3 \) - the group for software verification and testing;
4. \( A_4 \) - the management group.

The students were divided into the subgroups in the following way: \( A_1 = \{\omega_1, \omega_3, \omega_4\} \), \( A_2 = \{\omega_2, \omega_3, \omega_5, \omega_6\} \), \( A_3 = \{\omega_1, \omega_2, \omega_6, \omega_7\} \), \( A_4 = \{\omega_4, \omega_6, \omega_7\} \).

After some time the students presented all three variants of the project \( (d_1, d_2, d_3) \). We had to choose the best one with the objective of optimal realization of the problem. We had to evaluate the utilities of students concerning each version. So we had to study the projects in detail. Also we had to consider students’ competences and knowledge in given topics, the quality and reliability of the realization of projects, the ability to work in groups, etc.

The first estimations were made just according the brief overview of projects. The results are as follows (results are normalized in \([0,1]\) interval and normalized utilities present some possibilistic levels, Table 1):

Table 1. The utility evaluations on the initial step of temporalization – \( \hat{U}_0 \).

<table>
<thead>
<tr>
<th>( D )</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>( \omega_3 )</th>
<th>( \omega_4 )</th>
<th>( \omega_5 )</th>
<th>( \omega_6 )</th>
<th>( \omega_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>0.4</td>
<td>0.7</td>
<td>0.6</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>0.6</td>
<td>0.8</td>
<td>0.5</td>
<td>0.9</td>
<td>0.6</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>0.3</td>
<td>0.9</td>
<td>0.4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.7</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Obviously, the comparison of projects \( d_i \), \( i = 1, 2, 3 \) and selection of best one by their utilities is impossible. Let’s use temporalization. Let us construct the extremal valuations \( \{\hat{U}_0, \hat{U}_0^*\} \) (formula (13)) on \( \Omega \) by introducing \( A_1^0, A_2^0, A_3^0, A_4^0 \) etalons (Table 2):

Table 2. Extremal utilities on initial step of temporalization – \( \{\hat{U}_0, \hat{U}_0^*\} \).

<table>
<thead>
<tr>
<th>( D )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>[0.4; 0.8]</td>
<td>[0.4; 0.7]</td>
<td>[0.4; 0.7]</td>
<td>[0.6; 0.7]</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>[0.5; 0.9]</td>
<td>[0.5; 0.8]</td>
<td>[0.6; 0.8]</td>
<td>[0.7; 0.9]</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>[0.3; 0.7]</td>
<td>[0.3; 0.7]</td>
<td>[0.3; 0.9]</td>
<td>[0.6; 0.7]</td>
</tr>
</tbody>
</table>
On initial step let us suppose, that focal probabilities have uniform distribution $m_0(A_i) = 0.25, i = 1, 2, 3, 4$. Then, if we use formula (13), we will obtain the following extremal utilities for the project variants (Table 3):

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.45, 0.725]</td>
<td>[0.575, 0.850]</td>
<td>[0.375, 0.800]</td>
</tr>
</tbody>
</table>

As seen from Table 3, it is impossible to choose the best decision among $d_1, d_2, d_3$, which would be natural, because their expected utility intervals $(\hat{K}_*, \hat{K}^*)$ intersect.

Let us move on the first step of temporalization: from the contents of problem and without the loss of generality we suppose that etalons $A'_1, A'_2, A'_3, A'_4, t = 1, 2, \ldots$ don’t change on each step of temporalization and the possible interval of their focal probabilities is $0.10 \leq m_1 = m(A_i) \leq 0.50$.

On the first step of temporalization we suppose, that the table of $\hat{U}$ – utilities (Table 1) don’t change. Then the system (14) obtains the concrete form, the numerical solution of which gives us the results: $m_1(A'_1) = 0.14; m_1(A'_2) = 0.16; m_1(A'_3) = 0.33; m_1(A'_4) = 0.37$.

The expected extremal utilities are as follows (Table 4):

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.47, 0.715]</td>
<td>[0.605, 0.850]</td>
<td>[0.405, 0.800]</td>
</tr>
</tbody>
</table>

We continue the temporalization: After obtaining additional information on the competences and professional knowledge of the students, on the second step of temporalization the table of utilities $\hat{U}$ changed in the following way (Table 5):

Table 5. The utility evaluations on the second step of temporalization – $\hat{U}_2$.

The table of extremal utilities looks the following way (Table 6):

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
<th>$\omega_6$</th>
<th>$\omega_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>0.45</td>
<td>0.75</td>
<td>0.55</td>
<td>0.65</td>
<td>0.6</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.65</td>
<td>0.85</td>
<td>0.85</td>
<td>0.55</td>
<td>0.7</td>
<td>0.75</td>
<td>0.7</td>
</tr>
<tr>
<td>$d_3$</td>
<td>0.4</td>
<td>0.85</td>
<td>0.45</td>
<td>0.65</td>
<td>0.7</td>
<td>0.75</td>
<td>0.6</td>
</tr>
</tbody>
</table>

On this step we obtain $m_3(A'_1) = 0.14$; $m_3(A'_2) = 0.16$; $m_3(A'_3) = 0.40$; $m_3(A'_4) = 0.30$.

The expected extremal utilities are as follows (Table 7):

<table>
<thead>
<tr>
<th>$D$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>[0.45; 0.65]</td>
<td>[0.5; 0.75]</td>
<td>[0.45; 0.75]</td>
<td>[0.5; 0.65]</td>
</tr>
<tr>
<td>$d_2$</td>
<td>[0.55; 0.85]</td>
<td>[0.7; 0.85]</td>
<td>[0.65; 0.85]</td>
<td>[0.55; 0.75]</td>
</tr>
<tr>
<td>$d_3$</td>
<td>[0.4; 0.65]</td>
<td>[0.45; 0.85]</td>
<td>[0.4; 0.85]</td>
<td>[0.6; 0.75]</td>
</tr>
</tbody>
</table>

On the third step of temporalization we estimate that, the table of $\hat{U}$ – utilities (Table 5) does not change, and the the numerical solution of (14) gives us: $m_4(A'_1) = 0.14$; $m_4(A'_2) = 0.36$; $m_4(A'_3) = 0.17$; $m_4(A'_4) = 0.33$.

The expected extremal utilities are (Table 8):

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\hat{d}_1$</th>
<th>$\hat{d}_2$</th>
<th>$\hat{d}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>[0.475, 0.700]</td>
<td>[0.607, 0.815]</td>
<td>[0.4775, 0.785]</td>
</tr>
<tr>
<td>$d_2$</td>
<td>[0.485, 0.700]</td>
<td>[0.617, 0.815]</td>
<td>[0.485, 0.748]</td>
</tr>
<tr>
<td>$d_3$</td>
<td>[0.485, 0.700]</td>
<td>[0.617, 0.815]</td>
<td>[0.485, 0.748]</td>
</tr>
</tbody>
</table>

The process of précising the information enabled us to continue the temporalization until the 7-sth step ($s = 6$). The trajectory of précising the knowledge streams is given in the Table 9.
Table 9. The extremal fuzzy process of expert knowledge precision

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\hat{K}_s(\cdot, 0); \hat{K}_N^*(\cdot, 0)]$</td>
<td>[0.450, 0.725]</td>
<td>[0.575, 0.850]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 1); \hat{K}_N^*(\cdot, 1)]$</td>
<td>[0.47, 0.715]</td>
<td>[0.605, 0.850]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 2); \hat{K}_N^*(\cdot, 2)]$</td>
<td>[0.475, 0.700]</td>
<td>[0.607, 0.815]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 3); \hat{K}_N^*(\cdot, 3)]$</td>
<td>[0.485, 0.700]</td>
<td>[0.617, 0.815]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 4); \hat{K}_N^*(\cdot, 4)]$</td>
<td>[0.485, 0.689]</td>
<td>[0.635, 0.779]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 5); \hat{K}_N^*(\cdot, 5)]$</td>
<td>[0.485, 0.572]</td>
<td>[0.692, 0.725]</td>
</tr>
<tr>
<td>$[\hat{K}_s(\cdot, 6); \hat{K}_N^*(\cdot, 6)]$</td>
<td>[0.486, 0.532]</td>
<td>[0.692, 0.725]</td>
</tr>
</tbody>
</table>

Table 10. The modelled extremal fuzzy process of expert knowledge precision

<table>
<thead>
<tr>
<th>Modeling step $t$</th>
<th>$[\tilde{K}_sN(d, t); \tilde{K}_N^*(d, t)]$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$[\tilde{K}_sN(\cdot, 0), \tilde{K}_N^*(\cdot, 0)]$</td>
<td>[0.450, 0.725]</td>
<td>[0.575, 0.850]</td>
<td>[0.375, 0.800]</td>
</tr>
<tr>
<td>1</td>
<td>$[\tilde{K}_sN(\cdot, 1), \tilde{K}_N^*(\cdot, 1)]$</td>
<td>[0.468, 0.723]</td>
<td>[0.623, 0.847]</td>
<td>[0.413, 0.786]</td>
</tr>
<tr>
<td>2</td>
<td>$[\tilde{K}_sN(\cdot, 2), \tilde{K}_N^*(\cdot, 2)]$</td>
<td>[0.469, 0.711]</td>
<td>[0.628, 0.824]</td>
<td>[0.478, 0.761]</td>
</tr>
<tr>
<td>3</td>
<td>$[\tilde{K}_sN(\cdot, 3), \tilde{K}_N^*(\cdot, 3)]$</td>
<td>[0.471, 0.689]</td>
<td>[0.632, 0.809]</td>
<td>[0.492, 0.753]</td>
</tr>
<tr>
<td>4</td>
<td>$[\tilde{K}_sN(\cdot, 4), \tilde{K}_N^*(\cdot, 4)]$</td>
<td>[0.481, 0.677]</td>
<td>[0.647, 0.781]</td>
<td>[0.507, 0.704]</td>
</tr>
<tr>
<td>5</td>
<td>$[\tilde{K}_sN(\cdot, 5), \tilde{K}_N^*(\cdot, 5)]$</td>
<td>[0.484, 0.604]</td>
<td>[0.661, 0.748]</td>
<td>[0.516, 0.663]</td>
</tr>
<tr>
<td>6</td>
<td>$[\tilde{K}_sN(\cdot, 6), \tilde{K}_N^*(\cdot, 6)]$</td>
<td>[0.489, 0.572]</td>
<td>[0.668, 0.729]</td>
<td>[0.523, 0.619]</td>
</tr>
<tr>
<td>Prediction time period</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$[\tilde{K}_sN(\cdot, 7), \tilde{K}_N^*(\cdot, 7)]$</td>
<td>[0.491, 0.551]</td>
<td>[0.670, 0.723]</td>
<td>[0.525, 0.613]</td>
</tr>
<tr>
<td>8</td>
<td>$[\tilde{K}_sN(\cdot, 8), \tilde{K}_N^*(\cdot, 8)]$</td>
<td>[0.494, 0.549]</td>
<td>[0.674, 0.719]</td>
<td>[0.528, 0.607]</td>
</tr>
<tr>
<td>9</td>
<td>$[\tilde{K}_sN(\cdot, 9), \tilde{K}_N^*(\cdot, 9)]$</td>
<td>[0.497, 0.534]</td>
<td>[0.679, 0.715]</td>
<td>[0.532, 0.602]</td>
</tr>
<tr>
<td>10</td>
<td>$[\tilde{K}_sN(\cdot, 10), \tilde{K}_N^*(\cdot, 10)]$</td>
<td>[0.499, 0.521]</td>
<td>[0.682, 0.711]</td>
<td>[0.535, 0.598]</td>
</tr>
<tr>
<td>11</td>
<td>$[\tilde{K}_sN(\cdot, 11), \tilde{K}_N^*(\cdot, 11)]$</td>
<td>[0.502, 0.519]</td>
<td>[0.687, 0.709]</td>
<td>[0.541, 0.591]</td>
</tr>
<tr>
<td>12</td>
<td>$[\tilde{K}_sN(\cdot, 12), \tilde{K}_N^*(\cdot, 12)]$</td>
<td>[0.508, 0.518]</td>
<td>[0.691, 0.705]</td>
<td>[0.544, 0.587]</td>
</tr>
<tr>
<td>13</td>
<td>$[\tilde{K}_sN(\cdot, 13), \tilde{K}_N^*(\cdot, 13)]$</td>
<td>[0.511, 0.516]</td>
<td>[0.693, 0.704]</td>
<td>[0.548, 0.582]</td>
</tr>
<tr>
<td>14</td>
<td>$[\tilde{K}_sN(\cdot, 14), \tilde{K}_N^*(\cdot, 14)]$</td>
<td>[0.513, 0.515]</td>
<td>[0.697, 0.704]</td>
<td>[0.550, 0.576]</td>
</tr>
<tr>
<td>15</td>
<td>$[\tilde{K}_sN(\cdot, 15), \tilde{K}_N^*(\cdot, 15)]$</td>
<td>[0.51]</td>
<td>[0.698, 0.703]</td>
<td>[0.553, 0.573]</td>
</tr>
<tr>
<td>16</td>
<td>$[\tilde{K}_sN(\cdot, 16), \tilde{K}_N^*(\cdot, 16)]$</td>
<td>[0.51]</td>
<td>[0.698, 0.703]</td>
<td>[0.553, 0.573]</td>
</tr>
<tr>
<td>17</td>
<td>$[\tilde{K}_sN(\cdot, 17), \tilde{K}_N^*(\cdot, 17)]$</td>
<td>[0.51]</td>
<td>[0.698, 0.703]</td>
<td>[0.553, 0.573]</td>
</tr>
<tr>
<td>18</td>
<td>$[\tilde{K}_sN(\cdot, 18), \tilde{K}_N^*(\cdot, 18)]$</td>
<td>[0.51]</td>
<td>[0.698, 0.703]</td>
<td>[0.553, 0.573]</td>
</tr>
<tr>
<td>19</td>
<td>$[\tilde{K}_sN(\cdot, 19), \tilde{K}_N^*(\cdot, 19)]$</td>
<td>[0.51]</td>
<td>[0.698, 0.703]</td>
<td>[0.553, 0.573]</td>
</tr>
</tbody>
</table>
Using the prediction problem’s solution scheme (sections 1-2) we obtain the result presented in Table 10. The Table 10 shows modelling and prediction results (the parameters are \( \varepsilon = 10^{-2} \); 
\( s = 6 \); \( \tau_{opt} = 19 \); \( N = 2^{10} \).

As seen from the Table 10, we can take \( \tau_{opt} = 19 \). Let us apply the formulas (11) and (12) and for time period [7; 19] construct prediction continuous extremal fuzzy process \( \tilde{Q}_N^*(x, \tau_0); \tilde{Q}_N^*(x, \tau_0) \), \( \tau_0 \in [7; 19] \); more precisely, predicted optimal possibilistic distribution (12) for approximately \( \tau_0 = 19 \) time moment.

In the examples from [3] the technique for calculating extremal integrals of type (11) is well described. If we construct integrable functions of (11) and apply distribution (12) we obtain possibilistic distribution at approximately \( \tau = 19 \) time moment:

\[
\begin{bmatrix}
 d_1 \\
 d_2 \\
 d_3 
\end{bmatrix} = \begin{bmatrix}
 0.503:0.509 \\
 0.700:0.703 \\
 0.558:0.562 
\end{bmatrix}
\]

Obviously, \( d_2 > d_3 > d_1 \) and decision \( d_2 \), or the second variant of the project can be supposed as the optimal one with about \( [0.700 \div 0.703] \) possibility level (if we use the maximum principle of defuzzification).

6 Conclusion

The new approach developed in the paper has conceptual character. Several heuristic methods of expert knowledge representation and decision making may be considered as a method for modeling of more precise decisions. Discrimination analysis, connectivity analysis, fuzzy covering analysis, fuzzy grades analysis, decision aggregated operators, A. Kaufman’s theory of expertons etc. can be modified and constructed in Dempster-Shafer temporalized structure (analogically to the non-dissonant theory of Utility described in this paper).

For each method: 1) dual representation of input expert data stream has to be constructed; 2) the more precise temporalized criteria of decision making has to be developed; 3) the relation of information precision has to be constructed as the solution of the problem of some mathematical programming problem.

As a second step of a construction of more precise valuations of decision criteria on the stream of temporalized belief structure, the modeling of a finite possibilistic model is used. At the output of the second step we receive the preference ordering of possible decisions by their précised levels of possibilities.

The results of this research enable us to create software technology, thus making performed research more valuable.

As an application of this approach we consider non-probabilistic theory of Utility. The discussion in this paper shows that heuristic methods can obtain more fundamental basis by introducing temporalization and possibilistic modeling of expert knowledge streams together, enabling them to be successfully used in real applications.

References: