LQG Control of a Semi-active Suspension System equipped with MR rotary brake

Asha Dharan, Silje Helene Olsen Storhaug, Hamid Reza Karimi
Department of Engineering, Faculty of Engineering and Science, University of Agder,
N-4898 Grimstad, Norway

Abstract- This paper deals with analyzing LQG control to mitigate vibrations in a semi-active suspension system (SAS) equipped with an MR rotary brake. The results of various simulations are studied and compared to the real system.

Keywords: LQG control, MR damper, semi-active suspension system

I. Introduction

The fundamental goal of a vehicle suspension is the isolation of the automobile from the road by means of a spring and damper –like arrangement attached to the wheel that will compensate for the uneven and bumpy parts of the road. This gives the passenger a smoother ride. Because of the various limitations of a normal suspension system, semi-active and active suspension systems are introduced. However, semi-active suspension systems are widely used because of their higher reliability, lower cost and comparable performance [1]. This paper includes studying different linear control methodologies for vibration control of the semi active suspension system (SAS) equipped with a MR brake.

II. Problem Description

Mechanical vibration in vehicles is generally tackled by the use of suspension systems. They are mostly used in mobile applications, such as terrain vehicles, or in non-mobile applications such as vibrating machinery or civil structures. The elastic element of a suspension is constituted by a spring whereas the damping element is typically of a viscous type. The damping action is obtained by throttling a viscous fluid through orifices. Depending on the physical properties of the fluid, the geometry of the orifices and of the damper, a variety of force versus velocity characteristics can be obtained. SAS systems can be considered as a combination of passive and active suspension systems. Semi-active control devices offer reliability comparable to that of passive devices, yet maintaining the versatility and adaptability of fully active systems, without requiring large power sources. In SAS systems the amount of damping can be tuned in real time.

The controllable part of the SAS system is its damper. Hence it is very important to model it accurately. Applying a feedback to the control can radically alter the dynamics of a system. It affects the natural frequency, transient response as well as the stability of the system.

There are many types of control methods for SAS systems. A few of them have been discussed here. The first example would be the PID Controllers. These controllers are frequently designed in the frequency domain, assuming that the system is linear. But it works well only if the assumption is close to the actual behaviour of the system. The performance of a PID controller can be optimized using a robust design or by including an adaptive loop.

The next example would be the adaptive type of controllers. These are used if the parameters of a process are time varying and a fixed-parameter controller would not yield acceptable results. Here the parameters are estimated on the basis of the measured plant signals and then used in the control action. Systems with constant or slowly varying parameters are best suited for this type of control [1].

The next type would be robust controllers. These controllers are meant to provide a good level of performance in systems with uncertain parameters, no matter how fast they vary. But it needs the information on the boundary conditions. These controllers have low sensitivities, are stable and continuous to meet its nominal specification over a typical range of parameter variations. The desirable features of a robust design in the frequency domain are the largest possible bandwidth and the largest possible loop gain [1].

Heuristic and Back Stepping methods are particularly used in SAS systems. With a heuristic control strategy the damper model is not needed. Information about the vibration measurements is enough. Knowing the position, x, and the velocity, v, of the deflection of the car body and the vibration induced by the road unevenness , it can be decided whether to give a control signal to the MR damper or not [3].The back stepping technique was developed around 1990 to design stabilizing controllers for special nonlinear dynamical systems. These systems are built from subsystems that radiate out from an irreducible subsystem that can be controlled or stabilized with other methods. The backstepping control is a recursive method of stabilizing the origin of the system, where the design process starts at a known-stable system. One can “back out” new controllers that stabilizes outer subsystems. In this way several control steps is designed until you get the ultimate controller [3].

LQR (Linear Quadratic Regulator Control) and LQG (Linear Quadratic Gaussian Control) are another type of controllers that would be studied in details here. The LQR control gives an optimal state feedback gain, which gives a minimum phase margin of 60° [4]. Using LQR control the cost function (1) should be minimized. This is done by solving the Riccati equation (2) and giving the control gain K as in (3).

$$J(u) = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (1)$$

$$0 = SA + A^T S - SBR^{-1}B^T S + Q \quad (2)$$

$$K = R^{-1} B^T S \quad (3)$$

The control gain is given to the input from the states, x, such that u = - K*x.

Q and R are the weighing matrices of the input and the states. They are to be positive definite, symmetrical with only positive eigenvalues.

The LQG [5] is a combination of a LQR controller with gain K and an observer (the Kalman Filter) with observer gain L [6]. This method has been used for experiment purpose in this paper.

III. Control Design

1. *Representation of Nonlinear Model for the SAS System*

For experiments in the design of a control system, a SAS system is set up. Amongst other components it contains a wheel and body connected. An electric motor is running the wheel, and a spring and a MR damper connects the upper and lower lever. Figure 1 shows SAS system at the laboratory at University of Agder with a rotary damper.

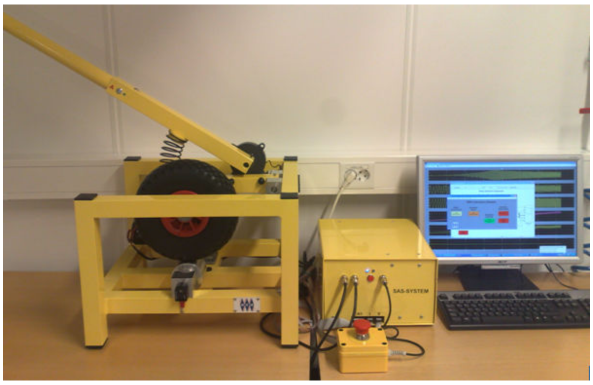

Figure 1. SAS with rotary damper in the lab [2].

A Simulink model of the SAS system is shown in Figure 2. Here the system is considered without Bouc-Wen model as it creates lot of complexities during the design of a control system.

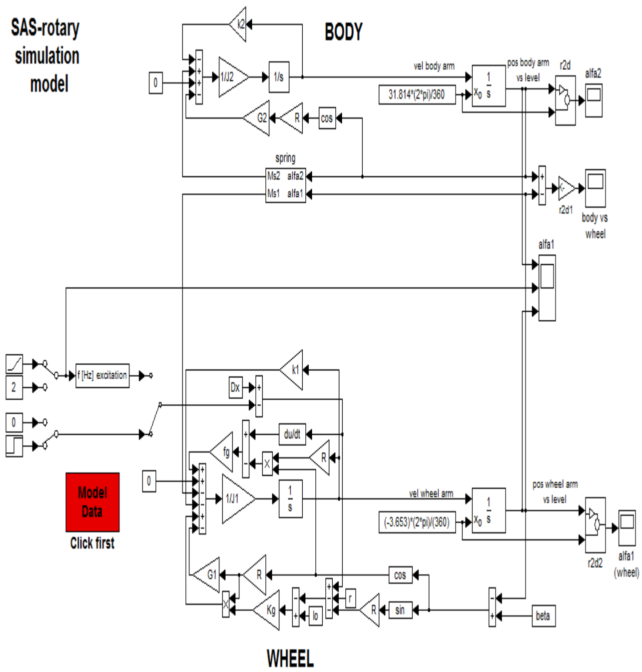

Figure 2. The SAS system Simulink model

2. *Linearization of the Model*

The input and output of the system needs to be defined before doing this step. It can be defined as a SISO (Single Input Single Output), SIMO (Single Input Multiple Output) or MIMO (Multiple Input Multiple Output) system. Here only SIMO systems are under consideration. The torque to the system is taken as the input to the system or plant and the angles of the body and wheel are taken as the output from the system. The model in Figure 2 is a nonlinear model. It is complex to make control systems for such systems. Hence it is converted into a linear system. This is done by using the matlab command:

```
[A B C D]=linmod ('sasr_model')
```

These values are the coefficients of the steady state equation:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \tag{4}$$

where x is the state variables and u is the input to the system.

2.1 Linearization of SIMO System:

For the SIMO (single-input multiple-output) system, the input u and output y are selected as follows:

$$u = [torque]$$

$$y = \begin{bmatrix} body\ angle \\ wheel\ angle \end{bmatrix}$$

The *linmod* command returns, respectively, the matrices of A, B, C and D as:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1.0 \\ 0 & 0 & 1.0 & 0 \\ 1192.0 & -5812.6 & -0.1281 & 0 \\ -86.2 & 91.2 & 0 & -.30 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 7.8585 \\ -0.8732 \end{bmatrix}$$

$$C = \begin{bmatrix} 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 57.2958 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = 0$$

The linearized model calculated by the *linmod*-command in Matlab gives a nonminimal representation of the system. We can minimize it by deleting the second and the last row in matrix C, which only contain zeroes. The new C obtained is:

$$C = \begin{bmatrix} 57.2958 & 0 & 0 & 0 \\ 0 & 57.2958 & 0 & 0 \end{bmatrix}$$

The values in matrix C are decided based on the type of output. Since here the angle is taken in degrees the values in the diagonal matrix C is 57.29, however it is changed to radians in some of the simulations in this project. In the case when the output is given in radians, the matrix C is:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

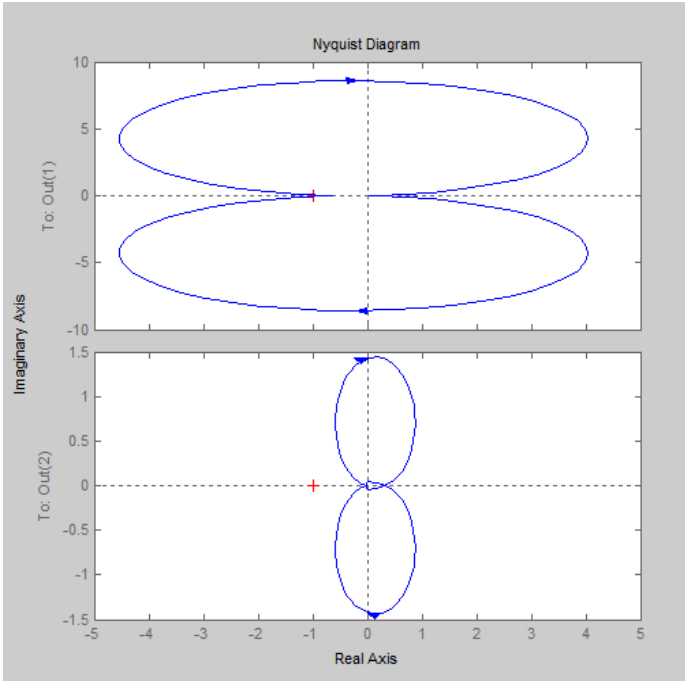

Figure 3. Nyquist plot of the open loop system

The Nyquist plot, Figure 3, shows that there are no encirclements around -1. This information together with no unstable open loop poles, gives a stable system. However it may not be fast, and a control methodology is needed to improve the performance.

ISBN: 978-1-61804-085-5

3. *Control Design of SIMO System*

The controllability of the SIMO plant is investigated according to the rank of the matrix C_M in the following:

$$C_M = 10^5 \begin{bmatrix} 0 & -0.0 & 0.0 & 0.008 \\ 0 & 0.0 & -0.011 & 1.057 \\ 0.0 & -0.011 & 1.057 & -72.711 \\ -0.0 & 0.0 & 0.008 & -1.031 \end{bmatrix}$$

The rank of the controllability matrix is found to be 4, which means that the system is controllable. Thus the rank is full.

The lqr control strategy is used for the controller. Then the weighing matrices Q and R have to be determined. When not knowing Q and R, a rule of thumb, Bryson's rule [7], may be to give them values according to (5) and (6) before tuning.

$$Q_{ii} = \frac{1}{Max(x_{ii}^2)} \tag{5}$$

$$R = \frac{1}{Max(u^2\ )} \tag{6}$$

The maximum value of the states is found by simulating with no input. R can initially be set to 1 and then tuned by finding the maximum input when a controller is included in the simulation.

Using this method the matrices Q and R are obtained as follows:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 39{,}0625 & 0 & 0 \\ 0 & 0 & 0{,}029727 & 0 \\ 0 & 0 & 0 & 0{,}020408 \end{bmatrix}$$

$$R = 16$$

However by simulating with the gain obtained from this, the result showed little improvement in damping. These weighing matrices were not so optimal.

To get a better result we tuned Q and R manually, and found that a dramatically different Q and R gave a far better result. The undamped system response is given in Figure 4 and the nice response in Figure 5 is with the tuned K. This gain is found as

$$K = [-43.17 \quad 480.63 \quad 28.197 \quad -47.1555]$$

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$R = 16$$

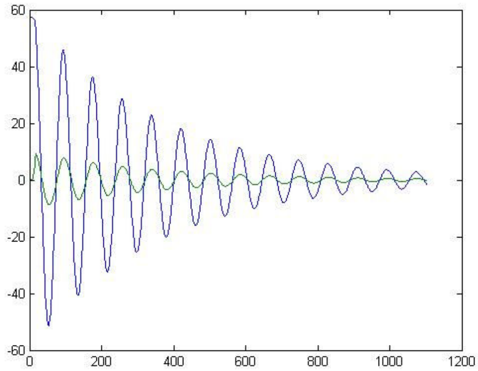

Figure 4. Uncontrolled SIMO system

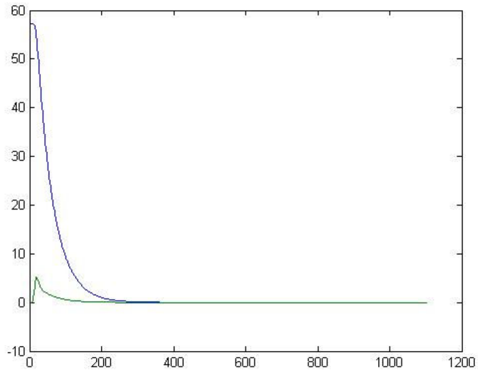

Figure 5. Controlled SIMO system

Using the SIMO system with radians instead of the degree outputs and the initial conditions expressed in the nonlinear model, a controller is design using lqr.

$$x_0 = \begin{bmatrix} 31.814 * (2 * pi)/360 \\ (-3.653) * (2 * pi)/(360) \end{bmatrix}$$

The matrix Q is at first estimated with Bryson's rule. However this alone did not give a satisfying result. Q11 and R is then modified to give a higher weighing on the first state and lower on the input. An increased q11 should give a faster response of x_1, which is exactly what is observed. The Q, R and K is then:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 129.132231 & 0 & 0 \\ 0 & 0 & 0{,}036559 & 0 \\ 0 & 0 & 0 & 0{,}051653 \end{bmatrix}$$

$$R = 0.1$$

$$K = [-43.127 \quad -16.608 \quad -0.102 \quad -9.9195]$$

With this control the damped response is shown in Figure 7, compared to the undamped response in Figure 6.

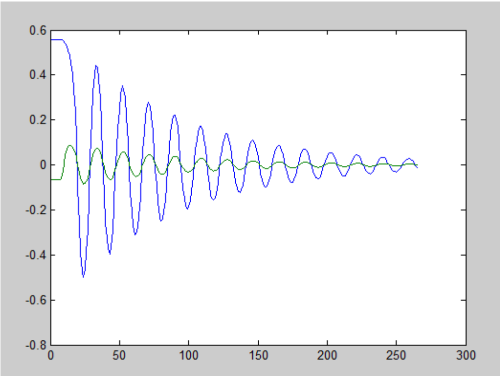

Figure 6. Uncontrolled SIMO system

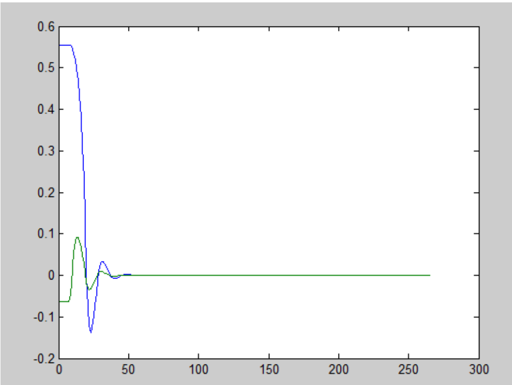

Figure 7. Controlled SIMO system from lqr

An observer is designed using the lqe-command in Matlab. Setting the covariance of the control and measurement noise to identity, an observer gain L is obtained (7).

$$L = \begin{bmatrix} 0.5850 & 0.0955 \\ 0.0955 & 0.0301 \\ 0.2328 & -0.4950 \\ -0.3243 & -0.1741 \end{bmatrix} \qquad (7)$$

This observer is used in experimental part.

IV. Experimental Results

After simulating the state space model of the plant, the controller is tested with the real system as shown in Figure 1 and Figure 8.

1. Evaluating the Original System

The first step during the experimental task was to run the system without any control signal to the damper. This was to make it possible to compare the controlled experiments with the uncontrolled.

Running the system without any damper signal, the body angle in Figure 9 was plotted. One can see that the system still damps itself. This is because of the spring and the passive part of the semi active damper. The body angle measured in this uncontrolled experiment reached amplitude above 10. This is used for the next comparisons.

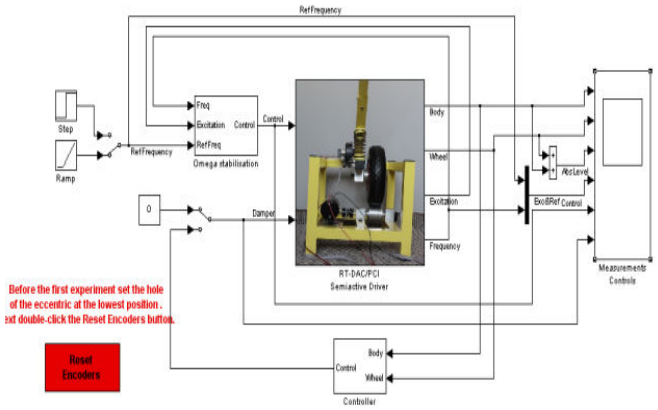

Figure 8. The experimental setup

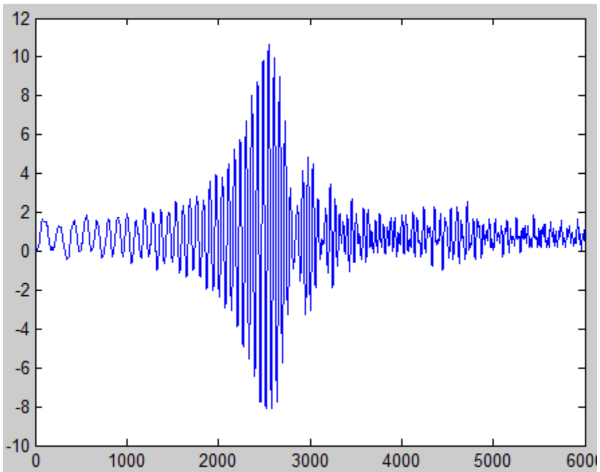

Figure 9. Experimental plot of original plant

2. Experiments with the SIMO System

The SIMO system (with output angles in radians) was connected to the SAS system to control it. At first the only control was the gain matrix K, and the results are given in Figure 10. The gain was calculated and is repeated in (8).

$$K = [-43.127 \quad -16.608 \quad -0.102 \quad -9.9195] \qquad (8)$$

The response was better than without control; the body angle is below 6 at all times. However the control does not seem to be optimal. The damper signal was very high, reaching above the limited value, making the saturation block limit it between 0 and 5.

ISBN: 978-1-61804-085-5

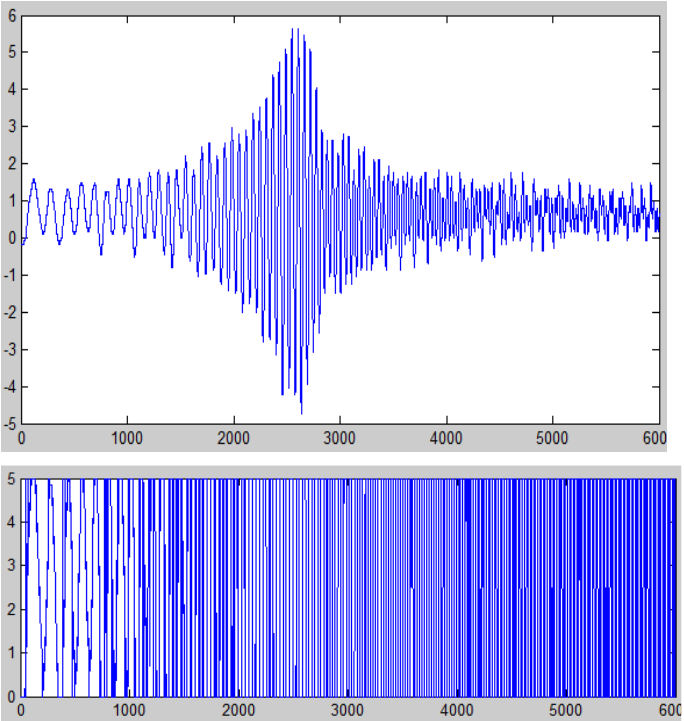

Figure 10. Body angle plot (upper) and damper signal (lower) for SIMO system

With an LQR control there is no observer. The disadvantage in this case is that all four states are needed. However only two of them are available from measurements, the body and the wheel angles. The other two states, the body and wheel velocities had to be calculated by differentiating the angles. This introduces noisy state-signals, which is a problem that would not have occurred by using a LQG control.

The LQG controller has the advantage of being able to control multivariable systems without measuring all the states. Because of this, and also to reduce the control and measurement noises, the LQG controller is introduced to the system. The kalman filter gain L was calculated and given in equation (7).

Introducing the observer and observer gain L in the control block, the results in Figure 11 were obtained. The linearized model was used as a copy of the plant for the observer.

The maximum body angle is reduced to about 2, which shows that it damps better with an observer than without. The damper control signal also seems much better, as it is smaller and gives a higher value when needed.

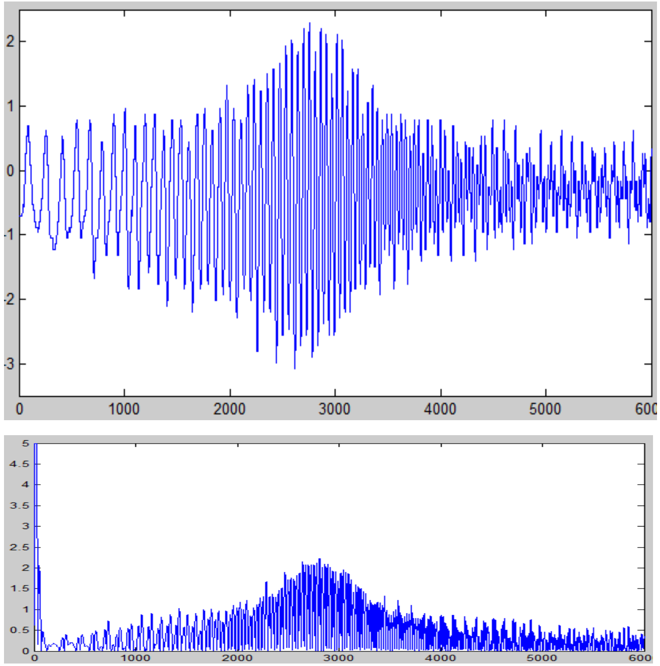

Figure 11. Body angle plot (upper) and damper signal (lower) for SIMO system with observer

When designing a controller, the aim is to get the nicest possible response with as little effort as possible. This means that the control current or voltage should be small, but give a high effect on the system. In this last experiment, including observer, the system damps much better and also the damping control signal is much smaller than before.

V. Conclusion

Based on the experiments conducted we can conclude that LQG controllers gave a satisfactory result in controlling the vibrations in the SAS system. This can be attributed to the presence of an observer.

VI. References

[1] Guglielmino, E.,Sireteanu,T.,Stammers,C.W.,Ghita,G.,Giuclea,M Semi-active Suspension Control-. 1st Edition, 2008.

[2] SAS System User's Manual, manufactured by INTECO Ltd. (Cracow, Poland).

[3] N. Luo, M. Zapateiro and H. R. Karimi, Heuristic ans backstepping control strategies for semiactive suspension in automotive systems equipped with mr dampers, 6th IFAC Symposium Advances in Automotive Control, (AAC 2010).

[4] C. Zhang and M. Fu, A Revisit to the Gain and Phase Margins of Linear Quadratic Regulators, Automatic Control, IEEE Transactions on, Oct 1996 Volume: 41, Issue:10.

[5] S. Chen, R. He, H. Liu, Y. Wang and M. Yao, An Optimal Design for Suspension Based on LQG Control, Electric Information and Control Engineering (ICEICE), 2011 International Conference on Issue Date : 15-17 April 2011.

[6] Q. Jin, S. Ren, L. Quan, Intelligent Computing and Intelligent Systems, 2009. ICIS 2009. IEEE International Conference on, Issue Date : 20-22 Nov. 2009 Volume : 3.

[7] J. P. Hespanha, Undergraduate lecture notes on lqg/lqr controller design, 2007.