Modeling and control design for a semi-active suspension system with magnetorheological rotary brake

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Abstract

In this paper we will first propose some methods for controlling a semi active suspension (SAS). The SAS with a magnetorheological rotary (MR) damper is highly non-linear and is hard to control. To make the controller, the system is slightly modified and linearized. The linearized model is then used to make an optimal controller for the system. LQR and LQG controllers are implemented and tested in Simulink. In addition to this a test on the physical system is made.

Keywords: MR damper; LQG control; SAS system

Introduction

The SAS system as shown in Figure 1 consists of an arm which works as the springmass (1), a spring (2) connects the wheel (4) and the springmass. In addition to the spring an angular MR damper (3) is mounted. The wheel tire works as a mass damper system when it is exposed to disturbance from the "ground", the eccentric metal wheel.

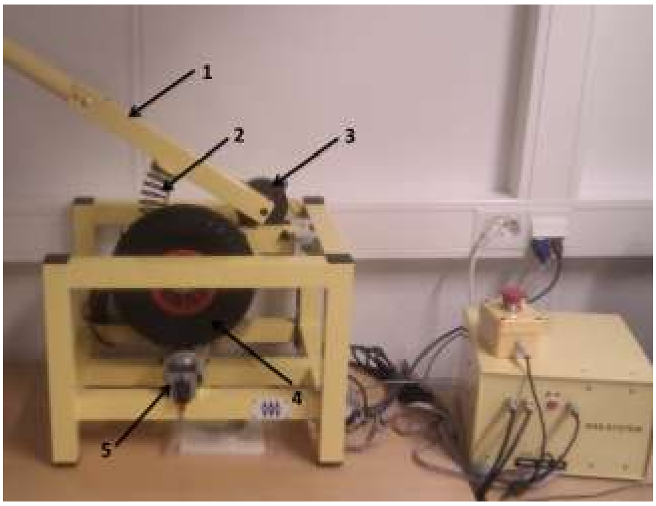

Figure 1: Semi active suspension system

Control methods

To control this SAS system these four methods could be used:

Semi active Skyhook Control Policy: [3]

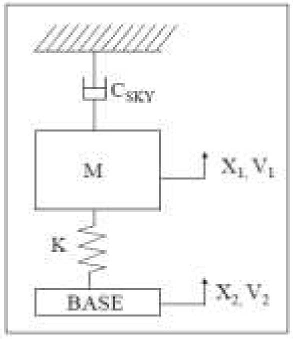

Figure 2: Skyhook damping model, **Skyhook**[1][2]

$$V_1 V_{12} > 0 \qquad F_{sa} = C_{SKY} V_1$$
$$V_1 V_{12} < 0 \qquad F_{SA} = 0$$

F_{SA} : Semiactive skyhook damper force
C_{SKY} : Skyhook damper
V_{12}: Velocity of suspended mass relative to base
V_1: Velocity of suspended mass.

We connect the damper to a inertial reference vertically fixed relative to a ground reference. [1]
This is switch controller and is often used in suspension systems in cars. The controller is switching between different gain-values dependent of the state variables.

LQR controller [7]: Optimal linear controller for plants with zero noise. The feedback controller is proven to be optimal when $u = -Kx$. This is a simple controller and easy to use when you don´t have any significant noise and all the states are visible.

LQG controller [7]: Optimal control of linear system with white Gaussian noise. The controller is common where linear system is uncertain (each state can be weighted by the quadratic matrix Q) and if the system has incomplete states (not all states is meshed). The LQG is a combination of a linear quadratic estimator (LQE) and a linear-quadratic regulator (LQR).

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Non-linear model of SAS [5]

T_{body} : Moment of inertia of upper beam due to MR damper

$$T_{body} = J_2 \frac{d^2\alpha_2}{dt^2}$$

$T_{lowbody}$: Potential energy of the lower body

$$T_{lowbody} = J_1 \frac{d^2\alpha_1}{dt^2}$$

$T_{viscous}$: Viscous friction damping force

$$T_{viscous} = k_2 \frac{d\alpha_2}{dt}$$

$T_{G\ lower}$: Potential energy of the lower beam

$$T_{G\ lower} = m_1 g_1 r_1 \cos(\beta - \alpha_1)$$

$T_{G\ upper}$: Potential energy of upper beam

$$T_{G\ upper} = m_2 g r_2 \cos\alpha_2$$

T_{DT} : Damping torque

$$T_{DT} = f_g \dot{x}$$

$$= f_g \left(\frac{d(D_x - U_{kin})}{dt} - \omega r\right) = f_g \left(\frac{d(D_x - U_{kin})}{dt} - \frac{d\alpha_1}{dt} R\cos(\beta - \alpha_1)\right)$$

T_{ST} : Actuating kinematic torque transferred through tire:

$$T_{ST} = -armk_g(x) = -k_g R\cos(\beta - \alpha_1)\left(l_{0g} + R\sin(\beta - \alpha_1) + r - D_X + u_{kin}\right)$$

T_{spring} : Spring torque

$$T_{spring} = r_2 k_s \left(l_{0s} - \sqrt{(r_2\cos\alpha_2 - r_1\cos\alpha_1)^2 + (r_2\sin\alpha_2 - r_1\sin\alpha_1)^2}\right)$$

T_{MR} : Torque in MR damper

The total torque equation for the upper beam:

$$\sum T = J\dot{\omega} = T_{body} + T_{G\ upper} - T_{spring} - T_{viscous} = T_{MR}$$

The total equation for the lower beam:

$$T_{lowbody} + T_{G\ lower} - T_{spring} - T_{viscous} - T_{ST} - T_{DT} = T_{MR}$$

We get two equations for the torque in MR damper. This model is pre constructed in Matlab Simulink and we will later try to linearize it to get a suitable controller/observer.

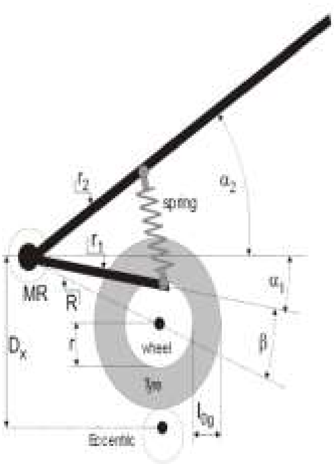

Figure 3: SAS system

α_2:	Angle between the upper beam and level
α_1:	Angle between the lower beam and level
$J_{1,2}$:	Moment of inertia of the lower and upper beam with respect to its axis of rotation
$k_{1,2}$:	Viscous friction coefficient of the lower and upper beam
$m_{1,2}$:	Mass of lower and upper beam
r_2,r_1:	The distances between the spring mount and the lower and upper beam
r:	Radius of the wheel rim
l_{0s}:	Length of the no-loaded spring
l_{0g}:	Length of the loaded spring
k_s:	The elasticity coefficient of the spring
k_g:	The elasticity coefficient of the tire
f_g:	The absorption coefficient of the tire
i:	The current of the rotary MR damper
R:	The distance between the beam pivots
u_{kin} :	The kinetic sinusoidal excitation
D_x:	The distance between the beam and the pivot of the eccentric wheel
β:	The angle between lower beam and the horizontal line

Linearization of SAS model

To simplify the SAS, the MR damper is described as a pure torque which is applied from our controller. The controlled torque (torque due to the change in oil viscosity in MR-damper) is simply added instead of the damping torque. The linearized model is obtained in Matlab Simulink by using the command "linmod". The model structure is:

$$\dot{x} = Ax + Bu$$

$$\dot{y} = Cx + Du$$

With linearization we get, A, B, C, D:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -84.7 & -0.3078 & 90.31 & 0 \\ 0 & 0 & 0 & 1 \\ 1181 & 0 & -7121 & -144.7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0.8732 \\ 0 \\ 7.8585 \end{bmatrix}, \qquad C = I_4, \qquad D = 0$$

The states are identified as:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \qquad \begin{matrix} x_1: angle, arm \\ x_2: velocity, arm \\ x_3: angle, wheel \\ x_4: velocity, wheel \end{matrix}$$

Simulated results in Simulink

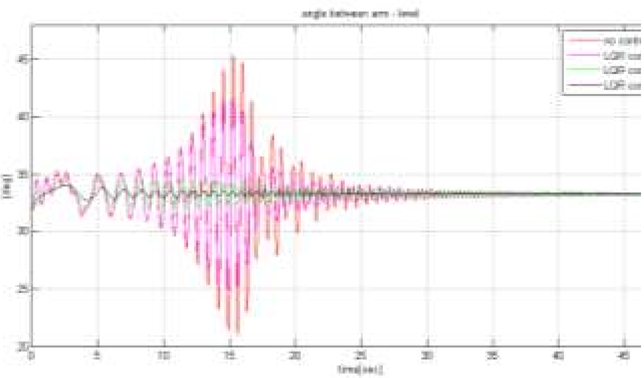

Figure 4: Response with different controllers

We simulate the SAS model in Simulink with a disturbance (rotation of the eccentric wheel) formed like a sine signal with increasing frequency from 0 to 5Hz, in 50 seconds. This

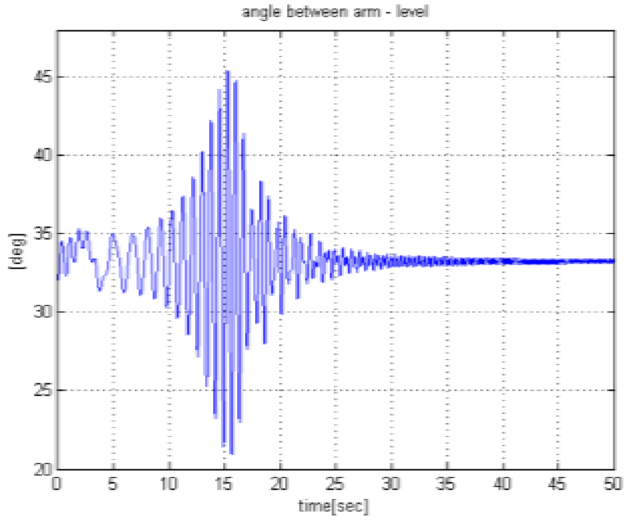

Figure 5: System response with ramped disturbance

gives us a varying response of the sprung mass. From Figure 3 we easily detect a critical area around the Eigen frequency. By implementing different control systems we want to reduce this effect.

- **Implementing a LQR controller**

Since we have the possibility to measure all our states in the Simulink model we choose to start with a simple LQR controller.

$$J = \int_0^\infty (x^T Q x + u^T R u) dt$$

Controller 1 has this cost function matrixes:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 1$$

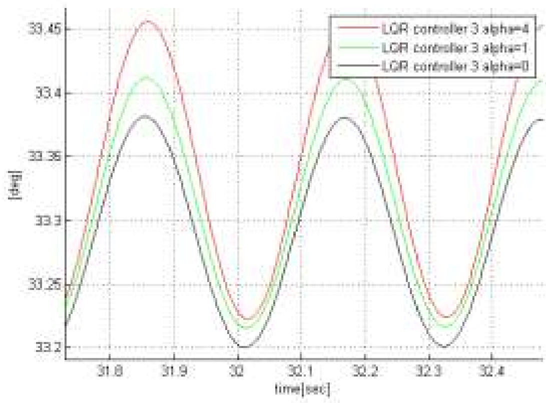

Figure 6: Different alpha

Controller 2 has this cost function matrixes:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 1$$

Controller 2 has this cost function matrixes:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 0.1$$

From these different cost functions, we manage to improve our damping. The results (see Figure 5) shows that from these three cost functions, the third controller has significant better results than the rest.

- **Implementing a LQR controller with exponential cost function**

The total cost is given by:

$$J = \int_0^\infty e^{2\alpha t}\,(x^T Q x + u^T R u) dt$$

α: Exponential constant, positive real

Implementing different values of α we got some new results (Figure 7). The new cost function was tested with alpha values from 0 and up to 4. The results show that the original controller 3 still is the best solution (see Figure 8).

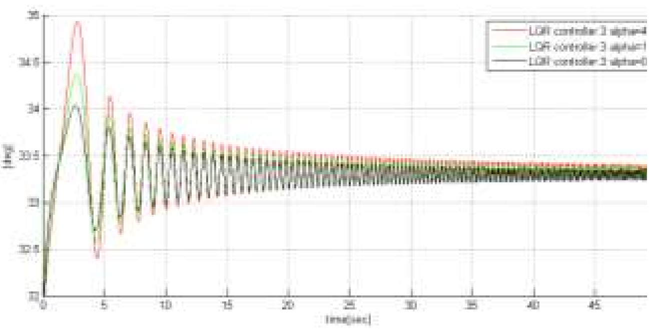

Figure 7: Different exponential values

- **Implementing an observer based controller**

The LQR controller works with the two measured angular positions and their derivatives for velocity. Because of the low resolution of the sensor, the velocity values are far from continuous.

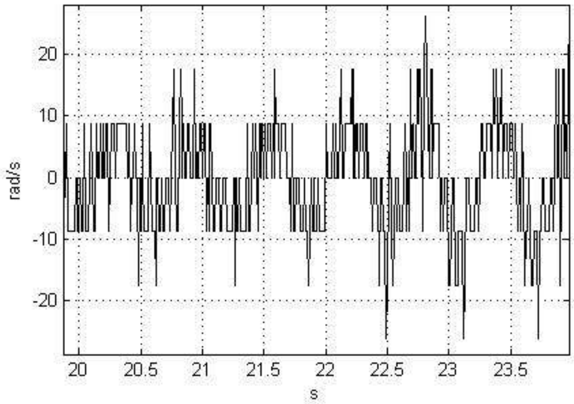

Figure 9: Velocity derived from position

An observer was created to check if this would result in a more accurate controller of the SAS system. This is a relatively high frequent system that requires a fast observer.

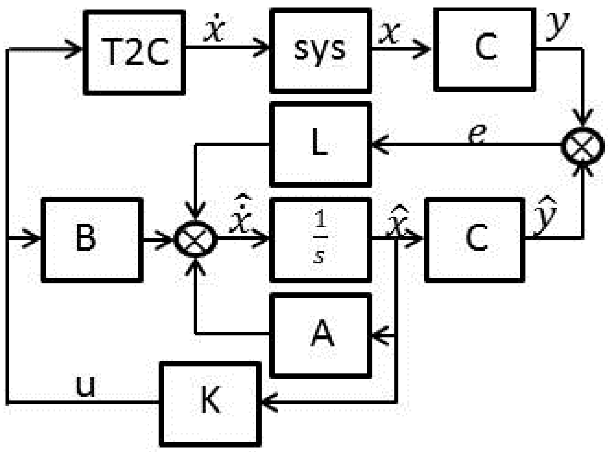

Figure 10: An observer based control.

To the observer was created using Matlab's PLACE-function which places the poles of the input system for higher, or lower, frequency. By placing the second-order poles of the observer 10 times higher than the dominant pole pair of the system the observer (green) becomes 10 times faster[8].

$$L = 10^3 \cdot \begin{bmatrix} 304 & 21.1 \\ 16.6 & -0.6 \\ 1.16 & 0.54 \\ 0.40 & 0.06 \end{bmatrix}$$

This observer was tested by simulating the SAS system with the wheel velocity increasing (top system).

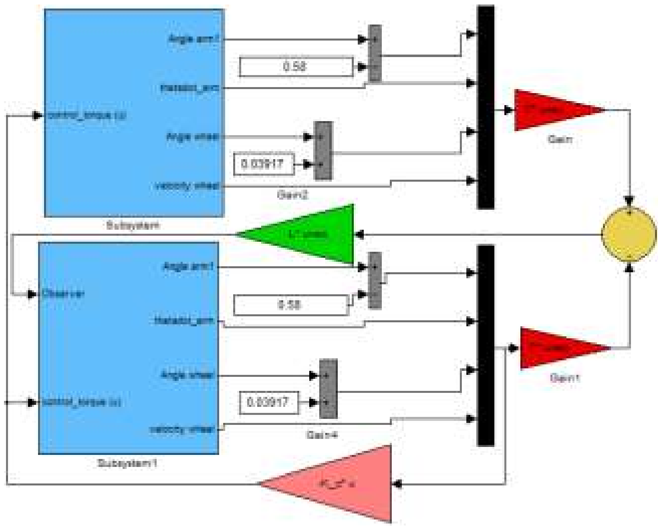

Figure 11: Test bench of observer based control

This system was controlled by a parallel observer (bottom system). The observer had no input on the wheel turning, only the two angles of the system. The observer was capable of emulating the system to a high degree of accuracy.

Experiment, control of real SAS system

- **Torque to current converter**

The controller is designed to control the damping torque. In the physical model, the control signal is the current. The current controls the mageetical field in the damper which controls the damping torque. Since our signal used in simulink is the control torque we have to convert the signal to the corresponding current. In this case we have chosen to use the bouc-wen model to represent the damper. Other models like Legre and dahl could also be used.

Bouc-Wen model equation[6]:

$$T_{MR} = (\alpha_0 i + \alpha_1) \cdot z(\dot{\theta}) + c_0 i + c_1$$

The Bouc-Wen equation with corresponding values for our specific damper:

$$T_{MR} = (8.0802 \cdot i + 1.8079) \cdot z(\dot{\theta}) + 0.0055 \cdot i\dot{\theta} + 0.0055 \cdot \dot{\theta}$$

$\dot{\theta}$: Relative angular velocity between arm and wheel.

z:: The Bouc-Wen parameter.

By changing this equation with the respect of the current:

$$\rightarrow i = \frac{T_{MR} - 0.0055 \cdot \dot{\theta} - 1.8079 \cdot z}{8.0802 \cdot z + 0.0055 \cdot \dot{\theta}}$$

From this equation a converter block in Simulink is made (Figure 12).

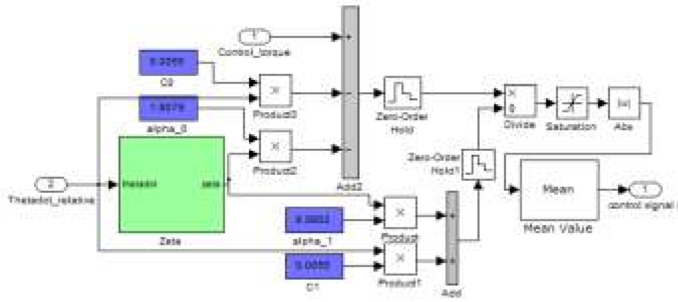

Figure 13: Block diagram, converter

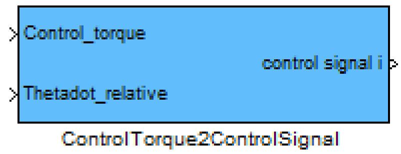

Figure 14: subsystem converter

- **State variables and SAS system outputs**

The physical sas system has only two outputs, relative posistion between body arm and wheel, and wheel position. The controller is dependent on the states: body arm vs level (position and velocity) and wheel position/velocity. These values needs to be estimated when an observer is not in use. Body arm position is found by subtracting the wheel pos from the body arm pos. This value is the adjusted to oscillate across the equilibrium. The obtained position is then derivated to obtain the velocity. Because of poor resolution in the position encoder, the values we get are chopped and inaccurate. To reduce this choppy signal we filter the signal wih a estimated mean value for every 0.5s(see Figure 11).

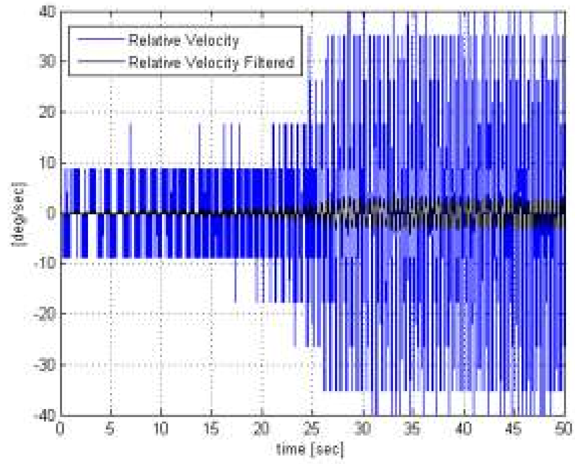

Figure 15: Velocity values, filtered

The filtered value loose amplitude gain because of the filtering. To take this into account we add a gain to the cunrrent into the controller.

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- **Results with pure lqr**

The lqr controller 3(see Figure 5) is used we have the controller:

$$K = [-3.7658 \;\; 9.4138 \;\; 15.4084 \;\; 0.1248]$$

The result gave us a amplitde of 2deg (see Figure 15).

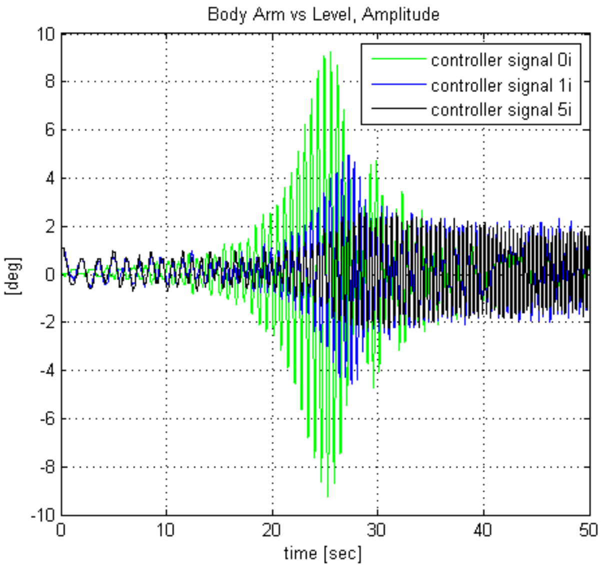

Figure 16: different controller gain

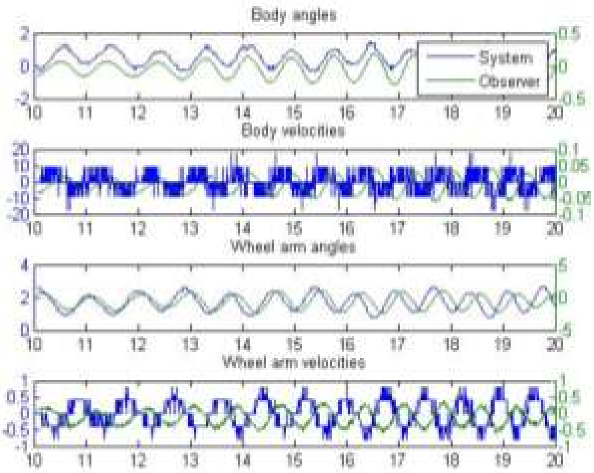

Figure 17: the effect of implementing the observer

- **Results with an observer included**

We did not have as much success with the observer design using the PLACE-function. This might have to do with the fact that the response of the system simulated does not correspond with the physical system. A new approach to L was done using the LQE-function.

$$L = \begin{bmatrix} 1.23 & 0.219 \\ 0.219 & 0.049 \\ 0.280 & -0.035 \\ -0.685 & -0.475 \end{bmatrix}$$

The similiarities using this matrix are clear.

At low frequencies, the observer is able to follow the system with smooth results, but at higher frequencies the observer lags. This can most likely be improved by placing the poles of the observer. The results were satissfactory. As predicted, the states did smoothen out, especially the velocities. The values, on the other hand, are wrong and the observer can therefore not be used. This is most like because our SAS model with the given parameters did not correspond with the physical system.

Conclusion

During this project, we have created an LQR controller through the use of Matlab Simulink. The model of the SAS system was retrieved and linearized. Using different cost functions, the optimal LQR controller was found for four outputs. Implementing this controller on the SAS model reduced the amplitude around the critical frequency (eigenfreq.). The best results on the physical system gave an amplitude of 2 degrees. An observer was also implemented and tested on the physical system. This gave state values with better resolution.

References

[1] Xubin Song, “Cost-Effective Skyhook Control for Semiactive Vehicle Suspension Applications. The Open Mechanical Engineering Journal, 2009, 3, 17-25.

[2] Hongkun Zhang, Hermann Winner, Wenjun Li, “Comparison between Skyhook and Minimax Control Strategies for Semi-active Suspension System”, World Academy of science, Engineering and Technology 55, 2009.

[3] Reichert, Brian Anthony Jr: APPLICATION OF MAGNETORHEOLOGICAL DAMPERS FOR VEHICLE SEAT SUSPENSIONS. Link: http://scholar.lib.vt.edu/theses/available/etd-11997-164941/unrestricted/Chapter2.pdf

[4] Skogestad,Postletwaite. Multivariable Feedback Control, chap. 9

[5] Ghorbany, Daruish "MR damper hysteresis characterization for the semi-active suspension system", UIA, Master Report 2011.

[6] Ching-han Ho Andrew N. Vavreck. Characterization of a commercia magnetorheological brake/damper in oscillatory motion. Smart structures and materials, 1(1):256–267, 2005

[7] N Nise, “Control Systems Engineering”, 2011, chap.9

[8] N Nise, “Control Systems Engineering”, 2011, chap.12