A numerical approach for modeling squeeze-film damping in rigid microstructures including rarefaction effects

SALVATORE NIGRO¹, LEONARDO PAGNOTTA² and MARIA F. PANTANO²
¹Department of Medical Sciences
University Magna Graecia
Viale Europa, 88100 Germaneto (CZ)
ITALY
²Department of Mechanical Engineering
University of Calabria
Ponte P. Bucci, 44C, 87036 Rende (CS)
ITALY
s.nigro@unicz.it, pagnotta@unical.it, mf.pantano@unical.it

Abstract: - In several MEMS applications, it may happen that the fluid causing squeeze-film damping cannot be considered as continuum. In such cases, the fluid behavior cannot be modeled through the classical Navier-Stokes equation, and other approaches should be preferred. During the years, many models have been proposed. However, in spite of the intense work already done by the scientific community, there is still need of further validation and improvement. In this paper, we adopt a numerical approach to solve some squeeze-film damping problems involving rigid non-perforated plates, provided with linear movement with respect to the substrate. We compare the numerical results with those obtained through analytical models, included in the literature, and the experimental data, which are already available. In all the examined cases, numerical simulations show good agreement with experiments, providing a difference from the experimental data, which is 10-30% less than that obtained with the corresponding analytical results.

Key-Words: Squeeze-film damping, rarefaction, MEMS, Navier-Stokes equation, FEM, Multiphysics analysis

1 Introduction
Squeeze-film damping has gained significant attention throughout the last years, as a consequence of the advent of microelectromechanical systems (MEMS). In fact, in several MEMS-based applications, like microaccelerometers, microgyroscopes, micromirrors and microswitches, squeeze-film damping is the main source of energy dissipation. Such phenomenon arises when a thin film of fluid, like air, is confined between two solid walls, which move with respect to one another. Because of this movement, the fluid results to be sucked in/pulled out of its tight channel. As a consequence, inside the fluid a pressure field is generated, which causes a resistive force to hinder the reciprocal movement of the plates.

The fluid flow can be modeled through the classical Navier-Stokes equation, which sometimes can be further simplified in the Reynolds equation under some assumptions (e.g., inertial effects, thermal gradients, and out-of-plane movement are negligible). However, for the Navier-Stokes equation to be valid, the fluid should be considered as continuum. Such hypothesis is stringent and does not apply in those cases, where either the fluid thickness is much smaller than the plate dimensions, or the ambient pressure is small compared to the atmospheric value.

The parameter conventionally used to quantify the rarefaction level of the fluid is the Knudsen number ($K_n$), defined as the ratio of the mean free path ($\lambda$) of the fluid molecules to the characteristic length of the fluid channel, where $\lambda$ is [1]:

$$\lambda = \frac{R \cdot T}{\pi \sqrt{2} d^2 \cdot N_A \cdot p}$$  (1)

in the above, $R$ is the gas constant, $T$ is the temperature, $d$ the diameter of the fluid molecules, $N_A$ the Avogadro number and $p$ the ambient pressure. For small $K_n$ ($K_n<0.01$: continuum regime), the Navier-Stokes equation can be used successfully; for medium ($0.01<K_n<1$: slip regime/transition regime) and high ($K_n>1$: free molecular regime) values, other approaches should be considered [2].
In spite of the great work already done by the scientific community, there is still need of further improvement and validation of the existing way to model rarefaction effects. In fact, the models included in the literature cannot allow to well predict squeeze-film damping throughout all the $K_n$ regimes.

In this paper, we adopt a numerical approach to solve some squeeze-film damping problems involving rigid non-perforated plates, for which experimental data are already provided in the literature. We will compare those experimental values with the results obtained by both numerical and analytical methods.

2 Approaches to model fluid rarefaction in squeeze-film damping

Two different approaches have been considered to model the behavior of rarefied air generating squeeze-film air damping [1]. The first approach is based on the introduction of a slight modification in the Reynolds equation. In particular, the standard viscosity term ($\mu$) is substituted with an effective viscosity ($\mu_{eff}$), which accounts for the effects related to the rarefied character of the fluid. In all the models already included in the literature, $\mu_{eff}$ is related to $\mu$, as:

$$\mu_{eff} = \frac{\mu}{Q_{PR}} \quad (2)$$

where the flow rate coefficient, $Q_{PR}$, is function of the Knudsen number. During the years, many semiempirical expressions for $\mu_{eff}$ have been proposed. Here, we consider those, which have been demonstrated to work better [3-4].

Veijola et al. computed the effective viscosity as [5]:

$$\mu_{ev} = \frac{\mu}{1 + 9.638 \cdot K_n^{1.159}} \quad (3)$$

Li proposed instead the following expression [6]:

$$\mu_{el} = \frac{\mu}{1 + 3a \sqrt{\pi}/D + 6bD^c} \quad (4)$$

where $a=0.01807$, $b=1.35355$ and $c=-1.17468$, and $D$ is:

$$D = \sqrt{\pi}/(2K_n)$$

Pandey et al. improved Li’s model to achieve better agreement with experimental data and proposed [3]:

$$\mu_{ep} = \frac{\mu}{1 + 3a \sqrt{\pi}/(1.4D) + 6b(1.4D)^c} \quad (5)$$

where $a$, $b$, $c$, and $D$ are defined as before.

However, it has not been yet proved that these models are effective independently on the movement of the plates causing squeeze-film damping (e.g., either torsional or normal movement) [7].

The second approach is based on the molecular dynamics (MD). In this case, collisions between molecules are neglected, and squeeze-film damping is considered as the result of the interaction of the molecules with the solid walls, confining the fluid. Based on this approach, Christian [8] proposed the first model, then validated [9] and improved by other authors [10-12]. All these models should be more effective in the free molecular regime, and less accurate in the other regimes, as they neglect the viscous character of the fluid flow (e.g., interaction between molecules). On the contrary, models following the first approach are based on a modified Reynolds equation, which necessarily leads to consider the viscous fluid flow as the main component of squeeze-film damping. This assumption is no longer valid as rarefaction becomes more significant (e.g., $K_n$ becomes higher). However, usually, because of its simplicity the first approach is preferred over the second one. For this reason, only the models based on a modified Reynolds equation will be further considered in this paper.

Usually the expression of the effective viscosity is implemented in simplified analytical formulas, valid for regular geometries, where the walls defining the fluid channel have rectangular shape, provided with either torsional or normal movement [2]. Anyway, it was verified [13] that performance of numerical simulations can be more effective than simplified analytical models, due to their versatility (e.g., also complex geometries can be studied) and the computational power of modern computers, which makes numerical simulations less time consuming than during the past.

In our paper, we consider a numerical approach to study squeeze-film damping, as described in the following section.
3 Numerical and analytical modeling of squeeze-film damping in rarefied regime

We adopted a numerical approach, based on the finite element method (FEM), implemented within a commercial software, Comsol Mutiphysics, which is particularly suitable to study squeeze-film damping problems. In fact, it allows for performance of multiphysics analysis, involving different physical domains, as required by squeeze-film damping, where both fluid dynamics and structural mechanics are contemporarily needed.

All our numerical simulations were carried out on a workstation with the following technical features: RAM 16 GB, Intel(R) CORE(TM) i7 CPU 860 @ 2.80 GHz.

The software automatically generated the 3D mesh, made by tetrahedral elements for both the moving plate and the air in its surroundings. Figure 1 shows a typical mesh generation (fig. 1a), and some exemplary results referring to a typical pressure distribution (fig. 1b) over the moving plate, as a consequence of its movement with respect to the substrate.

![Fig. 1 Typical mesh (a) and pressure distribution over one fourth of the analyzed geometry.](image)

The damping coefficient \( c_{nb} \), associated to squeeze-film damping, was identified from the integration of the pressure field over the moving plate, divided by its velocity.

Since the considered cases involved symmetrical plates, we simulated only one fourth of them to reduce the computational time.

In order to take into account fluid rarefaction, we solved full incompressible 3D Navier-Stokes equation, where the standard viscosity was replaced by the effective viscosity. Expressions (3), (4), and (5) for the effective viscosity were implemented, and the corresponding results were then compared with the experimental data, which can be found in the literature. The numerical results were also compared with the values obtained through the analytical approaches described in the following.

According to the first analytical approach, the damping coefficient \( c_{aB} \) for a rectangular non-perforated plate can be calculated by the following expression [2]:

\[
c_{aB} = \mu \frac{L W^3}{h^3} \beta \tag{6}
\]

where \( \mu \) is the fluid viscosity or the effective viscosity, which can be computed with (3), (4), or (5), giving three different results; \( L \) and \( W \) are the plate length and width, respectively, \( h \) is the gap thickness, and \( \beta \) is a correction factor, depending on the ratio \( W/L \).

The second analytical approach [14] allows for determination of the damping coefficient \( c_{av} \) in case of rectangular non-perforated plates, which move perpendicular to the substrate:

\[
c_{av} = Re \left[ \sum_{m=1,3,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{1}{Q_{PR}G_{mn} + j\omega C_{mn}} \right] \tag{7}
\]

where \( \omega \) is the frequency of the plate movement, \( Q_{PR} \), \( G_{mn} \), and \( C_{mn} \) are defined as:

\[
Q_{PR} = \frac{12\mu}{j\omega h^2 q} \left[ \frac{q h - (2 - q^2)1.016\lambda h \tanh(q h/2)}{1 + 1.016q \tanh(q h/2)} \right] \tag{8}
\]

\[
G_{mn} = \frac{\pi^6 h^3 (mn)^2 / (m^2 + n^2)}{768\mu ab} \left( \frac{a^2 + b^2}{2} \right) \tag{9}
\]

\[
C_{mn} = \frac{\pi^4 h (mn)^2}{64abn_f P} \tag{10}
\]

here, \( h \) and \( \mu \) are the same as before; \( q = \sqrt{jo\rho/\mu} \), \( \rho \) is the fluid density, \( \lambda \) is the mean free path, \( p \) is the ambient pressure, \( a \) and \( b \) are the plate sides, and \( n_f \) is a coefficient depending on the heat conduction and temperature boundary conditions. As opposite to the first approach, here it is not possible to identify an effective viscosity term, which can be neither changed with another expression nor implemented within numerical analysis.

4 Results and discussion

To evaluate the effectiveness of the present numerical approach for the solution of squeeze-film damping problems, we considered some cases experimentally investigated in [4]. In [4], the authors determined the damping coefficient characterizing the gold non-perforated plate, with
The profile\textsuperscript{1} reported in figure 2, at different pressures (from thousands Pa to few Pa).

Fig. 2 Geometry of the plate investigated in [4].

As regards the numerical analysis, because of the plate symmetry, we studied only one fourth of it. Figure 3 shows a typical pressure distribution of the analyzed geometry.

Fig. 3 Pressure distribution over one fourth of the geometry experimentally investigated by [4].

Full 3D incompressible Navier-Stokes equation was solved, with the viscosity of the fluid into the gap given by the expressions reported in equations (3), (4), and (5). A comparison between the damping coefficient obtained from numerical analysis \( (c_n) \) and the experimental damping coefficient \( (c_e) \) is shown in the log-log graph of figure 4. At high and medium pressure, all the effective viscosity models give equivalent good results; but at low pressures, Pandey & Pratap’s model (5) seems to work better. These squeeze-film damping cases were also studied through the analytical approaches mentioned above. However, the analytical formulas expressed by (6) and (7) are effective to study only rectangular plates. For this reason, they are applied to study an equivalent rectangular area with the same area as the one of the experiments. This procedure is suggested in [4] and also further verified by our numerical analysis.

Table 1 summarizes the characteristic dimensions of the equivalent rectangular plate proposed in [4].

<table>
<thead>
<tr>
<th>Characteristic Dimension</th>
<th>Value [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>154.3</td>
</tr>
<tr>
<td>Width</td>
<td>192.6</td>
</tr>
<tr>
<td>Thickness</td>
<td>5.7</td>
</tr>
<tr>
<td>Gap</td>
<td>4.1</td>
</tr>
</tbody>
</table>

A comparison between the analytical value for the damping coefficient obtained with equations (6) and (7) and the experimental data is shown in figure 5. In particular, three sets of analytical results coming from (6) are reported. They are obtained substituting the fluid viscosity in (6) with the effective viscosity computed by (3), (4), and (5).

Fig. 4 Comparison between the numerical damping coefficient \( (c_n) \) and the experimental value \( (c_e) \) at different pressures. Three different sets of numerical data are reported, each referring to a specific expression for the effective viscosity.

Fig. 5 Comparison between the analytical damping coefficient computed by (6) and (7) and the experimental data. In equation (6), the fluid viscosity was computed by (3), (4), and (5), giving three different sets of results.

\textsuperscript{1}The plate dimensions are known from personal communication with the author of the experiments.
From figure 5, it can be noticed that at high and medium pressures, both the analytical approaches work well, whereas at low pressures, (7) gives significant better agreement with experiments. For the sake of further comparison, table 2 collects all the analytical and numerical data we found. Numerical results show significant better agreement with respect to the experiments than the results obtained with equation (6). This is true for every expression of the effective viscosity herein considered. This can be explained since numerical analysis takes into account border effects, which are instead neglected by analytical models. During the past, some authors tried to compute a correction factor to take into account such border effects [15]. Nevertheless, the values they provided were verified for the slip regime only. Very good agreement with experiments is provided by model (7), which however does not provide an expression for the effective viscosity to be easily implemented in numerical analysis. Thus, for this case, it is not possible to have a direct comparison between numerical and analytical results.

5 Conclusion
A common way to study squeeze-film damping, including rarefaction effects, is to introduce in the Navier-Stokes equation (or the Reynolds equation) an effective viscosity instead of the standard fluid viscosity. Several models to compute the effective viscosity have been proposed during the years and implemented inside analytical formulas to determine the damping coefficient. We considered some squeeze-film damping problems at varying pressure, for which experimental data are already provided in the literature. We adopted both analytical and numerical approaches to determine the damping coefficient for such cases, and we compared our results with experiments. In particular, we found that if the same effective viscosity expression is used within numerical analysis and a simple analytical model, numerical results show better agreement with experiments. Furthermore, the numerical approach is more versatile compared to the analytical one, since it allows for study of more complicated geometries. However, the difference between the numerical and experimental values suggest that the already existing expressions for the effective viscosity could be further improved in future work.

References:
[3] AK Pandey and R Pratap, A semi-analytical model for squeeze-film damping including rarefaction in a MEMS torsion mirror with

<table>
<thead>
<tr>
<th>Pressure [Pa]</th>
<th>$c_{ab}$ with $\mu_{av}$ [%]</th>
<th>$c_n$ with $\mu_{av}$ [%]</th>
<th>$c_{ab}$ with $\mu_{av}$ [%]</th>
<th>$c_n$ with $\mu_{av}$ [%]</th>
<th>$c_{ab}$ with $\mu_{av}$ [%]</th>
<th>$c_n$ with $\mu_{av}$ [%]</th>
<th>$c_{av}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>89185</td>
<td>-19</td>
<td>-4</td>
<td>-19</td>
<td>-4</td>
<td>-17</td>
<td>-4</td>
<td>-19</td>
</tr>
<tr>
<td>72678</td>
<td>-18</td>
<td>-2</td>
<td>-17</td>
<td>-2</td>
<td>-15</td>
<td>1</td>
<td>-17</td>
</tr>
<tr>
<td>46873</td>
<td>-17</td>
<td>-1</td>
<td>-17</td>
<td>0</td>
<td>-12</td>
<td>5</td>
<td>-16</td>
</tr>
<tr>
<td>19497</td>
<td>-20</td>
<td>-2</td>
<td>-19</td>
<td>0</td>
<td>-8</td>
<td>11</td>
<td>-16</td>
</tr>
<tr>
<td>8854</td>
<td>-24</td>
<td>-1</td>
<td>-23</td>
<td>1</td>
<td>-5</td>
<td>20</td>
<td>-14</td>
</tr>
<tr>
<td>3792</td>
<td>-27</td>
<td>9</td>
<td>-26</td>
<td>12</td>
<td>0</td>
<td>40</td>
<td>-6</td>
</tr>
<tr>
<td>1143</td>
<td>-36</td>
<td>-1</td>
<td>-36</td>
<td>0</td>
<td>-7</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>376</td>
<td>-50</td>
<td>-19</td>
<td>-51</td>
<td>-19</td>
<td>-27</td>
<td>7</td>
<td>-2</td>
</tr>
<tr>
<td>78</td>
<td>-63</td>
<td>-35</td>
<td>-64</td>
<td>-36</td>
<td>-46</td>
<td>-16</td>
<td>-5</td>
</tr>
<tr>
<td>28</td>
<td>-70</td>
<td>-44</td>
<td>-71</td>
<td>-45</td>
<td>-57</td>
<td>-29</td>
<td>-9</td>
</tr>
<tr>
<td>13</td>
<td>-74</td>
<td>-49</td>
<td>-75</td>
<td>-50</td>
<td>-63</td>
<td>-36</td>
<td>-11</td>
</tr>
<tr>
<td>7</td>
<td>-74</td>
<td>-49</td>
<td>-76</td>
<td>-34</td>
<td>-64</td>
<td>-20</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>-75</td>
<td>-65</td>
<td>-77</td>
<td>-50</td>
<td>-66</td>
<td>-37</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-88</td>
<td>-73</td>
<td>-89</td>
<td>-74</td>
<td>-84</td>
<td>-68</td>
<td>-42</td>
</tr>
</tbody>
</table>

Table 2 Percentage difference of the numerical and analytical results with respect to the experiments reported in [4]. Pressure values were read from the graph reported in [4].


