Stability Analysis of Inverter for Renewable Energy

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Abstract: Dc power sources such as photovoltaic cells and fuel cells need inverter to connect into an existing electric system. The performance of the inverter is determined by the control, then the inverter control is important. The suitable gain in the control must be determined systematically. This paper proposes a Bode analysis approach for determination of inverter control parameters for renewable energy. The result of frequency response analysis is compared with ATPDraw simulation.

Key-Words: Inverter, Control, Stability, Linearization, Frequency Response Analysis, ATP Draw

1 Introduction

Distributed Generators such as photovoltaic power generation have been adopted in all over the world, and recently large capacity called as a mega-solar has been constructed. Power of the photovoltaic power generation is produced in a direct current, so it must be changed to an alternating current through an inverter to supply electricity to a load.

If the inverter capacity is large compared with short circuit power level of ac system, there is possibility of instability in the inverter operation. The stability of the inverter depends on control characteristic also [1].

This paper proposes a evaluation method for stability check based on conventional Bode analysis of frequency response technology. We assume a full bridge inverter and linearization is applied for equations representing the inverter and ac system. We have been developing this tool [2][3][4], we report here that frequency response is compared with ATPDraw simulation and good agreement in viewpoint stability is obtained. The capacity of inverter and short circuit power level is assumed 1MW and 5 MW respectively in the study. Linearization of equations is described fully in the paper. Frequency responses between Bode analysis and ATPDraw are compared each other in scales of gain and phase angle. In this paper, analysis results of not only active power control but also reactive power control are reported.

2 Inverter Circuit

The inverter of Voltage Source Converter (VSC) with Pulse Width Modulation (PWM) is treated for the study [5]. The inverter connected to a distribution network is assumed. However, there are many types of dc power sources, photovoltaic cell, fuel cell, and battery, a constant dc voltage source is assumed here for simplicity.

![Fig.1 Inverter model](image-url)

The inverter model for the application is shown in Fig.1 which consists of a voltage source of 400 Vdc, an inverter of VSC with 1 MW capacity, a 6.6kV/200V transformer of 1 MVA rating with 5%
impedance, and a set of control. The inverter is connected to 6.6 kV ac network which has a short circuit capacity of 5 MVA. The inverter is 6-arms full bridge configuration and a pulse width modulation with 2 kHz carrier wave is applied. It is assumed that the control is consisted of a constant active power control and a constant reactive power control.

3 Linearization

3.1 Ac-system

A simple ac-system model consisted by a resister and a reactor is shown in Fig.2.

![Fig.2 Ac-system](image)

The equation of relation in this model is Eq.(1).

$$\mathbf{V}_A - \mathbf{V}_B = R_A \mathbf{I}_A + X_A \frac{d}{dt} \mathbf{I}_A \cdots \cdot (1)$$

In Eq.(1), the voltage and the current are represented as follows with rotating angle $\theta$.

$$\mathbf{V}_A = (v_{Ad} + jv_{Aq}) e^{j\omega \theta_0} \cdots \cdot (2)$$

$$\mathbf{I}_A = (i_{Ad} + ji_{Aq}) e^{j\omega \theta_0} \cdots \cdot (3)$$

$\mathbf{V}_A, \mathbf{I}_A$ is rotating on $\theta_{DQ} = \omega \tau$. Eq.(2) and Eq.(3) are substituted into Eq.(1). The equation can be rewritten on the DQ axis plane by eliminating $e^{j\omega \theta_0}$.

$$\{(v_{Ad} + jv_{Aq}) - (v_{Bd} + jv_{Bq})\} = R_A (i_{Ad} + ji_{Aq}) + X_A \frac{d}{dt} (i_{Ad} + ji_{Aq})$$

$$+ X_A (i_{Ad} + ji_{Aq}) \cdot j \frac{d\theta_{DQ}}{dt} \cdots \cdot (4)$$

Following expression is substituted into Eq.(4)

$$\frac{d}{dt} = p$$

Eq.(1) becomes Eq.(5), and Eq.(5) can be derived into a d-q axis flame as follows.

$$\mathbf{V}_A - \mathbf{V}_B = R_A \mathbf{I}_A + X_A (p + jp \theta_{DQ}) \mathbf{I}_A \cdots \cdot (5)$$

$$\{(v_{Ad} + jv_{Aq}) - (v_{Bd} + jv_{Bq})\}$$

$$= R_A (i_{Ad} + ji_{Aq}) + X_A (i_{Ad} + ji_{Aq}) (p + jp \theta_{DQ})$$

$$= R_A (i_{Ad} + ji_{Aq}) + X_A (i_{Ad} + ji_{Aq}) (p + jp \theta_{DQ} + ji_{Aq} p - i_{Ad} p \theta_{DQ})$$

$$= R_A (i_{Ad} + ji_{Aq}) + X_A (i_{Ad} + ji_{Aq}) (p + jX_{Ad} p \theta_{DQ} + jX_{Aq} p - X_{Ad} i_{Ad} p \theta_{DQ})$$

$$= R_A (i_{Ad} + ji_{Aq}) + X_A (i_{Ad} + ji_{Aq}) (p + jX_{Ad} p \theta_{DQ} + jX_{Aq} p - X_{Ad} i_{Ad} p \theta_{DQ})$$

$$= R_A (i_{Ad} + ji_{Aq}) + X_A (i_{Ad} + ji_{Aq}) (p + jX_{Ad} p \theta_{DQ} + jX_{Aq} p - X_{Ad} i_{Ad} p \theta_{DQ})$$

$$\cdots \cdot (6)$$

The Eq.(6) is divided into a real part and an imaginary part.

$$v_{Ad} - v_{Bd} = (R_A + X_A p) i_{Ad} - X_A i_{Ad} p \theta_{DQ} \cdots \cdot (7)$$

$$v_{Aq} - v_{Bq} = (R_A + X_A p) i_{Aq} + X_A i_{Ad} p \theta_{DQ} \cdots \cdot (8)$$

Linearization applying $V_{ad0}/\Delta V_{ad}$ at an operation point is carried out and relationships with small perturbed values, $\Delta$, are obtained as shown in Fig. 3. Where $p=s/\omega_0$ is applied.

![Fig.3 Linearized model of ac-system](image)

The node B is assumed as an infinite bus and the node A is a bus that the inverter transformer is connected.

3.2 Inverter

The inverter model is shown in Fig. 4.

![Fig.4 Inverter model](image)

The voltage generated by the inverter is Eq.(9), where C is a control factor and kVbase is actual value of the secondary side line voltage of the transformer.

$$|V_I|^{PU} = \frac{0.612 \cdot E_j^{[kV]} \cdot C}{kVbase} \cdots \cdot (9)$$

The complex voltage of the inverter is Eqs.(10),(11)

$$\mathbf{V}_{I} = \mathbf{V}_{I0} + j \mathbf{V}_{Iq}$$

$$\mathbf{V}_{ad} = \mathbf{V}_{I}^{[PU]} \cos(\beta_A + \theta_I) \cdots \cdot (10)$$

$$\mathbf{V}_{aq} = \mathbf{V}_{I}^{[PU]} \sin(\beta_A + \theta_I) \cdots \cdot (11)$$
Where $\beta_A$ is a phase angle of $V_A$, and $\theta_i$ is a phase angle of $V_i$. Linearization applying $V_{dc}^0 + \Delta V_{id}$ at an operation point is carried out as follows.

$$V_{dc}^0 + \Delta V_{id} = \left[ V_{dc} + \Delta V_{id} \right] \cos(\beta_{dc0}^0 + \theta_{dc0}) + \left[ \Delta V_{id} \right] \cos(\Delta \beta_{id} + \Delta \theta_{id})$$

$$= \left[ V_{dc} + \Delta V_{id} \right] \cos(\beta_{dc0}^0 + \theta_{dc0}) \cos(\Delta \beta_{id} + \Delta \theta_{id}) - \sin(\beta_{dc0}^0 + \theta_{dc0}) \sin(\Delta \beta_{id} + \Delta \theta_{id})$$

$$= \left[ V_{dc} + \Delta V_{id} \right] \cos(\beta_{dc0}^0 + \theta_{dc0}) - \sin(\beta_{dc0}^0 + \theta_{dc0}) \sin(\Delta \beta_{id} + \Delta \theta_{id})$$

$$\Delta V_{id} = \left[ V_{dc} \right] \cos(\beta_{dc0}^0 + \theta_{dc0}) - \left[ \sin(\beta_{dc0}^0 + \theta_{dc0}) \sin(\Delta \beta_{id} + \Delta \theta_{id}) \right] (12)$$

The perturbed value is

$$\Delta V_{id} = \left[ V_{dc} \right] \cos(\beta_{dc0}^0 + \theta_{dc0}) - \left[ \sin(\beta_{dc0}^0 + \theta_{dc0}) \sin(\Delta \beta_{id} + \Delta \theta_{id}) \right] (12)$$

From Eq.(9)

$$|V_{dc}| + \Delta |V_{dc}| = \frac{0.612 \cdot (E_{dc} + \Delta E_{dc}) (C_0 + \Delta C)}{kV_{base}}$$

$$\Delta |V_{dc}| = \frac{0.612 \cdot E_{dc} \cdot C_0}{kV_{base}} + \frac{0.612 \cdot E_{dc} \cdot \Delta C}{kV_{base}}$$

$$\Delta |V_{dc}| + \frac{0.612 \cdot E_{dc} \cdot \Delta C}{kV_{base}} = \frac{0.612 \cdot E_{dc} \cdot C_0}{kV_{base}} + \frac{0.612 \cdot \Delta E_{dc} \cdot C_0}{kV_{base}} \cdot \ldots \cdot (13)$$

Eq.(13) is substituted for Eq.(12).

$$\Delta V_{id} = \frac{0.612 \cdot E_{dc} \cdot \cos(\beta_{dc0}^0 + \theta_{dc0}) \Delta C}{kV_{base}} + \frac{0.612 \cdot C_0 \cdot \cos(\beta_{dc0}^0 + \theta_{dc0}) \Delta E_{dc}}{kV_{base}}$$

$$- \frac{0.612 \cdot E_{dc} \cdot \sin(\beta_{dc0}^0 + \theta_{dc0}) \Delta \theta_{dc0}}{kV_{base}} - \frac{0.612 \cdot \Delta E_{dc} \cdot C_0 \cdot \sin(\beta_{dc0}^0 + \theta_{dc0}) \Delta \theta_{dc0}}{kV_{base}} \cdot \ldots \cdot (14)$$

The perturbed value $\Delta V_{id}$ in a similar manner is applied.

$$\Delta V_{id} = \frac{0.612 \cdot E_{dc} \cdot \sin(\beta_{dc0}^0 + \theta_{dc0}) \Delta C}{kV_{base}} + \frac{0.612 \cdot C_0 \cdot \sin(\beta_{dc0}^0 + \theta_{dc0}) \Delta E_{dc}}{kV_{base}}$$

$$- \frac{0.612 \cdot E_{dc} \cdot \cos(\beta_{dc0}^0 + \theta_{dc0}) \Delta \theta_{dc0}}{kV_{base}} - \frac{0.612 \cdot \Delta E_{dc} \cdot C_0 \cdot \cos(\beta_{dc0}^0 + \theta_{dc0}) \Delta \theta_{dc0}}{kV_{base}} \cdot \ldots \cdot (15)$$

A linearized relationship of the inverter is obtained as shown in Fig.5, where $\Delta E_{dc}$ is a dc voltage deviation, $\Delta \theta_{dc0}$ and $\Delta C$ are control signal deviation. $\Delta \beta_{id}$ is an ac voltage phase deviation.

![Fig.5 Linearized model of inverter](image)

**4 Configuration of control**

The configuration of the control is shown in Fig.6. It has an active power (P) control, a reactive power (Q) control, a PQ detector, and a voltage phase detector. Outputs of P and Q controls are changed to a phase angle signal and an amplitude signal of the inverter voltage through a signal conversion. Three phase voltage signals are generated by the signal conversion outputs and the phase signal $\beta_A$ of the ac voltage. Three phase voltage signals are fed to the PWM logic to get on/off pulse signals for switches.

![Fig.6 Control block](image)

The P and Q controls are shown in Fig.7. A signal P corresponds to an angle and a signal Q corresponds to an amplitude of the inverter output voltage. $K_{PP}$ and $K_{QQ}$ are proportional gains, and $K_{PI}$ and $K_{QI}$ are integral gains. These control blocks except constant reference values are combined with linearized models of ac system and inverter.

![Fig.7 P and Q control](image)

The linearized model of Fig.1 is analyzed by comparing Bode analysis tool and ATPDraw simulation to confirm equivalency [6][7].

**5 Frequency Response Analysis**

The Frequency response analysis tool for linearized model is developed by authors representing on a spreadsheet program including a power flow calculation to get the operation point and a complex...
matrix calculation for each frequency. An open loop and a closed loop responses can be calculated in this tool. Two cases are shown in this study.

5.1 P control loop
The open loop characteristic on the active power control loop is calculated. The control gains are listed in Table 1. Gain and phase diagram of the open loop are shown in Fig. 8 and margins are listed in Table 2.

Table 1. Gains

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<table>
<thead>
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<tbody>
<tr>
<td>$K_{pp}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_{pi}$</td>
<td>10</td>
</tr>
<tr>
<td>$K_{qp}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_{qi}$</td>
<td>10</td>
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</tbody>
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Table 2. Result of frequency response analysis

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<tbody>
<tr>
<td>Gain crossover</td>
<td>1.75</td>
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<tr>
<td>frequency [Hz]</td>
<td></td>
</tr>
<tr>
<td>Phase crossover</td>
<td>5.37</td>
</tr>
<tr>
<td>frequency [Hz]</td>
<td></td>
</tr>
<tr>
<td>Gain margin [dB]</td>
<td>-4.79</td>
</tr>
<tr>
<td>Phase margin [deg]</td>
<td>-100.9</td>
</tr>
</tbody>
</table>

From Table 2, this case is stable because the gain crossover frequency less than the phase crossover frequency, and gain margin is negative value. The active power control closed loop calculated by the same developed program is shown in Fig. 9.

5.2 Q control loop
Parameters of Q control are set to higher values than the gain of P control. The reason why is to get relatively rapid response of reactive power control. Calculation results of the open loop are shown in Fig. 10. In addition, the calculation results of the closed loop are shown in Fig. 11.

Table 3. Gains

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<tbody>
<tr>
<td>$K_{pp}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_{pi}$</td>
<td>10</td>
</tr>
<tr>
<td>$K_{qp}$</td>
<td>17</td>
</tr>
<tr>
<td>$K_{qi}$</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 4. Result of frequency response analysis

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<tr>
<td>Gain crossover</td>
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<tr>
<td>frequency [Hz]</td>
<td></td>
</tr>
<tr>
<td>Phase crossover</td>
<td>40.41</td>
</tr>
<tr>
<td>frequency [Hz]</td>
<td></td>
</tr>
<tr>
<td>Gain margin [dB]</td>
<td>-0.153</td>
</tr>
<tr>
<td>Phase margin [deg]</td>
<td>-137.9</td>
</tr>
</tbody>
</table>

Fig. 9 (a) Gain diagram of the closed loop

Fig. 9 (b) Phase diagram of the closed loop

Fig. 10 (a) Gain diagram of the open loop
From Table 4, this case is stable because the gain crossover frequency is less than the phase crossover frequency, and the gain margin is negative value.

6 Confirmation of Frequency Response Analysis Results

The ATPDraw simulations are carried out and are compared with the frequency response analysis. An example calculation by ATPDraw is selected in marked condition in Fig. 11 that corresponds to 30Hz. In the ATPDraw calculation, reactive power reference value of 0 pu is applied and additionally a sinusoidal signal of 0.05 pu amplitude of 30 Hz is superimposed. Fig. 12(a) shows the reactive power reference value and the response of the inverter, and enlarged results are shown in Fig. 12(b). Fourier analysis is applied for these curves and gain and phase corresponding to a closed loop are calculated. The obtained gain and phase are close to the marked points in Fig. 11.

The comparison of closed-loop of the active power controller is shown in Fig. 13. The comparison of closed-loop reactive power control is shown in Fig. 14. From Fig. 13 and Fig. 14 the results of ATPDraw and the developed program are mostly agreed with each other. Therefore, it can be said that the
calculation result of frequency response analysis is reasonable.

Fig. 14 Comparison of reactive power control

7 Conclusion
This paper proposes a linearized model of the inverter and the frequency response analysis is applied for it. The frequency response analysis is compared with ATPDraw simulation and validity of analysis is demonstrated. This method will be useful and effective for control design of renewable energy inverter.

References: