The relationship between changes of deflection and natural frequencies of damaged beams

GILBERT-RAINER GILLICH
Department for Machine dynamics & Vibrations
"Eftimie Murgu" University of Resita
P-ta Traian Vuia 1-4, 320085 Resita
ROMANIA
gr.gillich@uem.ro   http://www.uni-resita.eu

ZENO-IOSIF PRAISACH
SC HYDROENGINEERING SA,
Calea Caransebesului 16A, 320168, Resita
ROMANIA
zpraisach@yahoo.com

IONICA NEGRU
Department for Machine dynamics & Vibrations
"Eftimie Murgu" University of Resita
P-ta Traian Vuia 1-4, 320085 Resita
ROMANIA
i.negru@uem.ro

Abstract: - The paper describes the relationship between changes of deflection of beams due damages and the natural frequency changes. Previous papers relate the influence of damage location and severity upon the natural frequency changes of weak-axis bending vibration modes, for which the authors have found a correlation. It base on the reduction of stiffness in a slice of the beam and consequently on the reduction of the potential stored energy in that slice. While the stiffness reduction affect also the deflection under the own mass of the beam, we concluded that it can be found a relationship between deflection and frequency changes. Researches performed this direction revealed that it is a clear dependency between deflection and frequency changes, what makes deflection an important feature of beam behavior, usable in damage assessment.

Key-Words: - deflection, natural frequency, finite element method, damages

1 Introduction

The use of natural frequency shifts for damage detection is largely presented in literature [1], [2], [3] and [4]. The methods based on these changes are classified into two important groups [5]: methods limited to damage detection and methods which supplementary permit to locate and to quantify damages.

The dynamic behavior of structures is influenced by damages, changing their mechanical and dynamic characteristics such as natural frequencies [6], mode shapes and their curvatures [7], damping ratio and stiffness or flexibility [8]. Former approaches fit particular cases; recent researches made by the authors found a phenomenon permitting the development of a robust, general method, able to detect, locate and assess open cracks in all types of beams, for a large range of severity levels [9], [10] and [11]. In includes a relation composed by two terms: one linked with the damage severity and the second to its position.

The work presented in this paper bring new knowledge in this field, by revealing the way how a coefficient can be determined using the deflection of the beam under own mass in damaged and undamaged case.

2 Damage detection method

Methods for detecting damage in beams were presented in the works [12], [13]. In this paper is firstly made a brief description of this method.

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The considered damage is open type damage; on the width of the beam and the damage can be located anywhere along the length of the beam. The depth of the damage can take values of up to half the height of the cross section.

Method for detecting the damage in beams consists of measuring the natural frequency for the first ten modes of vibration on weak bend axis of the beam by using a single accelerometer, having already measured the natural frequencies for the undamaged beam and comparison of relative differences with specified value obtained analytically or numerically, values stored in database. By comparison, result the damage location and the assessment of damage depth [14].

Analytical calculation of natural frequency for the damaged beam, established by the authors is:

Relatia analitica pentru calculul frecventelor proprii la bara cu defect, stabilita de autori este de forma:

$$f_D(x,\partial)_i = f_{U\,i}\left(1 - c(\delta)\cdot\left(\frac{d^2\phi(x)_i}{dx^2}\right)^2\right) \quad [Hz] \tag{1}$$

where,

$f_D(x,\partial)_i$ *[Hz],* represents the natural frequency of vibration mode *i* for the damaged beam with damage located in the *x* position and damage depth δ;

$f_{U\,i}$ *[Hz],* represents the natural frequency of vibration mode *i* for the undamaged beam;

$\left(\frac{d^2\phi(x)_i}{dx^2}\right)^2 \in 0\ldots1$, is the second derivative of the mode shape. It is the law of natural frequency changes for damaged beam at vibration mode *i*;

$c(\delta)$ a coefficient that depends on the depth of the damage. For a damaged beam with the geometry, mechanical parameters and boundary condition known, the coefficient $c(\delta)$ has the same value regardless of mode of vibration *i*.

In other words, for a damaged beam, knowing the value of the coefficient $c(\delta)$ and the damage location, by changing the value of natural frequency of undamaged beam in relation (1), for other vibration mode, by calculation, get its natural frequency for the damaged beam at considered vibration mode.

The relationship (1) has been verified and validated by numerical methods and experimental measurements.

Verification of relationship by numerical methods, modal analysis by using FEM, was made on beams of rectangular section with three type of boundary conditions: clamped beam, double clamped beam and simply supported beam.

Validation of the method was made by experimental methods on clamped beams with different beam length and various cross sections.

For example, it is considered the clamped beam in figure 1.

The results obtained by analytical, numerical and experimental methods for the undamaged beams are presented in table 1 and 2, the results obtained for damaged beam with damage depth $\delta=0.25H$, coefficient $c(0.25H) = 0.009507$, damage location at $x_1=0.098L$ are presented in table 3 and the results obtained for the damage location at $x_2=0.31L$ are presented in table 4. The damage width is of *0.002L*.

The real analyzed beam was a steel one, having the following geometrical characteristics: length $L = 1000$ *mm*; width $B = 50$ *mm* and height $H = 5$ *mm*.

Consequently, for the undamaged state the beam has the cross-section $A = 250\cdot10^{-6}\ m^2$ and the moment of inertia $I = 520.833\cdot10^{-12}\ m^4$.

The material parameters of the specimens are: mass density $\rho = 7850\ kg/m^3$; Young's modulus $E = 2.0\cdot10^{11}\ N/m^2$ and Poisson's ratio $\mu = 0.3$.

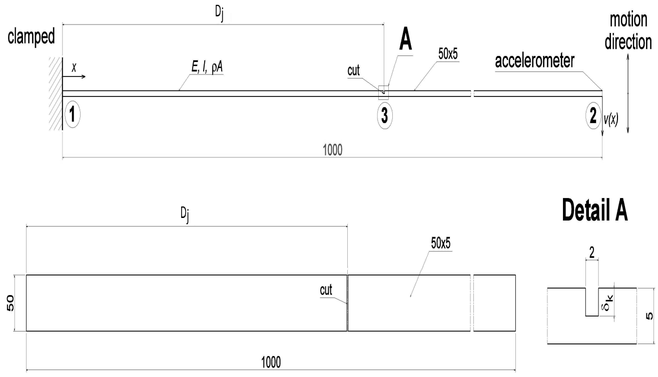

Fig. 1. Cantilever damaged beam

Table 1

Natural frequencies for undamaged beam [Hz]			
Vibration Mode	Analytic	FEM	Measured
i	f_{Ai}	f_{FEMi}	f_{Mi}
1	4.077	4.08986	4.035
2	25.550	25.6266	23.284
3	71.539	71.7545	70.970
4	140.189	140.6275	139.090
5	231.742	232.5200	230.336
6	346.182	347.4518	344.196
7	483.511	485.4578	481.809
8	643.727	646.5624	641.261
9	826.832	830.7827	823.897
10	1032.825	1038.1089	1030.068

Table 2

Natural frequencies for undamaged beam [Hz]			
Vibration Mode i	Analytic	FEM	Measured
	f_{Ai}	f_{FEMi}	f_{Mi}
1	4.077	4.08986	4.060
2	25.550	25.6266	25.439
3	71.539	71.7545	71.426
4	140.189	140.6275	139.902
5	231.742	232.5200	231.038
6	346.182	347.4518	344.750
7	483.511	485.4578	482.503
8	643.727	646.5624	641.823
9	826.832	830.7827	824.910
10	1032.825	1038.1089	1030.707

In table 5, are given the relative frequency shift from tables 1 and 3 and in table 6 are given the relative frequency shift from tables 2 and 4.

The relative frequency shifts have been determined with the relationship (2)

$$\Delta f(x,\partial)_i = \frac{f_{U\,i} - f_D(x,\partial)_i}{f_{U\,i}} \cdot 100 \quad [\%] \qquad (2)$$

Table 3

Natural frequencies for damaged beam with damage at 0.098L and δ=0.25H [Hz]			
Vibration Mode i	Analytic	FEM	Measured
	f_{DAi}	f_{DFEMi}	f_{DM}
1	4.061	4.059	4.005
2	25.512	25.551	25.211
3	71.518	71.711	70.911
4	140.188	140.649	139.87
5	231.652	232.438	230.175
6	345.797	346.907	343.458
7	482.631	484.084	480.231
8	642.303	644.241	638.874
9	825.057	827.813	821.035
10	1031.109	1035.40	1026.917

The results presented in tables 5 and 6, it is apparent that for a certain damage depth, if it is know the coefficient $c(\delta)$ for the first vibration mode, it can be determined the natural frequencies in analytical way for other vibration modes.

Table 4

Natural frequencies for damaged beam with damage at 0.31L and δ=0.25H [Hz]			
Vibration Mode i	Analytic	FEM	Measured
	f_{DAi}	f_{DFEMi}	f_{DM}
1	4.070	4.076	4.046
2	25.533	25.604	25.411
3	71.380	71.483	71.158
4	140.108	140.487	139.702
5	231.622	232.321	230.803
6	345.306	345.924	343.336
7	482.947	484.459	481.417
8	643.638	646.435	641.515
9	824.976	827.596	821.543
10	1031.051	1035.032	1027.791

Table 5

Relative frequency shift for damaged beam with damage located at x_1=*0.098L* [%]			
Vibration Mode i	Analytic	FEM	Measured
	Δf_{Ai}	Δf_{FEMi}	Δf_{Mi}
1	0.712	0.761	0.732
2	0.270	0.295	0.288
3	0.056	0.060	0.083
4	0.001	-0.015	0.002
5	0.071	0.035	0.070
6	0.206	0.157	0.214
7	0.337	0.283	0.328
8	0.410	0.359	0.372
9	0.398	0.358	0.347
10	0.308	0.286	0.306

Table 6

Relative frequency shift for damaged beam with damage located at x_1=*0.31L* [%]			
Vibration Mode i	Analytic	FEM	Measured
	Δf_{Ai}	Δf_{FEMi}	Δf_{Mi}
1	0.317	0.337	0.344
2	0.117	0.089	0.109
3	0.412	0.378	0.375
4	0.106	0.100	0.143
5	0.096	0.086	0.102
6	0.469	0.440	0.410
7	0.216	0.206	0.225
8	0.026	0.020	0.048
9	0.416	0.384	0.408
10	0.318	0.296	0.283

The question arises of determining the $c(\delta)$ coefficients.

3 Global stiffness reduction and changes in frequency

The natural frequencies for the undamaged cantilever beam are are given with relation:

$$f_{Ui} = \frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{EI}{\rho A L^4}} \quad [\text{Hz}] \tag{3}$$

where α_i represents the first ten solutions of equation: $1 + \cos\alpha \cdot \cosh\alpha = 0$, for undamaged cantilever beam and *g [m/s²]* is the acceleration due to gravity.

The maximum deflection for the cantilever beam is obtained with the relation:

$$v = \frac{\rho g A L^4}{8EI} \quad [\text{m}] \tag{4}$$

From relation (4) it can be obtain:

$$\frac{EI}{\rho A} = \frac{g L^4}{8v} \tag{5}$$

which is introduced in formula (3) and the natural frequencies for the undamaged cantilever beam can be calculated with the formula:

$$f_{Ui} = \frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{g}{8 \cdot v}} \quad [\text{Hz}] \tag{6}$$

By analogy, in the case of damaged beam, with the damage located near the clamped end (D_j~0 in figure 1), the natural frequencies shall be determined:

$$f_{Di} = \frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{g}{8 \cdot v_V}} \quad [\text{Hz}] \tag{7}$$

where v_D *[m]* represents the maximum deflection of the damaged cantilever beam.

Taking into consideration the relationship (2) and by replacing the relationships of natural frequencies (6) and (7), results:

$$\Delta f_i = \frac{f_{Ui} - f_{Di}}{f_{Ui}} = \frac{\left(\frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{g}{8 \cdot v}}\right) - \left(\frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{g}{8 \cdot v_D}}\right)}{\left(\frac{\alpha_i^2}{2\pi} \cdot \sqrt{\frac{g}{8 \cdot v}}\right)} \tag{8}$$

Finally, the relationship (8) can be written:

$$\Delta f_i = \frac{f_{Ui} - f_{Di}}{f_{Ui}} = \frac{\sqrt{v_D} - \sqrt{v}}{\sqrt{v_D}} \tag{9}$$

Consequently, the natural frequency for the damaged beam becomes:

$$f_{Di} = f_{Ui}(1 - \Delta f_i) = f_{Ui}\left(1 - \frac{\sqrt{v_D} - \sqrt{v}}{\sqrt{v_D}}\right) \tag{10}$$

Taking into consideration the second derivative of the mode shape and introduced it in (10), get the function for natural frequencies for the damaged beam:

$$f_{Di}(x,y) = f_{Ui}\left(1 - \frac{\sqrt{v_D(y)} - \sqrt{v}}{\sqrt{v_D(y)}} \cdot \left(\frac{d^2\phi(x)_i}{dx^2}\right)^2\right) \tag{11}$$

By comparing (1) to (11) shall be observed that:

$$c(\delta) = \frac{\sqrt{v_D(y)} - \sqrt{v}}{\sqrt{v_D(y)}} \tag{12}$$

is the coefficient that depends on the depth of the damage.

For a cantilever beam with two cross sections, loaded under its own weight, the maximum deflection is given by formula:

$$v_D(y) = \frac{\rho g A}{8EI_0(y)}\left(L^*\right)^4 \quad [\text{m}] \tag{13}$$

where, $I_0(y)$ [m⁴] is the moment of inertia in the damaged area.

The moment of inertia depends on the damage depth;

$$\left(L^*\right)^4 = (L - x_1)^4 \frac{I_0(y)}{I} + \frac{x_1}{3}\left(12L^3 - 22L^2x_1 + 18Lx_1^2 - 5x_1^3\right)$$

where x_1 [m] is the damage width multiplied with $\sqrt{\pi}$.

For the considered cantilever beam, in table (7) are presented maximum deflection obtained by analytical and numerical calculation for the undamaged beam.

Table 7

Maximum deflection for undamaged beam *v [mm]*	
Analytic	**FEM**
23.095	23.082

For damaged beam, the maximum deflection obtained by analytical and numerical calculations are presented in table 8. The damage width is of 2 mm and damage depth of 8%, 17%, 25%, 33%, 42%, 50% and 58% in respect to the beam height.

The natural frequencies for the considered examples, analytic obtained with formula (11) and numeric obtained with modal analysis, are presented in tables 1 and 2 for the undamaged beams, and tables 3 and 4 for the damaged beams.

Table 8

Maximum deflection for damaged beam v_D *[mm]*		
Damage depth δ	**Analytic**	**FEM**
0.08H	23.188	23.146
0.17H	23.339	23.269
0.25H	23.540	23.433
0.33H	23.852	23.693
0.42H	24.437	24.185
0.50H	25.373	24.919
0.58H	27.161	27.244

4 Conclusion

The damage detection method presented in the paper, applicable to beams with open cracks, is based on certain phenomena characteristic to the dynamic behaviour of beams, highlighted as a result of several analytical, numerical and experimental studies developed by the authors.

Observing the relationship (11) we concluded that it is a relationship between deflection and frequency changes.

The relationship (12) gives a solution for the previous works of authors.

For a level of damage depth, the value obtained for *c(δ)* allows us to calculate the natural frequency for damaged beam in any vibration mode, in any location of the damage on the beam.

Researches performed this direction revealed that it is a clear dependency between deflection and frequency changes, what makes deflection an important feature of beam behavior, usable in damage assessment.

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