Statistical Inference of Reliability Estimation of Industrial Processes

DUMITRASCU ADELA-ELIZA, NEDELCU ANISOR
Department of Manufacturing Engineering and Industrial Management
"Transilvania" University of Brasov
29 Eroilor Street, 500036 Brasov
ROMANIA
dumitrascu_a@unitbv.ro; a.nedelcu@unitbv.ro; http://www.unitbv.ro

Abstract: - Probability and statistical inference represents the starting point for development, modeling and elaboration of software program. In this respects, the presented paper describe statistical notions and reliability estimation concepts, and the case study consists of development a software program based on statistical inference using specialized MathCAD functions.

Key-Words: - statistical inference, reliability estimation, probability density function, Monte-Carlo simulation.

1 Introduction

Reliability is considered to be a performance attribute that is concerned with the probability of success and frequency of failures, and is defined as: "The probability that an item will perform its intended function understated conditions, for either a specified interval or over its useful life", [2].

Product reliability is a critical part of total product quality. Product reliability refers to the ability of a product to continue to perform its intended functions over time in the hands of the customer [7].

If a product has such poor reliability that it is seldom available for use, these other performance measures become meaningless. Reliability is also critical to safety and liability issues.

Superficially, measuring reliability is a simple matter. One merely counts failures and divides by operating time to come up with a mean time between failures (MTBF), the most common reliability measure.

2 Statistical Modeling

A random variable X is said to be normally distributed if it density function is specified by [3]:

$$f(t)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(t_i-\mu)^2}{2\sigma^2}\right], \qquad (1)$$

where σ and μ are two parameters (constants) that denote the mean and the standard deviation.

If random variables X is normally distributed, then its mean $E(X)=\mu$ and its variance $Var(X)=\sigma^2$, [3].

Proof:

$$E(x)=\int_{-\infty}^{+\infty} t f_x(t)dt=\int_{-\infty}^{+\infty}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]dt\,. \qquad (2)$$

Let $y=t-\mu$, then

$$E(x)=\int_{-\infty}^{+\infty}\frac{(\mu+y)}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2\right]dy=0+\mu=\mu\,. \qquad (3)$$

The variance is:

$$Var(x)=\int_{-\infty}^{+\infty}(t-\mu)f_x(t)dt$$

$$=\int_{-\infty}^{+\infty}\frac{y^2}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2\right]dy$$

$$=\frac{2}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty}y^2\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2\right]dy$$

$$=\frac{2\sigma^2}{\sqrt{\pi}}\Gamma\left(\frac{3}{2}\right)=\sigma^2. \qquad (4)$$

Suppose now we wish to find $P\{X\le a\}$, where X is a normal variate, then:

$$P\{X\le a\}=\int_{-\infty}^{a}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]dt\,. \qquad (5)$$

By a simple transformation the computation of relation (5) it can be accomplished with a single table. Let

$$z=\frac{t-\mu}{\sigma}, \qquad (6)$$

whence

$$P\{X\le a\}=\int_{-\infty}^{(a-\mu)/\sigma}\frac{\exp\left[-\left(1/2z^2\right)\right]}{\sqrt{2\pi}}dz=\int_{-\infty}^{(a-\mu)/\sigma}f(z)dz$$

$$=\phi\left(\frac{a-\mu}{\sigma}\right). \qquad (7)$$

3 The Estimation Algorithm

In development of products are involved many factors. Variability of these factors makes the reliability to be a probabilistically and statistically modeling.

Development of a simulation program is complicated, but the results are similar to real situation.

Considering normal distribution the Monte-Carlo simulation program is detailed in figure 1.

In the case of estimation of reliability using exponential distribution, it calculates the failure rate - λ, and for Weibull repartition, we determine β, η and γ parameters.

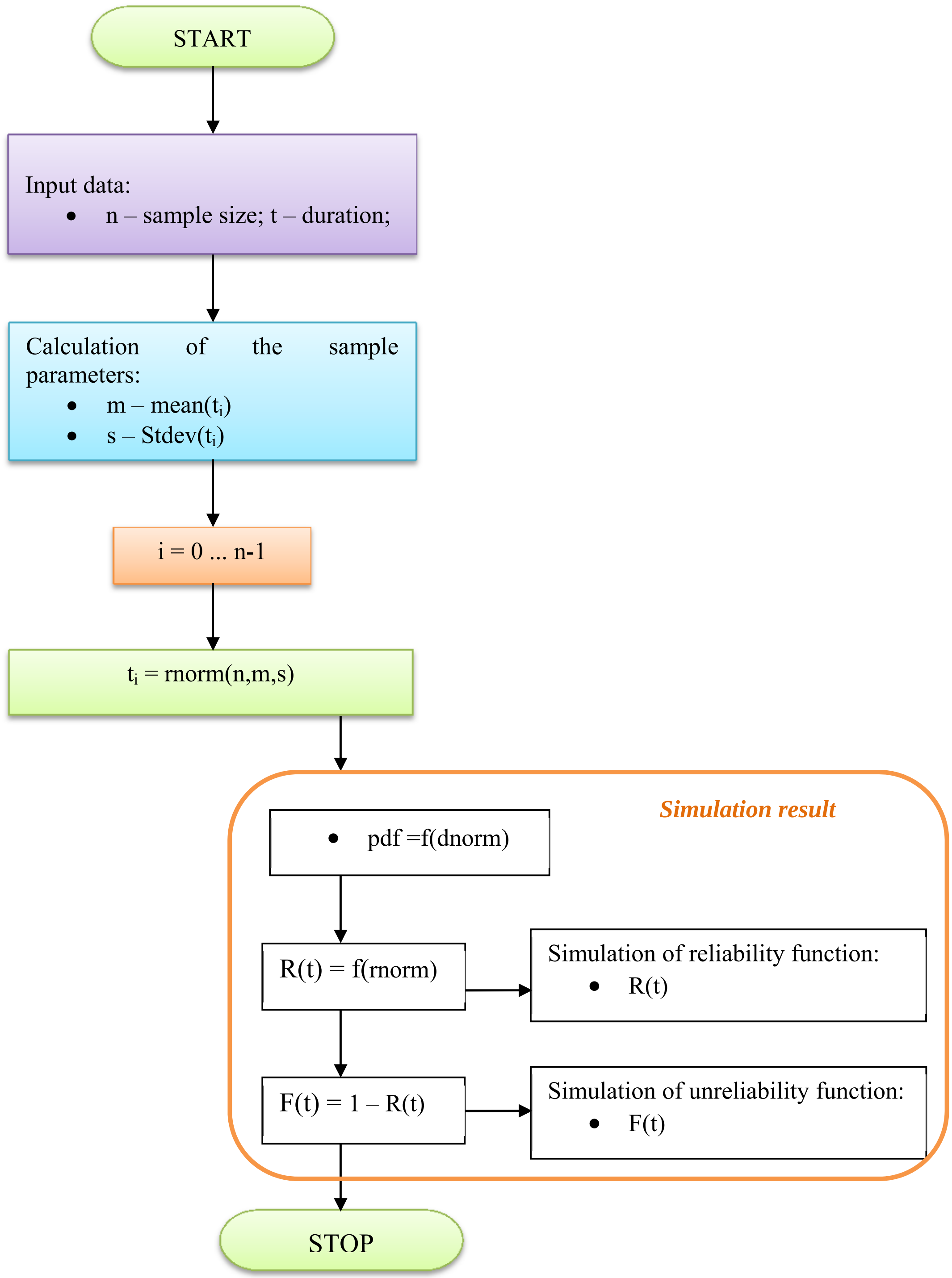

Figure 1. Monte-Carlo simulation program

About 2/3 of all cases fall within one standard deviation of the mean that is:

$$P(\mu - \sigma \le X \le \mu + \sigma) = 0.6826. \quad (8)$$

About 95% of cases lie within 2 standard deviations of the mean, that is:

$$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = 0.9544. \quad (9)$$

The unreliability function, [3] is given by relation

$$F(t) = \int_{-\infty}^{t} f(t)dt = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left[-\frac{(t_i - \mu)^2}{2\sigma^2}\right] dt, \quad (10)$$

and reliability function is:

$$R(t) = 1 - \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left[-\frac{(t_i - \mu)^2}{2\sigma^2}\right] dt. \quad (11)$$

In the case of exponential distribution, the density function is given by:

$$f_x(t) = \lambda \exp[-\lambda \cdot t]. \quad (12)$$

The distribution function is given by relation:

$$F(t) = 1 - \exp[-\lambda \cdot t]. \quad (13)$$

A random variable X is said to have the Weibull distribution if its probability density function is given by, [3]:

$$f_x(t) = \frac{\beta}{\eta} \cdot \left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \cdot \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]. \quad (14)$$

The distribution function is given by relation:

$$F(t) = 1 - \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]. \quad (15)$$

3 Monte-Carlo Simulation - Case Study

The methods used for estimation of products reliability should be those that meet the customer's needs in accordance with the strategy of the organization making the measurements [8]. For this reason, a variety of measurement methods, including test methods and specialized analytical techniques, it have been developed.

The case study consists of development of the simulation program in order to estimate the reliability of industrial processes.

The simulation program consists of:

- Generation of random values using the function of MathCAD software;
- Determination of the sample parameters;
  - For normal distribution: m - mean and s - standard deviation;
  - For exponential distribution: λ - failure rate;
  - For bivariate Weibull distribution: β, and η parameters.
- Estimation of probability density function;
- Estimation of reliability function using predefined MathCAD functions;
- Estimation of unreliability function using predefined MathCAD functions.

Considering normal distribution, the simulation results are illustrated in figure 2 and figure 3, respectively.

The probability density function and distribution function of the exponential distribution are shown in figure 4 and figure 5, respectively.

Simulation of probability density function for Weibull distribution is illustrated in figure 6, and reliability and unreliability functions in figure 7.

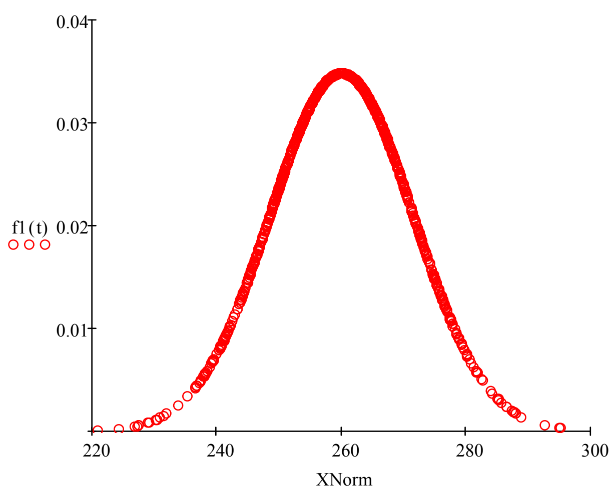

Figure 2. Simulation of probability density function - normal distribution

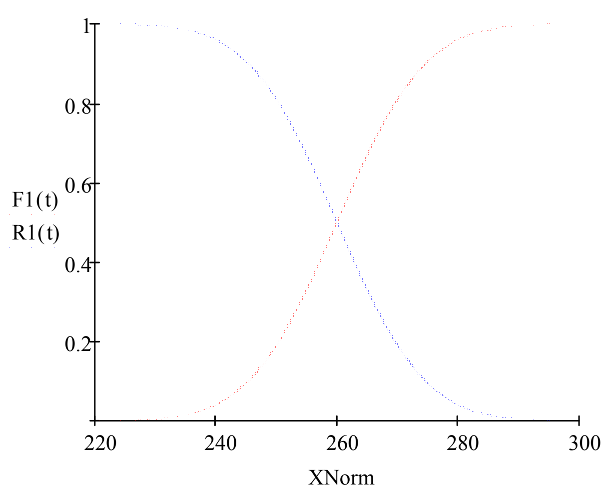

Figure 3. Simulation of reliability and unreliability functions - normal distribution

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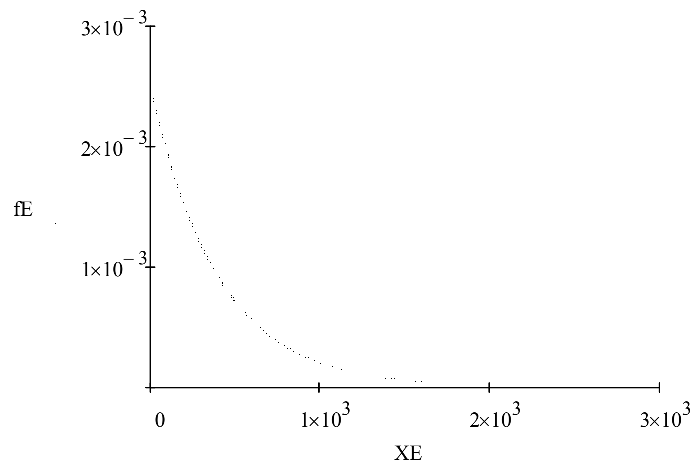

Figure 4. Simulation of probability density function - exponential distribution

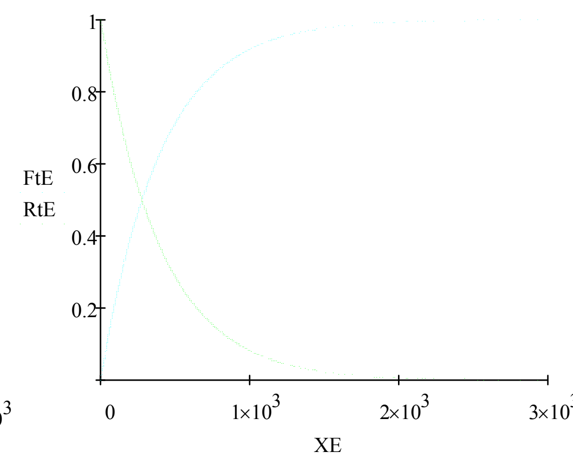

Figure 5. Simulation of reliability and unreliability functions - exponential distribution

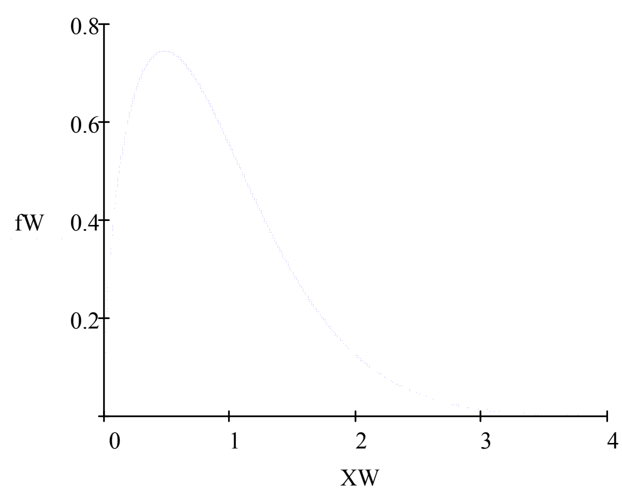

Figure 6. Simulation of probability density function - Weibull distribution

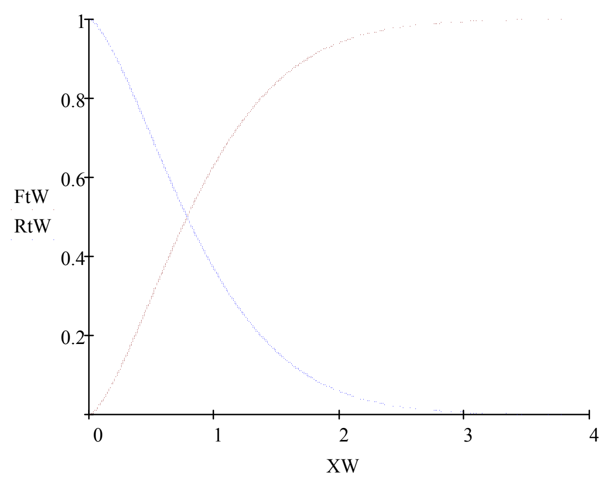

Figure 7. Simulation of reliability and unreliability functions - Weibull distribution

4 Conclusion

We have developed a simulation program for estimating the probability density, reliability and unreliability functions, using the programs Excel and Mathcad 14.0 with specialized functions.

Monte-Carlo simulation with Mathcad 14.0 software provides results comparable with experimental results. In the case study were obtained unreliability functions, reliability and probability density of industrial processes.

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