

# Formalization of Emergent Properties in Computing Agent Networks

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*Abstract:* - We propose notions and measures for the formalization and quantification of emergent properties in computing agent networks. Self-organization, homeostasis, autopoiesis and patterns are studied as emergent properties. Formal mathematic models for these emergent properties are developed with aspects related to equilibrium, self-production, order and topological indicators. The formality of the proposed representation can help in the better understanding of emergent properties in complex systems

*Key-Words:* - Emergence Quantification, Self-organization, Homeostasis, Autopoiesis, Graph Patterns, Multi-Agent Systems

## 1 Introduction

Properties such as emergence and self-organization have been used in order to explain complexity aspects in complex systems (CS) [1]. We understand by emergence, the properties of a higher scale that cannot be described based on properties in at low scale. We understand by self-organizing systems those that reach an “organized” state from the interaction of their element..

In spite of the useful of self-organization and emergence notions for CS study, it is appropriated the inclusion other aspects related with the adaptability and autonomy. That is the aspects related with homeostasis and autopoiesis. Homeostasis is referred to the establishment of a dynamic equilibrium among elements. Autopoiesis, as homeostasis extension, is related whit self-production and/or self-maintenance [2].

In addition, CS can show some structures or specific behaviors that could be observed as spatial-temporal patterns [3]. There is a particular case of topological patterns in a kind of network that represents a wide range of CS, the small-world networks (SW). These patterns are represented by the clustering coefficient ( $C_a$ ) and the characteristic path length ( $l$ ) [4].

Due to self-organization, homeostasis, autopoiesis and patterns (SHAP) are produce by

interactions and relationships among system elements, they can be seen emergences [3].

For SHAP modeling purposes, Computing Agent Networks (CAN) can be considered as a promising paradigm to be explored. CAN have the following structure  $C(N;K; a; F)$ , where, nodes ( $N$ ) can be described as agents and edges ( $K$ ) can be described as interactions. An algorithm ( $a$ ), can regulate the interactions between agents to reach a global state (using  $F$ ) [4].

This paper proposes the formal description of SHAP properties that can be used for CS modeling which are represented as CANs. Also, this approach enriches the approximations for emergent properties in CS systems defined as CAN. It is important to highlight that mathematical models presented here, for self-organization and emergence, expands the traditional way of quantification based on statistical entropy or information theory. This expansion permits a better analysis of a lot system founded in the universe.

## 2 Formal Aspects of Emergent Properties in Computing Agent Networks

### 2.1 Homeostasis

Homeostasis (Hm) can be defined as a general self-regulation mechanism that promotes the stability and flexibility in complex systems (CS) [4]. By means of Hm CAN facing up to changes, influences and/or perturbations from the endogenous or exogenous environment (Ev). Hm can considerer the following mechanisms and indicators.

### 2.1.1 Homesotatic Mechanisms

In CAN homoeostatic self-regulation processes can be occurs in both complementary ways, which can be active or passive as following is describe.

**Passive Mechanism:** Passive self-regulation is defined by tolerance range to one o more environmental factors, according to the following features:

- To face up to the influence of an environmental factor Ev, a computing network operates within a viability zone (Vz).
- The limits of Vz are defined by the viability ranges (RX) of nodes (N) and edges (K), to influences of the environment Ev.
- RX is calculated from the subtraction of the maximum ( $X_{max}$ ) and minimum ( $X_{min}$ ) values of response of Ev to the factor i. This way, the CAN viability range for a specific environmental factor ( $RX_i$ ) is obtained. Thus, for each i factor of Ev one node j has a viability range such that  $RX_i^j = X_{max} - X_{min}$ . Also, edges has a viability range for factor i of Ev, noted by  $RX_i^k$ . Consequently, the global viability range will be calculated as the mean of  $RX_i^j$  and  $RX_i^k$ .
- $RX_i$  is associated with the network response to the factor i of Ev. This response is named Tolerance to the factor i of Ev, which is  $TEv_i$ . It is assumed that  $TEv_i$  corresponds, in several cases, with a Gaussian function (Fig. 2). The choice of this function is based on the coincidence with the behavior of a great number of ecological and social systems. In case of different behaviors, the use of other kind of distributions is recommended.
- Inside of the viability zone Vz, an optimal zone of operation OZO can be defined. The OZO statistics limits are between  $\mu_i \mp \sigma_i$ .

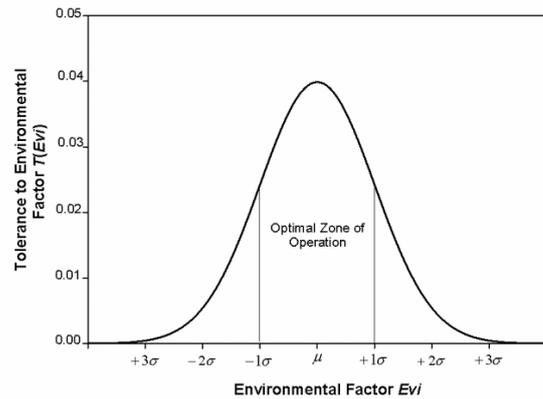


Fig. 2. Viability Zone Vz and Optimal Zone of Operation of a computing network facing an Ev<sub>i</sub> factor. Values in x axis is related with dominia of i factor and are expressed in standar deviations units. Values in y axis are expressed in probability interval [0,1]

- In order to the calculation of a tolerance value for a specific  $X_i \in RX_i$  of  $Ev_i$  factor, the standard deviation  $\sigma_i$  and the average  $\mu_i$  of  $RX_i$  should be considered. Equation 1 is appropriated to do this. Right there,  $\pi$  is equal to 3.1416 and  $e$  is the Euler's number.

$$T(Ev_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(Ev_i - \mu_i)^2}{2\sigma_i^2}\right) \quad (\text{Eq. 1})$$

- In self-organization terms, higher  $T(Ev_i)$  levels coincide with higher satisfaction levels for a node or agent. This happens when an agent reach a particular functional objective (e.g. survive, minimum energy, minimum delay, money, etc). At the same time, the friction (negative interactions) among agents, are low. This is in accordance with the sigma profile proposes by [4], in which minimizing friction in low scales increases benefits in high scales.
- An  $Ev_i$  factor can affects all nodes (global influence) or some nodes (partial influence). However the individual response of nodes or K for these influence, could be different due to the individual tolerances to  $Ev_i$ .
- If the operation of nodes or edges is outside of its viability zone (Vz), they will operate in a zone distinguish as critical (Cz). Operation in Cz can lead to the failure or degradation of CAN or their elements. This fact, in autopoietical terms, is a limitation for self-maintenance because of the structure and function will be affected. Nevertheless, the CAN could balance, in homeostatic (self-

regulation) and autopoietic (self-maintenance) sense, this effect.

**Active Mechanisms:** First active self-regulation is carried out by the combination of local functions  $f_i$  of each node. Combination is made by means of an operator, “o” in this case, giving as result a global function  $F$ . For instance,  $F = f_1 \circ f_2 \neq F = f_2 \circ f_1$ . The operator o is part of the algorithm a, which means that active self-regulation is specified in the algorithm.

The second active homeostatic mechanism to face up to the influence of environmental factors influence can be founded in algorithm a of CAN. This mechanism consists in choosing a proper action  $y \in Y$  to confront the environmental factor  $E v_i$ . To do this, an objective function ( $f_{ob}$ ) that restore CAN to Vz, will be executed. That is  $y(E v_i, f_{ob}, X_{max}, X_{min})$ .

For the  $f_{ob}$  execution is require that: (i) the action y controls the influence or imbalance in an effective way, and (ii)  $f_{ob}$  does not has a new imbalance in other components of the CAN. Thus,  $f_{ob}$  should cause a minimal imbalance and should compensate the  $E v_i$  influence.

Also, the above vision coincides with the maximization of satisfaction and friction reduction in the computing network expressed by [4].

The second way of active homeostatic mechanism can be extended to autopoiesis. This way, an autopoietic function  $f_{\omega}$  can be involved in the production (understood as balance of synthesis-decay) of its elements.

### 2.1.2 Homeostatic indicators

So far the stability and flexibility of CAN have been supported by tolerance function  $T(E v_i)$  that involves  $R X_i$  and in the mechanism of active self-regulation. However, in a complementary way we propose additional indicators that can help to determine the flexibility and fragility of the CAN. Accordingly, notions for resilience, persistence and resistance expressed by [7] were taken into account. From these three notions, we established a set of new mathematical formalisms that will support an integrated homeostasis measure. The proposed indicators are:

- Resilience ( $R s_i$ ): rate (velocity) of returning to media point  $\mu_i$  at the viability zone Vz. It is defined by

$$R s_i = |D_i|/t \text{ (Eq. 2)}$$

Where  $|D_i|$  or distance is the absolute value of the difference between a value  $X_i \in R X_i$  and its average  $\mu_i$ , such that  $D_i = |X_i - \mu_i|$ . t is the time

of returning to  $\mu_i$ . The indicator  $R s_i$ , also can be expanded to calculate the returning rate to specific value inside the optimal zone of operation OZO. Higher values of  $R s_i$  indicates higher homeostatic capacity.

- Persistence ( $P e_i$ ): measure of time that spends in  $\mu_i$ , or closer to  $\mu_i$ .

$$P e_i = t_r \text{ Eq. 3}$$

Where  $t_r$  es the residence time in that value. Higher values of  $P e_i$  indicates higher homeostatic capacity and stability.

$R s_i$  and  $P e_i$  indicators have different behavior with the increasing of time. That means that with higher returning time's resilience and the homeostatic capacity drop. When residence time in media point is higher, the persistence and homeostatic capacity are higher.

- Maximun Resistance ( $Max R_i$ ): measure of the degree of displacement from absolute value of  $\mu_i$ , such that the maximum displacement degree is the absolute value of  $X_i$ . That is represented by equation 4.

$$Max R_i = |R X_i|/|\mu_i|. \text{ (Eq. 4)}$$

Higher values of  $Max R_i$  indicates higher homeostatic capacity and stability. According with the  $X_i$  taken in account, variants of this measure will be calculated as OZO-resistance or instantaneous-resistance.

- Vulnerability ( $V u_i$ ): susceptibility or fragility of computing network to  $E v_i$ , is defined by an inverse function of the viability range  $R X_i$ , such that:

$$V u_i = 1/(1+R X_i) \text{ (Eq. 5)}$$

where  $V u_i \in [0,1]$ . Higher  $V u_i$  indicates low homeostatic capacity and stability. Consequently, fragility is higher.

By reason of the self-regulation mechanism of homeostasis is a dynamical process; the calculation of the above indicators should be considered its time changing.

In Synthesis, homeostasis as self-regulation capacity to facing up an environmental factor  $E v_i$ , can be formalized as a  $\theta$  function of tolerance, resilience, persistence, maximum resistance and vulnerability such that  $H m_i = g(T(A b_i), R s_i, P e_i, Max R_i, V u_i)$ . If  $R s_i, P e_i, Max R_i$  are normalized to  $[0,1]$  scale, the  $\theta$  function used to calculate the average of all homeostatic indicators.

## 4.2 Autopoiesis

Autopoiesis (A) is a particular self-regulation process of synthesis and decay of nodes (N) and/or edges (K) that compose the CAN structure [3]. This way, A is supported in homeostatic mechanism focused in elements constitution and promotes the “production of the system” and “structure preservation” [20]. By means of A the capacity of development, maintenance, production and establishment of CAN identity and unity at specific level is achieved. In particular, A is based on the hetero and self-referencing processes. This fact requires of a determined degree of cognition. Cognition, according with [4], is referred to the knowledge that the system has about how to act in its environment.

In formal terms, autopoiesis A corresponds with a function of self-maintenance (fa), which is part of active mechanism of homeostasis supported on the local combination of the (fi) combination. This way, fa regulates the natural network wearing. The wearing of the network, in homeostatic sense, is viewed as a variant of  $Ev_i$ .

On the basis of above notion and mechanism, it is assumed that autopoiesis can be measurable by mean of degree of self-maintenance of the CAN from production of nodes N and edges K. Thus, autopoietic self-maintenance can be expressed as mass balance [21] between synthesis (S) and decay (Dc) of the CAN elements. The change in number of nodes, expressed as mass balance is:

$$\frac{dN}{dt} = \frac{dS}{dt} - \frac{dDc}{dt} \quad (\text{Eq. 6})$$

Synthesis (S) of N is equal to  $S = \gamma N$ , where  $\gamma$  is the average rate of N synthesis. Decay (Dc) is equal

to  $D = -\lambda N^d$ , where  $\lambda$  corresponds with the

average rate of decay of N. Exponential  $d$ , is the decay order [9]. Now, equation 6 takes the following form:

$$\frac{dN}{dt} = \gamma N - \lambda N^d \quad (\text{Eq. 7})$$

If order  $d=1$ , N can be factorized as follow:

$$\frac{dN}{dt} = \gamma N - \lambda N^1 = N(\gamma - \lambda) = Nr p_N \quad (\text{Eq. 8})$$

Where  $r p$  is named reparability rate and is the result of subtraction of rates (synthesis and decay). If  $r p_N > 0$  means that self-production capacity is increasing and network is growing. If  $r p_N = 0$  self-production capacity is stable. If  $r p_N < 0$  self-production capacity is decreasing and network is decaying.

Previous argumentation can be instantiated for edges production. This way, for an edge  $K(i,j)$ , its balance of synthesis and decay is as follows:

$$\frac{dK}{dt} = \gamma' K - \lambda' K^1 = K(\gamma' - \lambda') = Kr p_K \quad (\text{Eq. 9})$$

Where  $\gamma'$  and  $\lambda'$  are the rates of synthesis and decay, and  $r p_K$  is the edges self-production rate.

For the overall computing network the average of  $r p_N$  y  $r p_K$  is equal to self-production rate  $r p_N$  and it is an expression of the computing network self-reparability and self-maintenance.

It is important to highlight that all autopoietic systems are homeostatic systems, but no all homeostatic systems are autopoietic systems. For this reason, only CAN with both characteristics will include the  $r p_N$  rate in the homeostatic measure.

### 2.3 Self-organization

Self-organization (SO) is a dynamic process that generates the structure and maintains computing network functionality, in order to reach its design goal. SO is achieved in terms of essential attributes and occurs without mediation of a central control and under changing conditions of the surroundings [3]. Essential attributes corresponds to (i) autonomy or no dependence of computing agents network to reach its function and goal; (ii) stability or balance; (iii) persistence as functionality maintenance; (iv) robustness as tolerance to external pressures or internal failures; (v) flexibility related to the susceptibility to change structurally according to the needs; (vi) integrity, which defines the unit in terms of the whole, the parts and relations [3].

As can be seen in these essential attributes, Self-organization SO is supported on the basis of homeostasis (Hm) and autopoiesis (A). For that reason and as explained in item II, there are a synergistic condition among SO, Hm and A. In consequence, a SO measure can consider a homeostatic component. If the system is homeostatic and autopoietic at the same time, Hm will include an autopoietic component represented in the reparability rate  $r p$ .

Including a homeostatic component (Hm) in Self-organization (SO) measure results in a novel notion of SO. It is well-known that traditionally SO has

been related to the statistics measure of entropy, for no imposed order representation. An example of this is the measures proposed by [[8] supported on the information theory. This expression is indicative of more or less order in a computing agent network that has a specific configuration. The Mathematical expression is:

$$Hc_t = -\sum_{c \in C} P(c_t) \log_2 P(c_t) \quad (\text{Eq. 10})$$

Where  $Hc_t$  is a measure equivalent to the information (in bits) containing in nodes and edges of a configuration  $c$ , in an instant of time  $t$ . Configuration  $c$  is the complete description of CAN in the time  $t$ . The order/information measure  $Hc_t$  including a normalize probability distribution  $P(c_t)$  that is equivalent to the CAN probability for  $c$  configuration in  $t$  time [8].

We propose the addition of the homeostatic component  $Hm$  with the order measure  $Hc_t$  for self-organizing process characterization in time  $SO_t$ . Thus,  $SO_t$  is a  $g$  function of information changing  $Hc_t$ , on the way through the different computing network configuration  $c_0 \rightarrow c_1 \rightarrow \dots \rightarrow c_n$ , and homeostatic capacity changing ( $Hm$ ), such that  $SO_t = g(Hc_t, Hm)$ . Giving the synergy between homeostasis and self-organization processes, it is presumed that  $g$  function could calculate the multiplication of  $\nabla Hc_t$  values.

### 2.4 Patterns in Computing Agent Network

In the field of networks theory is well-known the use of the clustering coefficient ( $C$ ) and the characteristic length ( $l$ ) as topological properties.  $C$  and  $l$  can use as patters especially in small world networks (SW), because it has been observed that large  $C$  are in correspondence with small  $l$  [5].

SW networks are an intermediate kind of network between regular and random networks. Also, SW are a particular case of networks with high level of complexity due to its relevant interactions and the difficulty of its state prediction. At the same time SW corresponds with a lot of social and ecological phenomena of reference [6].

From the perspective of the use of clustering coefficient  $C$  and  $l$  as patterns in computing networks, complex and emergent system can be characterized. According with [6]  $C$  is the media fraction of the neighbors of an  $i$  node which, in turn, are neighbors to each other. The local expression for  $C$  is:

$$C_i = \frac{2K}{k_i(k_i-1)} \quad (\text{Eq. 11})$$

Where  $K$  is the fraction of existing edges or connected neighbors and  $k_i$  is the degree of the  $i$  node, understood as the number of edges of this node.

Global expression for  $C$  will be equal to the average of the local  $C_i$ .

Characteristic length  $l$  is referred to the average distance (number of edges), among all pairs of nodes. That is:

$$l = \frac{1}{\frac{1}{2}N(N-1)} \sum d_{ij}$$

Where  $d_{ij}$  is the distance between nodes  $i$  y  $j$ . This fact is indicative of computing network structure as result of the nodes interactions.

From [5], is possible to obtain  $C$  and  $l$  averages and standard deviation, for small world networks. From them, maximum and minimum statistical limits can be established. All of these statistical parameters can be catalogued as “expected” for complex networks, and can be taken as nominative values of patterns.

The procedure for pattern quantification in computing network, from  $C$  and  $l$  observed values is made on the basis of statistical normalization using chi-square  $\chi^2$  distribution. The sequence to do that is: (i) observed  $C$  and  $l$  values ( $C_{obs}$ ,  $l_{obs}$  are transformed in statistical value of  $\chi^2$ , using the following equation:

$$\chi^2 = \frac{(C_{obs}-C_{exp})^2}{C_{exp}} + \frac{(l_{obs}-l_{exp})^2}{l_{exp}} \quad (\text{Eq. 12})$$

Where  $C_{exp}$  and  $l_{exp}$ , are the average listed in table I. (ii) Then,  $\chi^2$  values are transformed in probability values using statistical tables with  $n-1$  degrees of freedom and a confidence value of 95%. Probability values obtained represent the measure of topological patterns expression  $P(Pe)$ .

### 4.5 Emergence (E) as composed function

The strong relation of self-organization (SO), homeostasis (Hm) and autopoiesis (A) could be useful to synthesize the emergent behavior of the CAN. It means, the emergent condition of the network is supported in self-regulation, self-maintenance and no imposed order. However,  $C$  and  $l$  pattern expression ( $P(Pe)$ ) has in the same way of SO, Hm and A, an emergent feature. For that reason, it is possible to develop an expression for the quantification of the emergence in CAN that considers all emergent properties studied here. In consequence, the emergence (E) is proposed, in a dynamic sense, as  $g$  function such that  $E=g(SO_t,$

Ept) It means, the change of E is a function of the change in self-organization and patterns expression. It is estimated that function g could calculate the multiplication between these two indicators.

On the basis of the complexity measure proposed by [2], it is estimated that the relation between self-organization (SO) and patterns (P(Pe)) that defines the emergence measure (E), could generate the behavior depicted in figure 2. This is an intuitive behavior that considers the variation of SO, (P(Pe)) and E, for networks that goes from regular to irregular, where the randomness of interactions or wirings is progressively increasing. It is noticed that in regular networks the pattern expressions it would be higher (near to 1), while in irregular networks the expression would be low. Irregular networks show higher SO, because the amount of information required for its state description is higher as well. The E measure, as product of SO and P(P(e)), could be described similar to a Gaussian function. The higher values of E are located in an intermediate point, between regular and irregular networks. This is a zone, where the small world networks could be situated.

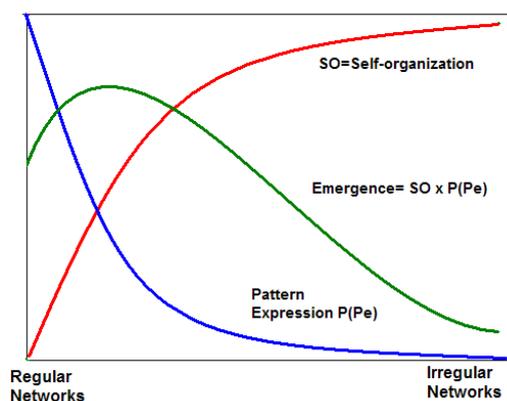


Fig 2. Intuitive notion of SO, P(Pe) and E magnitudes for networks from regular to irregular, where randomness is increasing.

## 5 Final Remark and Future Work

This document presents the conceptual and formal notions that suggest a wide approach for complex systems CS with emergence features. This was possible by considering the characterization and quantification of the components of no imposed order (self-organization), self-maintenance (autopoiesis) and pattern expression. These processes have as a common factor its emergent condition, due to the correspondance with an elemental way of complex behavior. This behavior comes from the local interactions, which is viewed

as global and synthetic conduct of the whole computing network.

Future work is directed towards planning instantiations of the formal models here explained, considering several study cases in order to clarify its usage. At the same time, a computational tool for easy calculus of all measures will be developed.

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