Fuzzy Connectivity of Graphical Fuzzy State Space Model of Fuzzy Graph Type 3

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Abstract: - Graphical Fuzzy State Space Model (GFSSM) incorporates the concept of fuzzy graph and Fuzzy State Space Model. Initial development for systems of Fuzzy State Space Model and fuzzy state space algorithm were introduced with the purpose of solving the inverse problem in multivariable dynamic system. The concept of Fuzzy graph theory was then applied to describe connections between subsystems, in particular interconnected system of graphical representation of Fuzzy State Space Model. In this paper, we focused on fuzzy graph type 3 to represent the connectivity in a multi-connected system. This new approach leads to the development of the definition of Single Fuzzy Edge Connectivity (SFEC) in a multi-connected system, in particular the boiler system.

Key-Words: - Fuzzy Graph, Fuzzy Algorithm, Power-plant, Graphical Representation

1 Introduction
In recent studies of complex control system, graphical Fuzzy State Space Model (GFSSM) have been developed to define and interpret the interconnection structure using graph theory. Subsystems were associated with vertices while interconnection with edges of the graph [6, 12]. Fuzzy set theory provides an important tool in dealing with various aspects of complexity, imprecision and fuzziness of the network structure of a multi-connected system [4, 11]. Hence, the initial development of Fuzzy State Space Model (FSSM) was discussed in [2, 5]. Meanwhile fuzzy graph is another extension of the application of fuzzy sets in its relation to graph theory [13]. Rosenfeld [8] has defined fuzzy graph to consist both fuzzy set for vertices as well as for the edges. Later, various types of fuzziness that exists in graphs were generalized and named as taxonomy of fuzzy graphs [14]. In this paper we will present on how fuzzy graph is used to define the connectivity between subsystems in a graphical representation for systems of FSSM.

2 Graphical Representation For Systems of FSSM
The transformation of Fuzzy State Space Model (FSSM) of multivariable controlled system to a graphical representation of FSSM (GFSSM) was discussed in [7]. A combination of two mappings was involved in this transformation. The first mapping embeds the FSSM to a point in Euclidean space. The second mapping transforms the point in Euclidean space to a graph. The definitions are as follows:

Theorem
Given a fuzzy state space system
\[ S_{gfj} = (A, B, C) \in A_{p \times p} \times B_{p \times n} \times C_{m \times p} \]

such that
\[ A = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pp} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{p1} & \cdots & b_{pn} \end{bmatrix} \]

\[ C = \begin{bmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mp} \end{bmatrix} \]

The transformation,
\[ \theta: A_{p \times p} \times B_{p \times n} \times C_{m \times p} \rightarrow \mathbb{R}^{p^2+pn+mp} \]

\[ \theta(S_{gfj}) = (a_{11}, \ldots, a_{pp}, b_{11}, \ldots, b_{pn}, c_{11}, \ldots, c_{mp}) \]

is a bijective map.

The above transformation embeds a single FSSM of a system to a point in a finite Euclidean space. The
idea of divisor and feeder were introduced to represent simple interconnection system of FSSM [6]. Based on both concepts, the connectivity between two systems of FSSM is defined as follows:

**Definition 1**

The connectivity between two fuzzy state space systems is defined as

\[ C_{GF} = \{ \left( S_{gFi}^{*}, S_{gFj}^{*} \right) : S_{gFi}^{*} \mid S_{gFj}^{*} \} \]

where \( S_{gFi}^{*} \) is a feeder of \( S_{gFj}^{*} \) (written as \( S_{gFi}^{*} \mid S_{gFj}^{*} \)) if at least one or all the output(s) of \( S_{gFi}^{*} \) is/are the input(s) of \( S_{gFj}^{*} \) and \( S_{gFi}^{*} \subseteq S_{GF} \).

In order to explore the properties of graphical Fuzzy State Space model, these points and links are then map from a finite Euclidean space to a graph representation using the following definition:

**Definition 2**

\( n^{th} \) Euclidean space to Graph representation

Given a set of point, \( S_{gF1}^{*}, S_{gF2}^{*}, \ldots, S_{gFk}^{*}, \in E^{p+2+pn+mp} \), we can map points to graph representation by the mapping \( \phi : E^{p+2+pn+mp} \rightarrow G(V, E) \) such that,

\[ V = \{ S_{gFi}^{*}, S_{gFj}^{*} \mid S_{gFi}^{*} \in E^{p+2+pn+mp}, i = 1,2,\ldots,k \} \cup \{ S_{gfE}^{*} \} \]

and,

\[ E = \{ S_{gFi}^{*}, S_{gFj}^{*} \mid S_{gFi}^{*} \in E^{p+2+pn+mp}, i = 1,2,\ldots,k \} \]

\[ \cup \{ S_{gfE}^{*}, S_{gfE}^{*}, S_{gfE}^{*}, S_{gfE}^{*} \} \]

where \( S_{gfE}^{*} \) represent the environment which connects to the outside system. The set of \( E \) and \( V \) represent the edges and vertices of the graph.

![Graphical connectivity in multi-connected system](image)

**3 Fuzzy Graph Type 3**

Even though Rosenfeld [8] and shortly Yeh and Bang [9] have given the definitions of a fuzzy graph, however, Tahir and Sabariah [13] had formalized five types of possible fuzziness in graphs that was described by Blue et al. [14] and stated in the following definition.

**Definition 3**

(Taxonomy of Fuzzy Graph)

Fuzzy graph is a graph \( G_{F} \) satisfying one of the fuzziness \((G_{Fi}) \) of the \( i^{th} \) type or any of its combination:

1) \( G_{1F}^{i} = \{ G_{1F}, G_{2F}, G_{3F}, \ldots, G_{nF} \} \)

where fuzziness is on \( G_{Fi}^{i} \) \( i = 1,2,3,\ldots,n \).

2) \( G_{2F}^{2} = \{ V, E_{F} \} \) where the edge set is fuzzy.

3) \( G_{3F}^{3} = \{ V, E(t_{F}, h_{F}) \} \)

where both the vertex and edge sets are crisp, but the edges have fuzzy heads and tails.

4) \( G_{4F}^{4} = \{ V, E_{F} \} \) where the vertex set is fuzzy.

5) \( G_{5F}^{5} = \{ V, E(w_{F}) \} \)

where both the vertex and edge sets are crisp but the edges have fuzzy weights.

The definition of fuzzy graph is explored to formulate the best appropriate fuzzy graph that are suitable to any multi-connected controlled system.
Fuzzy graph of type 3 has the same vertices and crisp edges, but unknown edge connectivity, that is, the edges have fuzzy heads and tails. In this section, the fuzzy graph of type 3 is described based on the definition of fuzzy head and fuzzy tail where $h(e_i)$ is the fuzzy head of the edge $e_i \in E$ and $t(e_i)$ is the fuzzy tail of the edge $e_i \in E$ [13]. This is shown in Fig. 2. Fuzzy head and tail are redefined in order to incorporate with FSSM of multi-connected systems. The definition is as follows:

Definition 4 (Fuzzy Head and Fuzzy Tail)
The fuzzy tail is defined as $t : E \rightarrow [0,1]$, The fuzzy head is defined as $h(e_i) = 1, \; \forall e_i \in E$.

The value of fuzzy tail is based on the fuzzy value obtained using the fuzzy state space algorithm [2, 4]. This fuzzy value determines the optimal input parameter to the interconnecting fuzzy state space systems of a multi-connected system where it designates the degree of the desirability [4]. The definition of fuzzy head indicates that there exists at least one link $S_{gFi}^* \rightarrow S_{gFj}^*$ in the multi-connected controlled system. The above idea leads to the development of the following definition of Single Fuzzy Edge Connectivity (SFEC).

Definition 5 (Single Fuzzy Edge Connectivity)
Fuzzy edge connectivity $C$ for $e_i \in E$, denoted as $C(e_i)$ is a tuple of $(t(e_i), h(e_i))$. Thus, the set of all ordered pairs of fuzzy edge connectivity is given as $C = \{(t(e_i), h(e_i)) : e_i \in E\}$.

Single fuzzy edge connectivity is suitable for determining the connectivity between vertices $S_{gFi}^*$ to $S_{gFj}^*$ or $e_i$ which involve only one edge. This is shown in Fig. 2. For any multi-connected systems, the connectivity between two systems can be represented by a weight. This weight is known as the membership value of the fuzzy edge connectivity of the system. In FSSM, the membership value is used to determine the optimal input parameter of a system. Hence, the membership value for a single fuzzy edge connectivity for graph $C$ of $e_i \in E$ is given as follows.

Definition 6
The membership value fuzzy edge connectivity $C$ of $e_i \in E$ is defined as $\mu(e_i) = \min\{t(e_i), h(e_i)\}$ where $\mu(e_i)$ is the membership value for fuzzy edge connectivity of $C$. Given, $h(e_i) = 1$, the membership value for the fuzzy connectivity for each $e_i$ is, $\mu(e_i) = t(e_i)$

Fig. 2. Single Fuzzy Edge Connectivity

5 Implementation and Discussion
To illustrate the fuzzy edge connectivity of GFSSM in multi-connected system, we consider the crisp state space representation of a furnace, riser and drum systems [7]. These are the components or subsystems of a Boiler system of a combined cycle power plant. $S_{gF1}$, $S_{gF2}$ and $S_{gF3}$ represent furnace, riser and drum systems respectively. There are five output parameters come from $S_{gF1}$, $Q_h$ (the heat supplied to the riser system) is the only output parameter of $S_{gF1}$, $S_{gF2}$ and $S_{gF3}$ represent furnace, riser and drum systems respectively. There are five output parameters come from $S_{gF1}$, $Q_h$ (the heat supplied to the riser system) is the only output parameter of $S_{gF1}$.

Fig. 3. Framework for Fuzzy Algorithm
Determination of membership value of graphical representation of FSSM will be based on the fuzzy algorithm approach introduced by Ismail [2]. The algorithm has been shown to give good parameter estimation for multivariable systems [4,10]. The main objective of this approach is used to determine the optimal combination of input-output parameters for multivariable dynamic system. The development of this algorithm is based on three phases of the fuzzy system, namely, fuzzification, fuzzy environment and defuzzification as shown in Fig. 3. In the first phase, the crisp values are converted to fuzzy values. These fuzzy values are, then, processed in the fuzzy environment. Lastly, in the final phase of defuzzification, the optimized value of the generated combination data is determined by Modified Optimized Defuzzified Value Theorem [5]. The optimized value that generated comes from the best fuzzy value which was processed in the fuzzy environment. The determination of fuzzy values, 0.7645 and 0.7270 for furnace and riser systems were discussed in detailed in [7]. Each fuzzy value will represent the fuzzy connectivity between multi-connected systems of FSSM. Based on the concept of Fuzzy graph type 3, \( \mu(e_1) \) represents the membership value for a single fuzzy edge connectivity of \( S_{gF_1} \mid S_{gF_2} \) or \( e_1 \). Since there are only one edge involve in \( S_{gF_1} \mid S_{gF_2} \) and \( S_{gF_3} \mid S_{gF_2} \), the membership value, \( \mu(e_1) \) and \( \mu(e_2) \) are 0.7645 and 0.7270 respectively. Hence, the graphical FSSM of Fuzzy Graph Type 3 as shown in Fig. 4 consists of \( v_1 \), \( v_2 \) and \( v_3 \) represent \( S_{gF_1} \), \( S_{gF_2} \) and \( S_{gF_3} \). \( e_1 \) and \( e_2 \) represent the edges.

Fig. 4. Graphical FSSM of Fuzzy Graph Type 3

6 Conclusion
In this paper we have successfully describe the fuzzy edge connectivity in a multi-connected system using the concept of fuzzy graph type 3. The fuzzy value that represents the weight of desirability is determined using the fuzzy state space algorithm. This new approach focused on the connectivity which involves only one edge that represent the connection of subsystems in a multi-connected system, in particular the boiler system. Thus, further discussion on the connectivity which involves multiple edges will be undertaken in the near future.

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References:


