Essential Properties of Abiyev's Balanced Squares

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Abstract : - Perfection of the algorithm, found for writing magic squares of any order from any numbers (rational, irrational, complex, symbols etc.), shows itself in the properties of these squares as well. The essential properties of Abiyev's balanced squares have been studied with the purpose of applying in science, technology, art, architecture, urban planning and other fields. These features, examined in the case of numbers, are fascinating the human mind.

Key-Words: - Magic square, Abiyev's balanced square, algorithm, optimal programs, invariant, property.

1 Introduction

Today by revealing a lot of secrets of magic squares, mathematicians and physicists prove Pythagorean's "All things are numbers" expression to have deep philosophical meaning. The number of variations of the magic squares unbelievably increases depending on their order. For example, for 3rd order N=9, 4th order N=880, 5th order N=275 304 224, 6th order N=1.775399× 10[19] etc. [1]. However, the perfect ones among them are very few. Certainly, this criterion of perfection may be conditional. It should be noted that, this criteria is not so easy to formulate [2]. Using only the definition of a magic square, Abiyev has discovered the more important features in the squares that are written only with the algorithm he invented. He has named the squares formulated with this algorithm as Abiyev's Balanced Squares. The fundamental features of these squares have been studied [3, 4, 5]. Today, scientists make more efforts to expand application fields of magic squares [6,7,8]. As an example to these areas can be shown cryptology, mathematical programming in optimal problem solution, genetics, architecture, music, urban planning and so on. Considering the above-said, in this paper we will examine the essential properties of Abiyev's balanced squares.

2 Properties

The main idea of the algorithm is the following: four arithmetical sequences are named as α, β, γ *and* δ, with constants +1, +n, -1 and -n (or a_0, b, c and d), respectively; the cells of constituent of each sequences were painted with orange (set α), red (set β), blue (set γ) and violet (set δ); the numbers in the cells of concentric frames were assigned by means of closed graphs [9]. Magic numbers equals to $S_n = \frac{n^2+1}{2}n$ [or $S_n = \frac{2a_0+(n-1)(b+c)}{2}n$], where n is order of square, a_0, b, c and d are any numbers [10]. In order to explain the features of these squares with examples, the balanced squares of 15th and 16th order, and the natural square of 16th order are used.

Now let's suggest some definitions: divide any even square to the sections, shown in fig. 1.The sign "+" in the figure is formed by 2 rows and columns that cross the centre. In the centre of the square from their crossing it is formed 2x2 square. And in the square of odd order the sign "+" will be formed by one row and one column crossing. The cells in the diagonals and sign "+" divide the entire square in the 8 regions (1-8).

Definitions: We will name:

-the numbers on the diagonal as diagonal numbers;

-4 numbers in the centre as central numbers;

- the numbers inside the sign "+" (except central numbers) as "plus-inner numbers";

-the numbers in the 1-8 regions as triangular-inner numbers (t-in).

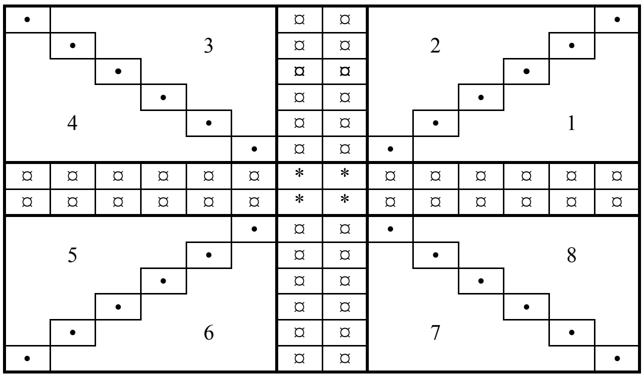

Fig.1.˙The regions square of even order; placing numbers in cells: *---central; •---diagonal; ¤---plus-inner; 1-8---triangle-inner.

2.1 First property

a-diagonal numbers symmetrical to the geometrical centre of the square;

b- triangle-inner numbers symmetrical to the vertical or horizontal central lines that are in the non-neighbouring regions;

c- for double-even and single-even frames of balanced square the plus-inner horizontal (vertical) numbers, symmetrical to central vertical (horizontal) line respectively ;

provide n^2+1 constant.

Examples from Fig. 3:

$a_{4,13}+a_{13,4}$= 61+196=257; $a_{4,2}+a_{4,15}$= 50+207=257; $a_{8,2}+a_{9,2}$= 114+143=257 and $a_{8,15}+a_{9,15}$= 127+130=257

2. 2 Second property

The sum of four vertical (horizontal) and diagonal neighbour numbers of any number in the 1-8 regions, is $2(n^2+1)$ (Fig.3). Examples:

$a_{5,15}+a_{7,15}+$
$a_{5,14}+a_{7,16}$=66+98+190+160=514=2(16^2+1);
$a_{13,6}+a_{15,6}+a_{13,7}+a_{15,5}$=198+230+58+28=514

In balanced square of odd order this property can be given as following. The sign "+" divides the square to four small squares of $\frac{n-1}{2}$ order. Here if to add 1 to number for balanced square of n order, we get single- or double-even numbers. In this case for double-even numbers there will be one cell in the centre of each small square, and for single-even number - a dot. (Fig.5,a,b). In a balanced square of 15^{th} order the numbers in these cells are 218, 120, 8 and 106 (Fig.5,b).

2. 3 Third property

Diagonal numbers and the numbers on the lines, adjacent to diagonals (including central numbers) form several certain arithmetical sequences (fig.3):

a) 17, 34, …,119,**136**, 153, 170,…,255;
b) 32, 47, …,122, **137**, 152, 167,…,242;
c) 15, 30, …,105, **120**, 135, 150,…,225;
d) 1, 18, …,103, **137**, **120**, 154,…,256;
e) 2, 19, …,104, **121**, 138, 155,…,240:
f) 16, 31, …,106, **136**, **121**, 151,…,241;

2. 4 Fourth property

If to take the numbers as electric charges in balanced square, then the electrical potentials $(\varphi_{2j}, \varphi_{2j+1})$, created in the centre by these charge frames, are calculated with the following formulas; here $4(4l^2+1)$ and $4[4l(l+1)+2]$ are constants of balanced squares of even and odd orders, respectively.

$\square_{2l}=\sum_{j=1}^{l}\varphi_{2j}=\varphi_2+\varphi_4+\cdots+\varphi_{2l}$ is electric potential, created by all charges in the centre of balanced square of even order.

$$\varphi_{2l}=4(4l^2+1)\left[\sum_{k=0}^{l-2}\frac{1}{\sqrt{l(l-1)+0,5+k(k+1)}}+\frac{1}{\sqrt{8l(l-1)+2}}\right];\ \text{Examples:}$$

$$\varphi_2==4(4l^2+1)\frac{1}{\sqrt{2}};\quad \varphi_4=4(4l^2+1)\left[\frac{1}{\sqrt{2,5}}+\frac{1}{\sqrt{18}}\right];\quad \varphi_6=4(4l^2+1)\left[\frac{1}{\sqrt{6,5}}+\frac{1}{\sqrt{8,5}}+\frac{1}{\sqrt{50}}\right].$$

$\phi_{2l+1}=\sum_{j=1}^{l}\varphi_{2j+1}=\varphi_3+\varphi_5+\cdots+\varphi_{2l+1}$ is electric potential, created by all charges in the centre of balanced square of odd order.

$$\varphi_{2l+1} = 4[4l(l+1)+2]\left[\sum_{k=1}^{l-1}\frac{1}{\sqrt{l^2+k^2}} + \frac{1}{2l}\left(1+\frac{1}{\sqrt{2}}\right)\right];$$

Examples:

$$\varphi_3 = 4[4l(l+1)+2]\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)\right]; \qquad \varphi_5 = 4[4l(l+1)+2]\left[\frac{1}{\sqrt{5}}+\frac{1}{4}\left(1+\frac{1}{\sqrt{2}}\right)\right];$$

$$\varphi_7 = 4[4l(l+1)+2]\left[\frac{1}{\sqrt{10}}+\frac{1}{\sqrt{13}}+\frac{1}{6}\left(1+\frac{1}{\sqrt{2}}\right)\right].$$

IV Conclusion

These unique properties of Abiyev's balanced squares are also true for magic squares, written from any numbers [9,10], and exist in almost none of the other magic squares. The properties here given and not given, are the indicators of the perfection of Abiyev algorithm. There is no doubt that, this algorithm will be widely applied in science and technology.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128
129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176
177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192
193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208
209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224
225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256

Fig.2. The natural square of 16th order

1	242	14	244	12	246	10	249	248	7	251	5	253	3	255	16
240	18	227	29	229	27	231	232	25	234	22	236	20	238	31	17
33	223	35	212	44	214	42	217	216	39	219	37	221	46	34	224
208	50	206	52	197	59	199	200	57	202	54	204	61	51	207	49
65	191	67	189	69	182	74	185	184	71	187	76	68	190	66	192
176	82	174	84	172	86	167	168	89	170	91	85	173	83	175	81
97	159	99	157	101	155	103	152	153	106	102	156	100	158	98	160
144	114	142	116	124	118	138	137	136	119	123	133	125	131	127	129
128	143	126	141	140	139	122	121	120	135	134	117	132	115	130	113
145	111	147	109	149	107	151	105	104	154	150	108	148	110	146	112
96	162	94	164	92	166	90	88	169	87	171	165	93	163	95	161
177	79	179	77	181	75	183	72	73	186	70	188	180	78	178	80
64	194	62	196	60	198	58	56	201	55	203	53	205	195	63	193
209	47	211	45	213	43	215	41	40	218	38	220	36	222	210	48
32	226	30	228	28	230	26	24	233	23	235	21	237	19	239	225
241	15	243	13	245	11	247	9	8	250	6	252	4	254	2	256

a) 17, 34, …,119,**136**, 153, 170,…,255;

b) 32, 47, …,122, **137**, 152, 167,…,242;

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f) 16, 31, …,106, **136**, **121**, 151,…,241.

Fig.3. The Abiyev's balanced square of 16th order

114	100	86	72	58	44	30	1	212	198	184	170	156	142	128
130	116	102	88	74	60	31	17	3	214	200	186	172	158	144
146	132	118	104	90	61	47	33	19	5	216	202	188	174	160
162	148	134	120	91	77	63	49	35	21	7	218	204	190	176
178	164	150	121	107	93	79	65	51	37	23	9	220	206	192
194	180	151	137	123	109	95	81	67	53	39	25	11	222	208
210	181	167	153	139	125	111	97	83	69	55	41	27	13	224
211	197	183	169	155	141	127	113	99	85	71	57	43	29	15
2	213	199	185	171	157	143	129	115	101	87	73	59	45	16
18	4	215	201	187	173	159	145	131	117	103	89	75	46	32
34	20	6	217	203	189	175	161	147	133	119	105	76	62	48
50	36	22	8	219	205	191	177	163	149	135	106	92	78	64
66	52	38	24	10	221	207	193	179	165	136	122	108	94	80
82	68	54	40	26	12	223	209	195	166	152	138	124	110	96
98	84	70	56	42	28	14	225	196	182	168	154	140	126	112

Fig.4. The Abiyev's balanced square of 15^{th}

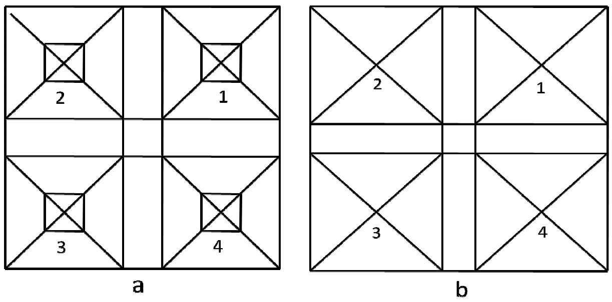

Fig.5. The special regions of square of odd order

a- If n+1 is a double-even number

b- If n+1 is a single-even number

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