

The Simple Algorithm Of Abiyev's Balanced Cubes Of Odd Order

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Abstract. - For the first time, a simple algorithm for writing the balanced cubes of odd order has been found. With the help of this algorithm we can write color balanced (magic) cubes from any numbers (rational, irrational, complex, symbols etc.) of any orders. Examples have been shown for these cubes and compared with Marian Trenkler's cube. This algorithm can be easily applied to informatics, optimal problems and other computer programs. Abiyev's balanced cube of 1001 order has been formed with this algorithm.

Key-Words: - Balanced cube, information, algorithm, optimal programs, magic cubes.

I. Introduction

The fact that famous mathematicians like P. Fermat, B. Pascal, L. Euler, A. Cayley and others were involved in magic squares, shows them to be a serious mathematical problem [1,2]. Understanding this seriousness the German thinker P. Carus has expressed his impressions on the magic squares by the following words: "There is no science that teaches the harmonies of nature more clearly than mathematics, and the magic squares are like a mirror which reflects the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos and in the construction of the atom not less than in the mysterious depth of the human mind" [1].

It should be noted that today the problem in magic squares is to reveal the compliance between their unique properties and the laws of nature, and to direct them towards the development of science and technology [3,4]. The correlation between Abiyev's balanced squares and periodic tables of elements can be an obvious example to confirm the above -said [5].

However, having more information than magic squares, magic cubes need a simple algorithm in order to expand their application spheres.

The algorithm proposed by M. Trenkler for writing magic cubes allows to write only the natural magic cubes [6,7]. By the algorithm described in this article, it is possible to write colored magic cubes of any order and from any numbers (rational, irrational, complex, symbols and etc).

II. Latin squares

Latin squares are used in the design of experiments, tournament scheduling and constructing magic squares and cubes. A Latin square is an arrangement of m variables x_1, x_2, \dots, x_m into m rows and m columns such that no row and no column contains any of the variables twice. Two Latin squares are called orthogonal if when one is superimposed upon the every ordered pair of variables occurs once in the resulting square.

In this site [8] the writing of magic squares from natural numbers according to its own order base was programmed. The perfection of this algorithm is that this square corresponds to a joint state of 2 orthogonal Latin squares. In Fig.1, the example of this square is shown. If we use coefficients of this square and formula $k_{ij}c + p_{ij}b + a_0 = m_{ij}$ as a result we will get a square from any numbers [9]; where k_{ij} and p_{ij} - are the elements of Latin square, $0 \leq i, j \leq n - 1$. It should be mentioned that these Latin squares have mirror symmetry. The magic number of this square is calculated with the formula: $M_n = \frac{2a_0 + (n-1)(c+b)}{2} n$, where c , b and a_0 are any numbers (rational, irrational, complex, symbols and etc).

If $a_0=1$, $b=1$, $c=n$, then this square is transformed to natural magic square and the magic number is calculated with the formula $M_n = \frac{n^2+1}{2} n$. This number is expressed with the formula $M_n = \frac{(n-1)n}{2} \times (1,1)_n$ according to order base of

the square. The magic number of Latin squares is

$$\text{equal to } M_n = \frac{(n-1)n}{2}.$$

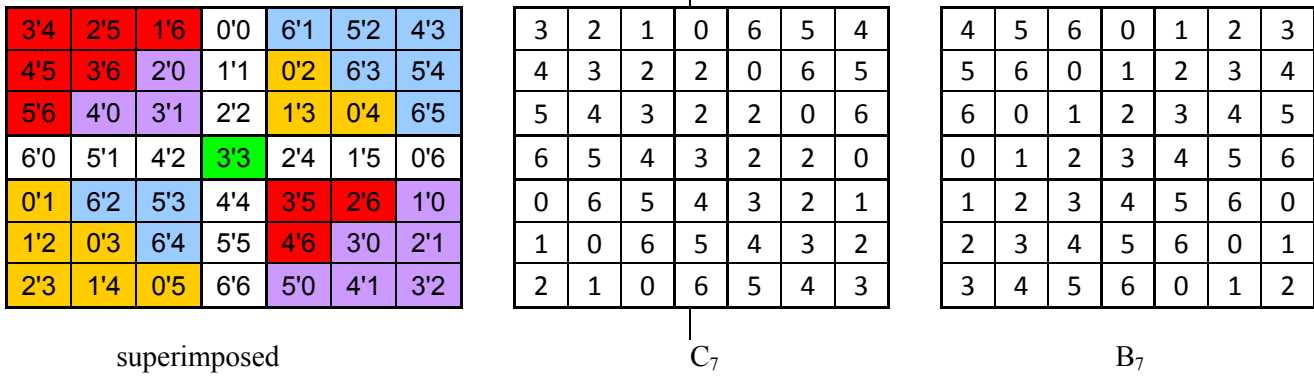


Fig.1. The Abiyev’s balanced square of 7th order in own base, B₇ and C₇ are orthogonal Latin squares of 7th orders

III. The Algorithm Of Balanced Cube Of Odd Order

In order to write the magic cube of odd order, three Latin squares are used. Let’s mark them with *A*, *B* and *C*. Accept the element (1,1) of Latin squares (Ls) *A*(*i,j*) and *B*(*i,j*) to be equal respectively to 0 and *n*-1, increase the first row elements of these two Ls from right to left and elements in columns top to down successively. And the Ls *C*(*i,j*) is the mirror reflection of Ls *B*(*i,j*). For writing Ls *A*_{*n-1*}, *B*_{*n-1*} and *C*_{*n-1*} the following simple method is used: The Ls *A*_{*n*} and *C*_{*n*} are copied and slipped one column right, and *B* is slipped one column left (Fig.2.). The last columns of Ls *A*_{*n*} and *C*_{*n*} are cut and pasted in front of the first column, and Ls *B*_{*n*} -the first column is cut and pasted next to the last column. Thus, the obtained Ls will form the Ls *A*_{*n-1*}, *B*_{*n-1*} and *C*_{*n-1*}. Using this algorithm we get Latin squares (*A*_{*n-2*}, *B*_{*n-2*}, *C*_{*n-2*}); (*A*_{*n-3*}, *B*_{*n-3*}, *C*_{*n-3*}); ..., (*A*₁, *B*₁, *C*₁). Here *A*_{*k*}=*B*_{*n+1-k*}. For example *A*_{*n*}=*B*₁ and *A*_{*(n+1)/2*}=*B*_{*(n+1)/2*}.

If we multiply the elements of Latin squares, *A*_{*k*}, *B*_{*k*} and *C*_{*k*} respectively by *d*, *c*, *b* and add *a*₀, that is $m_n = A_k(i, j) d + B_k(i, j) c + C_k(i, j) b + a_0$ then we will get horizontal cuts of the magic cube, where *A*_{*k*}(*i, j*), *B*_{*k*}(*i, j*) and *C*_{*k*}(*i, j*) are the elements of *k*th Latin squares *A*, *B* and *C*, respectively. The magic number of this magic cube is $M_n = \frac{2a_0+(n-1)(d+c+b)}{2}n$, where *d*, *c*, *b* and *a*₀ are

$$\begin{aligned} m_5(4,4,3) &= a_0 + 2d + 2c + 4b = e; \\ m_5(1,5,2) &= a_0 + 2d + 2c + 4b = i; \\ m_5(2,2,5) &= a_0 + 4c + 2b = \pi; \\ M_5 &= 5[a_0 + 2(d + c + b)] = 1. \end{aligned}$$

The system equations .

any numbers (rational, irrational, complex, symbols and etc.). We call this magic cube Abiyev’s General Balanced Cube.

M. Trenkler’s natural magic cube is obtained with the formula $m_n = A_k(i, j) n^2 + B_k(i, j) n + C_k(i, j) + 1$ and magic number $M_n = \frac{n^3+1}{2}n$. Note that *A*_{*k*} = *C*_{*n+1-k*} and *A*_{*k*}, *B*_{*k*} are symmetrical by mirror reflection.

In figure 3 the general balanced cube of 5th order is shown as an example. In figure 4, Abiyev’s balanced (1), Trenkler’s magic (3) and natural cubes are displayed for comparison. The colours in these cubes have been obtained from natural cube in accordance with numbers. The white cells consist of two different colours. The algorithm of Abiyev’s General Balanced Cube allows us to write a magic cube upon request. Suppose that, *n*=5; *m*₅=(2,2,5)= π ; *m*₅=(4,4,3)=*e*; *m*₅=(1,5,2)=*i*= $\sqrt{-1}$ and the magic number *M*₅=1. According to the data, system equations are written and solved; and as a result algorithm parameters: *d* = 0,25*e* + 0,5*i* – 0,15; *c* = 0,5 *n* + 0,25*e* + 0,5*i* – 0,25; *b* = 0,5*e* – 0,1; *a*₀ = –(π + 2*e* + 2*i* – 1,2); *M*₅ = 5[*a*₀ + 2(*d* + *c* + *b*)] = 1 are found. Using these constants the third (medium) cut of balanced cube of fifth order has been written (fig.5). We can write the other cuts in the same way as well.

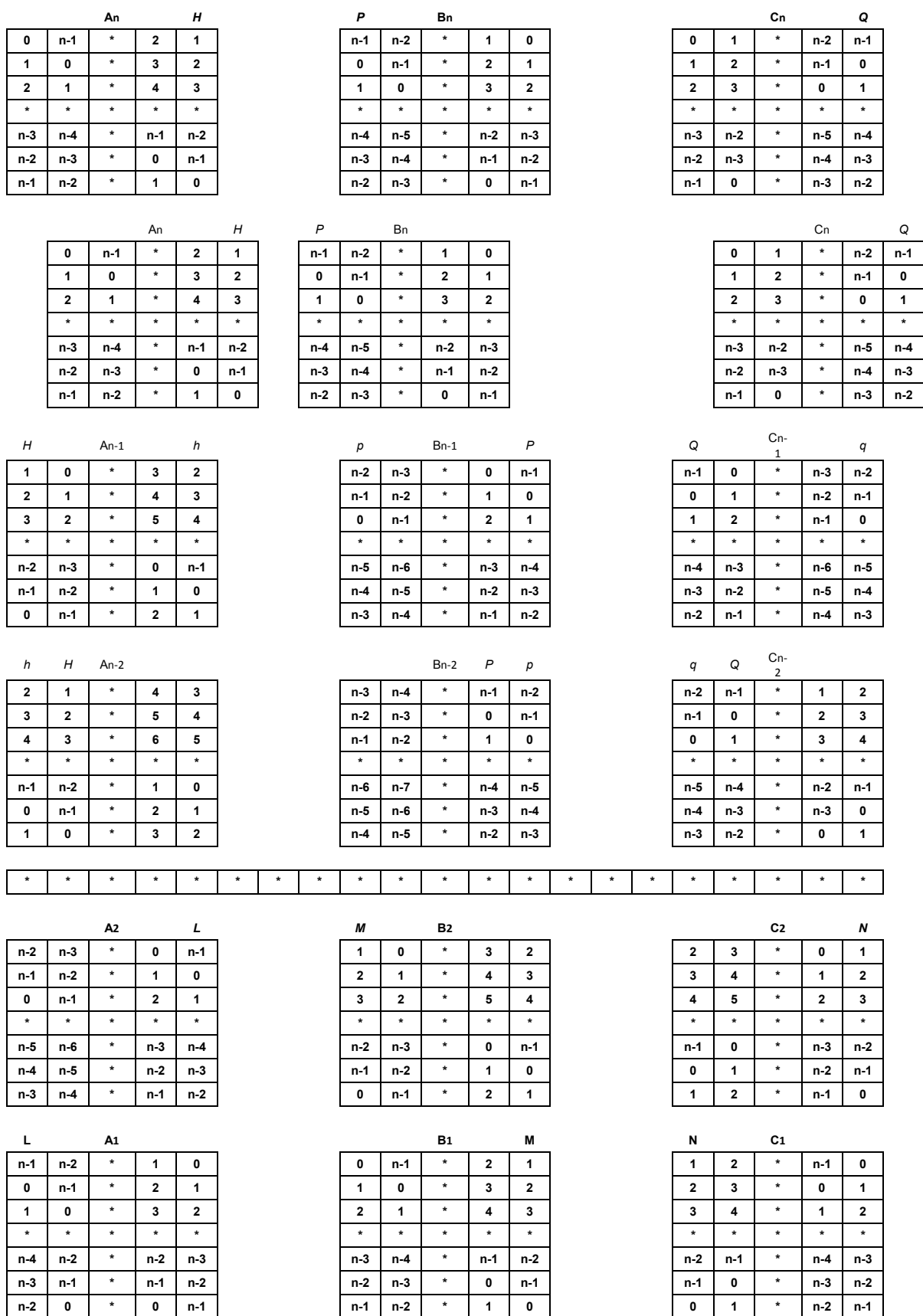


Fig.2. The Latin Squares of Abiyev's Balanced Cube of n order

| F | $\times d$ | $\times c$ | $\times b$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 1 | 0 | 4 | 3 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 1 | 0 | 4 | 3 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1 | 0 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 2 | 1 | 0 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 3 | 2 | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 2 | 3 | 4 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 4 | 0 | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 0 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 2 | 3 | 4 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 0d1c2b | 4d0c3b | 3d4c4b | 2d3c0b | 1d2c1b | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1d2c3b | 0d1c4b | 4d0c0b | 3d4c1b | 2d3c2b | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2d3c4b | 1d2c0b | 0d1c1b | 4d0c2b | 3d4c3b | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3d4c0b | 2d3c1b | 1d2c2b | 0d1c3b | 4d0c4b | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 3 | 2 | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 4 | 3 | 2 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 4 | 3 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1 | 0 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 2 | 1 | 0 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 4 | 3 | 2 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 4 | 3 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1 | 0 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 2 | 1 | 0 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 3 | 2 | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 2 | 3 | 4 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 3 | 4 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 4 | 0 | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 0 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Fig.3. The general balanced cube of 5th order

| 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| <table border="1"> <tr><td>21</td><td>117</td><td>88</td><td>59</td><td>30</td></tr> <tr><td>27</td><td>23</td><td>119</td><td>90</td><td>56</td></tr> <tr><td>58</td><td>29</td><td>25</td><td>116</td><td>87</td></tr> <tr><td>89</td><td>60</td><td>26</td><td>22</td><td>118</td></tr> <tr><td>120</td><td>86</td><td>57</td><td>28</td><td>24</td></tr> </table> | 21 | 117 | 88 | 59 | 30 | 27 | 23 | 119 | 90 | 56 | 58 | 29 | 25 | 116 | 87 | 89 | 60 | 26 | 22 | 118 | 120 | 86 | 57 | 28 | 24 | <table border="1"> <tr><td>101</td><td>102</td><td>103</td><td>104</td><td>105</td></tr> <tr><td>106</td><td>107</td><td>108</td><td>109</td><td>110</td></tr> <tr><td>111</td><td>112</td><td>113</td><td>114</td><td>115</td></tr> <tr><td>116</td><td>117</td><td>118</td><td>119</td><td>120</td></tr> <tr><td>121</td><td>122</td><td>123</td><td>124</td><td>125</td></tr> </table> | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 123 | 124 | 125 | <table border="1"> <tr><td>116</td><td>12</td><td>33</td><td>54</td><td>100</td></tr> <tr><td>90</td><td>106</td><td>2</td><td>48</td><td>69</td></tr> <tr><td>59</td><td>80</td><td>121</td><td>17</td><td>38</td></tr> <tr><td>28</td><td>74</td><td>95</td><td>111</td><td>7</td></tr> <tr><td>22</td><td>43</td><td>64</td><td>85</td><td>101</td></tr> </table> | 116 | 12 | 33 | 54 | 100 | 90 | 106 | 2 | 48 | 69 | 59 | 80 | 121 | 17 | 38 | 28 | 74 | 95 | 111 | 7 | 22 | 43 | 64 | 85 | 101 |
| 21 | 117 | 88 | 59 | 30 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 27 | 23 | 119 | 90 | 56 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 58 | 29 | 25 | 116 | 87 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 89 | 60 | 26 | 22 | 118 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 120 | 86 | 57 | 28 | 24 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 101 | 102 | 103 | 104 | 105 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 106 | 107 | 108 | 109 | 110 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 111 | 112 | 113 | 114 | 115 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 116 | 117 | 118 | 119 | 120 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 121 | 122 | 123 | 124 | 125 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 116 | 12 | 33 | 54 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 90 | 106 | 2 | 48 | 69 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 59 | 80 | 121 | 17 | 38 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 28 | 74 | 95 | 111 | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 22 | 43 | 64 | 85 | 101 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <table border="1"> <tr><td>45</td><td>11</td><td>107</td><td>78</td><td>74</td></tr> <tr><td>71</td><td>42</td><td>13</td><td>109</td><td>80</td></tr> <tr><td>77</td><td>73</td><td>44</td><td>15</td><td>106</td></tr> <tr><td>108</td><td>79</td><td>75</td><td>41</td><td>12</td></tr> <tr><td>14</td><td>110</td><td>76</td><td>72</td><td>43</td></tr> </table> | 45 | 11 | 107 | 78 | 74 | 71 | 42 | 13 | 109 | 80 | 77 | 73 | 44 | 15 | 106 | 108 | 79 | 75 | 41 | 12 | 14 | 110 | 76 | 72 | 43 | <table border="1"> <tr><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td></tr> <tr><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td></tr> <tr><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table> | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | <table border="1"> <tr><td>87</td><td>108</td><td>4</td><td>50</td><td>66</td></tr> <tr><td>56</td><td>77</td><td>123</td><td>19</td><td>40</td></tr> <tr><td>30</td><td>71</td><td>92</td><td>113</td><td>9</td></tr> <tr><td>24</td><td>45</td><td>61</td><td>82</td><td>103</td></tr> <tr><td>118</td><td>14</td><td>35</td><td>51</td><td>97</td></tr> </table> | 87 | 108 | 4 | 50 | 66 | 56 | 77 | 123 | 19 | 40 | 30 | 71 | 92 | 113 | 9 | 24 | 45 | 61 | 82 | 103 | 118 | 14 | 35 | 51 | 97 |
| 45 | 11 | 107 | 78 | 74 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 71 | 42 | 13 | 109 | 80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 77 | 73 | 44 | 15 | 106 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 108 | 79 | 75 | 41 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | 110 | 76 | 72 | 43 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 76 | 77 | 78 | 79 | 80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 81 | 82 | 83 | 84 | 85 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 86 | 87 | 88 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 91 | 92 | 93 | 94 | 95 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 96 | 97 | 98 | 99 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 87 | 108 | 4 | 50 | 66 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 56 | 77 | 123 | 19 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 30 | 71 | 92 | 113 | 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | 45 | 61 | 82 | 103 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 118 | 14 | 35 | 51 | 97 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 64 | 35 | 1 | 122 | 93 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 95 | 61 | 32 | 3 | 124 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 121 | 92 | 63 | 34 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 123 | 94 | 65 | 31 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 33 | 4 | 125 | 91 | 62 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 51 | 52 | 53 | 54 | 55 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 56 | 57 | 58 | 59 | 60 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 61 | 62 | 63 | 64 | 65 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 66 | 67 | 68 | 69 | 70 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 71 | 72 | 73 | 74 | 75 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 58 | 79 | 125 | 16 | 37 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 27 | 73 | 94 | 115 | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 21 | 42 | 63 | 84 | 105 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 120 | 11 | 32 | 53 | 99 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 89 | 110 | 1 | 47 | 68 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 83 | 54 | 50 | 16 | 112 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 114 | 85 | 51 | 47 | 18 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 20 | 111 | 82 | 53 | 49 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 46 | 17 | 113 | 84 | 55 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 52 | 48 | 19 | 115 | 81 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 26 | 27 | 28 | 29 | 30 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 31 | 32 | 33 | 34 | 35 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 36 | 37 | 38 | 39 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 41 | 42 | 43 | 44 | 45 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 46 | 47 | 48 | 49 | 50 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | 75 | 91 | 112 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 23 | 44 | 65 | 81 | 102 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 117 | 13 | 34 | 55 | 96 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 86 | 107 | 3 | 49 | 70 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 60 | 76 | 122 | 18 | 39 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <table border="1"> <tr><td>102</td><td>98</td><td>69</td><td>40</td><td>6</td></tr> <tr><td>8</td><td>104</td><td>100</td><td>66</td><td>37</td></tr> <tr><td>39</td><td>10</td><td>101</td><td>97</td><td>68</td></tr> <tr><td>70</td><td>36</td><td>7</td><td>103</td><td>99</td></tr> <tr><td>96</td><td>67</td><td>38</td><td>9</td><td>105</td></tr> </table> | 102 | 98 | 69 | 40 | 6 | 8 | 104 | 100 | 66 | 37 | 39 | 10 | 101 | 97 | 68 | 70 | 36 | 7 | 103 | 99 | 96 | 67 | 38 | 9 | 105 | <table border="1"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr> <tr><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td></tr> </table> | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <table border="1"> <tr><td>25</td><td>41</td><td>62</td><td>83</td><td>104</td></tr> <tr><td>119</td><td>15</td><td>31</td><td>52</td><td>98</td></tr> <tr><td>88</td><td>109</td><td>5</td><td>46</td><td>67</td></tr> <tr><td>57</td><td>78</td><td>124</td><td>20</td><td>36</td></tr> <tr><td>26</td><td>72</td><td>93</td><td>114</td><td>10</td></tr> </table> | 25 | 41 | 62 | 83 | 104 | 119 | 15 | 31 | 52 | 98 | 88 | 109 | 5 | 46 | 67 | 57 | 78 | 124 | 20 | 36 | 26 | 72 | 93 | 114 | 10 |
| 102 | 98 | 69 | 40 | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 104 | 100 | 66 | 37 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 39 | 10 | 101 | 97 | 68 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 70 | 36 | 7 | 103 | 99 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 96 | 67 | 38 | 9 | 105 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 7 | 8 | 9 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 12 | 13 | 14 | 15 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 16 | 17 | 18 | 19 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 21 | 22 | 23 | 24 | 25 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 25 | 41 | 62 | 83 | 104 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 119 | 15 | 31 | 52 | 98 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 88 | 109 | 5 | 46 | 67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 57 | 78 | 124 | 20 | 36 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 26 | 72 | 93 | 114 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Fig.4. Comparison of Abiyev's balanced (1), natural (2) and Trenkler's magic cubes of 5th order.

| | | | | |
|------------------------|------------------------|---------------------|-------------------|------------------------|
| $0,5e+0,1$ | $(-)0,5\pi+0,5e-i+0,4$ | $(-)\pi-2e-2i+1,2$ | $\pi+0,5e+2i-0,5$ | $0,5\pi+0,5e+i-0,2$ |
| $0,5\pi+1,5e+i-0,4$ | $(-)e+0,4$ | $(-)0,5\pi-e-i+0,7$ | $(-)\pi-e-2i+1$ | $\pi+1,5e+2i-0,7$ |
| $\pi+2i-0,4$ | $0,5\pi+i-0,1$ | $0,2$ | $(-)0,5\pi-i+0,5$ | $(-)\pi-2i+0,8$ |
| $(-)\pi-1,5e-2i+1,1$ | $\pi+e+2i-0,6$ | $0,5\pi+e+i-0,3$ | e | $(-)0,5\pi-1,5e-i+0,8$ |
| $(-)0,5\pi-0,5e-i+0,6$ | $(-)\pi-0,5e-2i+0,9$ | $\pi+2e+2i-0,8$ | $0,5\pi-0,5e+i$ | $(-)0,5e+0,3$ |

Fig.5. General balanced cube constructed upon request (third cut). Magic number equals to 1

IV. Conclusion

This algorithm, offered by us in order to write a magic cube of odd order, is very simple as it is seen and enables us to easily form a colored magic cube any order from any numbers. The algorithm can also be easily programmed. This programme can be used in many mathematical modellings. There is no doubt that colorful writing of magic cubes will enlarge the mathematical information spectrum. Abiyev’s balanced cube of 1001 order has been formed with this algorithm.

REFERENCES

[1] W. S. Andrews. Magic Squares and Cubes. (New York: Dover, 1960), originally printed in 1917

[2] Euler’s Science of Combinations Leonhard Euler Tercentenary: 1707-2007, Robert E. Bradley, C. Edward Sandifer.

[3] Peter D. Loly, Franklin Squares: A Chapter in the Scientific Studies of Magical Squares, Complex Systems, 17 (2007), 143-161.

[4] A. A. Abiyev, A. Baykasoglu, T. Dereli, I. H. Filiz, A. Abiyev, Investigation of center of mass by using magic square and its possible engineering applications, Robotics and Autonomous Systems, 49, (2004) 219-226.

[5] Abiyev A.A. The correlation of Abiyev’s balanced squares with periodic law, Proceedings of the 2nd International Conference on Applied Informatics and Computing Theory (AICT’11), Prague, Czech Republic, September 26-28, 2011, pp. 33-38.

[6] M.Trenkler, Magic p-dimensional cubes, Acta Arithmetica XCVI.4 (2001).

[7] M.Trenkler, An algorithm for making magic cubes, The PIME Journal, Vol.12, No. 2, pp.105-106, Spring 2005.

[8] <http://www1.gantep.edu.tr/~bingul/php/magic/>

[9] Mehmet Sahin, Asker Ali Abiyev, Azer Abiyev, The General Algorithm of Balanced Squares, Proceedings of Fourth International Conference on Soft Computing, Computing with Words and Perceptions in Systems. Analysis, Decision and Control, p. 155-162. Antalya, Turkey



Name: Asker Ali Abiyev¹

Place and date of birth: Baku, Azerbaijan, 28.06.1934.

Education: primary school No 79, and 208 (1944-1954), Baku; Azerbaijan State University (1954-1957); Moscow State University named M. V. Lomonosov, Physic faculty (1957-1961); post graduate student at Institute of Atomic Energy named after I. V. Kurchatov (1963-1966); Ph.D in physics-mathematics (1970); Doctor in physics-mathematics (1989); Professor (1990).

Scientific activity: neutron spectroscopy, radiation physics of semiconductors, magic squares and cubes.

Career/Employment:

1.Head of the laboratory of radiation physics and semiconductors at the Institute of Radiation Problems of Azerbaijan National Academy of Sciences, 1975-1993; Consultant at a scientific school, Ankara, Turkey, 1993-2000; Professor in Mathematics at Gaziantep University in Turkey, 2000-2007; Head of Experimental Department of the Electron Accelerator, 2009 up to today.

Participated Conferences: 1. IMS'2008 6th International Symposium on Intelligent and Manufacturing Systems “Feature, Strategies and Innovation”, October 14- 17, 2008, Sakarya, Turkey. 2. Fourth International Conference on Soft Computing, Computing with Words and Perceptions in Systems. Analysis, Decision and Control, Turkey, August 27-28, 2007.

3. School of Information Technology and Mathematical Sciences, University Ballarat (Australia). 14 -28 June 2006.

Turkey, August 27-28, 2007. b-Quadrat Verlag,

4. Joint International Scientific Conference “New Geometry of Nature”, August- September 5, 2003, Kazan State University, Kazan, Russia.

5. The Third International Conference On Mathematical And Computational Applications, September 4-6, 2002, Konya, Turkey.

6. 2nd International Conference On Responsive Manufacturing, Gaziantep, Turkey, 26-28, June, 2002.

7. The Second International Symposium on Mathematical and Computational Applications, September 1-3, 1999, Baku, Azerbaijan.

8. Research Conference on Number Theory and Arithmetical Geometry, San Feliu de Guixols, Spain 24-29 October, 1997.

9. International Conference on Radiation Physics of Semiconductors and Related Materials, TBİLİSİ, USSR, 1978.

Publications: Number of paper in refereed journals: 120

Number of communications to scientific meetings: 25

The Book:

A.K. Abiyev, Sayılı Sihirli Karelerin Doğal Şifresi-The Natural Code of Numbered Magic Squares, Enderun Publications, Ankara, (ISBN 975-95318-3-6), p.77, 1996, (in Turkish and in English).

<http://www1.gantep.edu.tr/~bingul/php/magic/>

Language Skills: Azeri, Russian, English, Turkish.

Contact: askeraliabiyevev@hotmail.com

Scientific activity: Automatic Control system at roll milling industry, automatic regulation thickness of steel strip, Mathematical research random noise signals in automatic control system, biology cybernetics, spectral analysis of entry information.

Career/Employment: Chief of department “Automation and industrial electronics” in the Lipetsk Technical University – 1971-1983 y. (Russia)

- Chief of department of “Electro technical and industrial electronic” of Azerbaijan building and architecture University,(Baku), 1983-1987
- Rector of Azerbaijan State Sport Academy Baku, - 1987 till present tense, department mathematics and computers system.

Publications: Number of paper in refereed journals :105

Number of scientific conference – 12

Invention certificate – 4 (automatic control system)

The book: The statistic research accuracy of thickness of the roll milling sheets.

Language skills: Azeri, Russian, Turkish, English

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Name: Azer Abiyev³

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Education: received his BSc and MSc degrees in civil engineering from the Azerbaijan State University in 1993 and 1997, respectively.

Head of Department of AZENCO ASC Azerbaijan, 2009 up today.

Participated Conferences: 1.

IMS'2008 6th International Symposium on Intelligent and Manufacturing Systems “Feature

- . 2nd International Conference On Responsive Manufacturing, Gaziantep, Turkey, 26-28, June, 2002.

- . The Second International Symposium on Mathematical and Computational Applications, September 1-3, 1999, Baku



Name: Aghajan Abiyev²

C Place and date of birth::

Baku, Azerbaijan 24.11.1937

Education: Primary school №

208, (1945-1955), Baku

Azerbaijan State Sport Academy

– 1956-1960

Azerbaijan Technical University, automation and computer faculty

(1961-1966)

Postgraduate at Moscow Steel and Alloys University, department of Automatic Control system (1967-1970) Ph.

Dr. in technical science – 1970

Professor – 1989