Abstract: The paper deals with the on-line parameter identification of transmission lines from synchronized measurements of current and voltage phasors provided by the phasor measurement units. Particularly because the current measurements contain a systematic error, which strongly influences the identified parameters, a technique for simultaneous parameter identification and measurement calibration will be proposed. The series and shunt line parameters are recursively estimated from the phasor measurements and weather conditions using the extended Kalman filter. Actual line temperature and reference resistance are estimated instead of actual series resistance, which is then computed from these estimates. Two case studies based on data measured from two 110kV overhead transmission lines are presented to demonstrate the effectiveness of the proposed approach.

Key–Words: On-line parameter identification, measurement calibration, overhead transmission line, phasor measurement unit, postive sequence component.

1 Introduction

As the demand for electricity transmission capacity increases, the knowledge of transmission line parameters involving series impedance and shunt admittance is crucial to determine the line ampacity. While construction parameters are commonly valid when the line is designed, actual values differ due to line aging, ambient conditions and the load of line. It is therefore necessary to compute the actual parameters to ensure effective and safe operation of the transmission line.

Development of the phasor measurement units (PMUs), which guarantee the high precision of mutual synchronization of phasor measurements from both ends of the transmission line, makes possible on-line parameter identification. A number of papers deal with the utilization of synchronized phasors in wide area measurement systems, e.g. [2] and other references mentioned therein.

Several approaches for identification of the line parameters from the measured phasors were discussed e.g. in [1]. The weighted least squares (WLS) method is widely used for power system state and parameter estimation, where the weights are commonly set to be the inverse of the measurement error variances [3]. At each time, the line parameters are estimated based on a redundant number of current and voltage measurements taken at different time instants. The number of measurements used is limited by numerical stability and must be sufficiently large to yield reliable instantaneous estimates of time varying parameters [4].

The accuracy of the identified parameters is significantly affected by random and systematic measurement errors. While the existing methods work with random errors, the suppression of the influence of systematic errors have not been adequately addressed yet. Therefore, a new technique using extended Kalman filter (EKF) [7] for parameter identification and simultaneous measurement calibration is proposed in this paper.

The paper is organized such that Section 2 describes the proposed model of the transmission line. Section 3 introduces the EKF based parameter identification and measurement calibration method. The proposed approach is illustrated in two case studies for 110kV overhead transmission lines in Section 4. The results are summarized in Section 5.

2 Model Description

Let us assume a three-phase transmission line, where PMUs are installed at both ends of each phase for synchronous measurement of voltage phasors, \( \vec{U} = U_{Re} + jU_{Im} \), and current phasors, \( \vec{I} = I_{Re} + jI_{Im} \). The objective is to estimate the line parameters \( \vec{Z} = R + jX \) and \( \vec{Y} = G + jB \), where \( R \) is series resis-
Figure 1: Model of the transmission line.

tance, X is reactance, G is shunt conductance and B is susceptance, from the measurements. Instead of independent estimation of the line parameters from the measurements on each of the three phases, the positive sequence component for the voltage and current phasors will be computed [6]. The transmission line model represented by equivalent π circuit is shown in Figure 1. Phasors of positive sequence components measured at one end will be marked with superscript (1) and at the other end with superscript (2).

As the current is usually measured in the lower part of the measuring range of the meter, it is appropriate to consider the influence of systematic errors in the current measurements. So, let the relation between the current measurements and calibrated current be considered in the following equation

\[ \hat{I} = k_I \cdot I + c_I, \]  

where \( \hat{I} \) is calibrated (true) value, \( I \) is measured value, \( k_I, c_I \in \mathcal{C} \) are calibration constants, which are considered for measuring devices at both ends of the line to be equal.

Using relation (1) in the line model, which is displayed in Figure 1, the following relations hold

\[ \vec{U}^{(1)} - \vec{U}^{(2)} = \vec{Z} \cdot \vec{I}, \]  

\[ (\vec{P}^{(1)} - \vec{P}^{(2)}) = \frac{1}{k_I} \left( \frac{\vec{Y}}{2} \cdot (\vec{U}^{(1)} + \vec{U}^{(2)}) \right), \]

where

\[ \vec{P} = \frac{1}{2} \left( k_I (\vec{P}^{(1)} + \vec{P}^{(2)}) - \vec{Y} \cdot (\vec{U}^{(1)} - \vec{U}^{(2)}) \right) + c_I. \]  

The line parameters change due to line currents and ambient weather conditions. To obtain accurate parameter estimates, it is desirable to include their effects to the model. Let us assume that we have measurements of ambient temperature and speed and direction of wind. Then, the dependence of resistance can be described by an average temperature of the line, which is given in the IEEE standard [5] by the differential equation

\[ \tau \frac{dT}{dt} = \frac{1}{I} R(T) |\vec{I}(t)|^2(t) - q_c(T) - q_r(T) + q_s, \]

\[ R(T) = R_{20}(1 + \alpha (T - 20)), \]

where

- \( \tau \) is total heat capacity of conductor,
- \( l \) is length of the transmission line,
- \( |\vec{I}(t)| \) is absolute value of current through the series impedance at time \( t \), see equation (4),
- \( q_c(T) \) is convected heat loss rate per unit length,
- \( q_r(T) \) is radiated heat loss rate per unit length,
- \( q_s \) is solar heat gain rate per unit length,
- \( R(T) \) is AC resistance of conductor at average temperature \( T \),
- \( R_{20} \) is AC resistance of conductor at reference temperature \( 20^\circ C \),
- \( \alpha \) is temperature coefficient of resistance.

Averaged conductor temperature is understood as the average between conductor core and surface temperatures.

Heat losses and gain are determined by the following expressions:

The solar heat gain:

\[ q_s = \alpha Q_{se} \sin \theta A, \]

where \( \theta = \arccos \left( \cos(H_c) \cos(Z_c - Z_1) \right) \), \( A \) is solar absorptivity, \( H_c \) is altitude of the sun, \( Z_c \) is azimuth of the sun, \( Z_1 \) is azimuth of the line, \( A \) is projected area of conductor, \( Q_s \) is total solar and sky radiated heat flux.

The radiated heat loss:

\[ q_r(T) = 0.0178 D \varepsilon \left( \frac{T + 273}{100} \right)^4 - \left( \frac{T_a + 273}{100} \right)^4 \]

where \( D \) is conductor diameter, \( \varepsilon \) is emissivity, \( T_a \) is ambient temperature.

The convected heat loss is determined according to the IEEE standard [5] as the maximum from convected heat losses computed for natural convection \( q_{cn} \), low wind speed \( q_{c1} \) and high wind speed \( q_{c2} \):

\[ q_c(T) = \max(q_{c1}, q_{c2}, q_{cn}), \]
where

\[
q_{c1} = \left[ 1.01 + 0.0372 \left( \frac{D \rho f V_u}{\mu_f} \right)^{0.52} \right] k_f K_a (T - T_a),
\]

\[
q_{c2} = \left[ 0.0119 \left( \frac{D \rho f V_u}{\mu_f} \right)^{0.6} \right] k_f K_a (T - T_a),
\]

\[
q_{cn} = 0.0205 \rho_f^{0.5} D^{0.75} (T - T_a)^{1.25}.
\]

Computation of coefficients \( K_a, \mu_f, \rho_f, \) and \( k_f \) is addressed in the IEEE standard [5]. Identification of the reactance \( X \) of the conductor. However, this impact is negligible and the reactance \( X \) will be assumed to be constant.

Similarly to relation (6), the impact of the conductor temperature on the reactance \( X \) could be modeled, which is reflected through thermal expansion of the conductor. However, this impact is negligible and the reactance \( X \) will be assumed to be constant.

Shunt parameters \( G \) and \( B \) obviously also depend on weather conditions. However, description of the dependence based on physical laws is not possible. So, their variability will be modeled by a random walk.

The next section focuses on description of a technique of identification of the line parameters and calibration coefficients using EKF.

### 3 Parameter identification by EKF

Let us have a measured positive sequence component of voltage and current phasors at discrete time instants \( k \) with sampling period \( \Delta t \). When equation (5) is discretized with period \( \Delta t \), the following difference equation will be obtained:

\[
T_{k+1} = T_k + \frac{\Delta t}{c_p} R_{20,k} (1 + \alpha (T_k - 20)) \cdot |I_k|^2 + \frac{\Delta t}{c_p} q_s - \frac{\Delta t}{c_p} (q_c + q_r).
\]

We define a vector \( s_k \) of the line parameters and calibration coefficients, which has to be identified, as

\[
s_k = \begin{bmatrix} T_k \\ R_{20,k} \\ X_k \\ G_k \\ B_k \\ k_{l,Re} \\ k_{l,Im} \\ c_{l,Re} \\ c_{l,Im} \end{bmatrix}
\]

Then, a system of equations describing the dynamics of changes of identified parameters is given as

\[
s_{k+1} = \begin{bmatrix} f_1(s_k) \\ \vdots \\ f_9(s_k) \end{bmatrix} + \begin{bmatrix} w_{1,k} \\ \vdots \\ w_{9,k} \end{bmatrix} = f(s_k) + w_k, \tag{13}
\]

where \( f_1(s_k) \) is the right hand side of equation (11) and \( f_i(s_k), i = 2, \ldots, 9, \) is the \( i \)-th element of the vector \( s_k \).

The noise \( w \) makes it possible to represent uncertainty in the development of parameter values. As the parameters \( T_k, G_k, \) and \( B_k \) are assumed to be a time varying, the elements \( w_{i,k}, i = \{1,4,5\}, \) are considered as random variables with zero mean and variances \( \sigma_i^2 \) and the remaining elements are set to zero for all \( k \).

The measurement equations are given in the following form

\[
z_k = \begin{bmatrix} \Delta U_{Re,k} \\ \Delta U_{Im,k} \\ \Delta I_{Re,k} \\ \Delta I_{Im,k} \end{bmatrix} = h_k(s_k) + e_k. \tag{14}
\]

Substituting from (2) and (3) to (14) we obtain

\[
z_k = \begin{bmatrix} R_k I_{k,Re} - X_k I_{k,Im} \\ R_k I_{k,Im} + X_k I_{k,Re} \\ \left\{ \left( U_k^{(1)} + U_k^{(2)} \right) G_k + j B_k \right\}_{Re} \frac{2 \pi}{2 k t} I_{Im} \\ \left\{ \left( U_k^{(1)} + U_k^{(2)} \right) G_k + j B_k \right\}_{Im} \frac{2 \pi}{2 k t} I_{Im} \end{bmatrix} + \begin{bmatrix} e_{1,k} \\ e_{2,k} \\ e_{3,k} \\ e_{4,k} \end{bmatrix}, \tag{15}
\]

where \( I_{k,Re} \) and \( I_{k,Im} \) are real and imaginary part of the current \( I_k \), which is computed according to (4) from the measurements obtained at time \( k \). The symbols \( \{\cdot\}_{Re} \) and \( \{\cdot\}_{Im} \) denote real and imaginary part of the expression in brackets. \( e_k \) denotes random errors of measurement devices with

\[
E\{e_k\} = 0, \quad \text{cov}(e_k) = R = \text{diag} \left( \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2 \right), \tag{16}
\]

where variances are given according to the degrees of precision of the measurement instruments. As the values of \( z_k \) are assumed to be the differences between measurements at line ends 1 and 2, the resulting variance is the sum of the variances of measuring instrument errors.

Uncertainty in the difference of phasors is a crucial issue in parameter estimation due to the low signal to noise ratio. Therefore it is necessary to use as much of data as possible. The EKF [7] takes into account the entire measurement history up to time \( k \) over the traditional WLS approach and is defined by the following
relations:

\[ s_{k|k} = s_{k|k-1} + K_{F,k} (z_k - h_k(s_{k|k-1})) \]  \hspace{1cm} (17)

\[ K_{F,k} = P_{k|k-1} H_k^T [H_k P_{k|k-1} H_k^T + R]^{-1} \]  \hspace{1cm} (18)

\[ P_{k|k} = P_{k|k-1} - K_{F,k} H_k P_{k|k-1}, \]  \hspace{1cm} (19)

\[ s_{k+1|k} = f(s_k), \]  \hspace{1cm} (20)

\[ P_{k+1|k} = F_k P_{k|k} F_k^T, \]  \hspace{1cm} (21)

where \( s_{k|k} \) is the parameter estimate at time \( k \) based on measurement history up to time \( k \), \( s_{k+1|k} \) is the one-step ahead prediction of the parameter values at time \( k + 1 \) based on measurement history up to time \( k \),

\[ H_k = \left. \frac{\partial h_k}{\partial s_k} \right|_{s_k = s_{k|k-1}} = \begin{bmatrix} \frac{\partial h_{1,k}}{\partial s_k} \\ \frac{\partial h_{2,k}}{\partial s_k} \\ \frac{\partial h_{3,k}}{\partial s_k} \\ \frac{\partial h_{4,k}}{\partial s_k} \end{bmatrix}^T, \]

\[ F_k = \left. \frac{\partial f}{\partial s_k} \right|_{s_k = s_{k|k}} = \begin{bmatrix} \partial f \partial s_k^T \\ 0_{8 \times 1} \end{bmatrix}, \]

\( 0_{8 \times 1} \) is an 8-by-1 vector of zeros and \( I_{8 \times 8} \) is identity matrix of dimension 8.

\section{4 Case studies}

This section is focused on two case studies demonstrating the effectiveness of the proposed approach. Both are performed on real data from 110kV transmission lines. Identification of the lines is performed recursively, in order to simulate the activity in real time. The proposed estimation algorithm was implemented in MATLAB.

\subsection{4.1 Identification of the transmission line A}

Let us consider the 110kV overhead transmission line with the maximum permissible current 483A. The aim is to identify the transmission line parameters based on measurements of positive sequence currents and voltages, respectively, from period of April 4 to September 9 with sampling period \( \Delta t = 1 min \).

Development of parameter estimates is displayed in Figure 2 and Figure 3. In Table 1, the final parameter estimates obtained with and without calibration of measurements of positive sequence current are compared to the designed values. Improved quality of the estimate of the line resistance can be seen if the simultaneous calibration is used in comparison with the case of the identification without the calibration.
Figure 4: Comparison of real and imaginary parts of measured voltage drop on the series impedance of the transmission line A (dotted line) with the voltage drop computed from measured current and designed parameters (dot-and-dash line) and identified parameters with measured current calibration (solid line), respectively.

Figure 5: Measurements of positive sequence current on the transmission line A.

4.2 Identification of the transmission line B

Let us consider another 110kV overhead transmission line with the maximum permissible current 700A, where the measurements of positive sequence currents and voltages, respectively, are obtained from period of June 15 to September 9 with sampling period $\Delta t = 1\text{min}$. As in the previous example, the aim is to recursively identify the line parameters.

Development of identification of the line parameters is shown in Figure 6 and Figure 7. Comparison of final parameter estimates with designed values is presented in Table 2. Again, the calibration of current measurement brings an improvement of quality of the parameter estimates.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designed value</th>
<th>Identified value with calibration</th>
<th>Identified value without calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{20},[\Omega]$</td>
<td>1.56</td>
<td>1.59</td>
<td>1.39</td>
</tr>
<tr>
<td>$X,[\Omega]$</td>
<td>7.21</td>
<td>7.32</td>
<td>7.08</td>
</tr>
<tr>
<td>$G,[\mu S]$</td>
<td>57.78</td>
<td>57.43</td>
<td>56</td>
</tr>
<tr>
<td>$B,[\mu S]$</td>
<td>1.55</td>
<td>1.11</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7: Development of the identified temperature of conductor and actual resistance of the transmission line B.

5 Conclusion

The paper was devoted to the online transmission line parameter identification. A new algorithm was proposed for online parameter identification and simultaneous calibration of the current measurement. It was shown, that measurement calibration significantly improves the quality of the parameter estimates.

The transmission line model incorporating average temperature of conductor is used. The estimation of reference resistance and actual conductor temperature instead of actual resistance allows to identify parameters of the entire data history and thus improves resultant estimates of the overhead transmission line parameters.

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References: