Overlapping Control Structure based on Adaptive Control for Water Distribution Network

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Abstract: -The present paper proposes and evaluates a decentralized control method for water distribution networks. With the objective of leakage reduction in distribution systems, it is hoped that pressure optimization control will be realized. The present paper proposes an overlapping control structure based on a sensitivity matrix. Convergence of control is evaluated from a mathematical perspective for a centralized control structure, an overlapping control structure, and a separated control structure. The results show that convergence of control using an overlapping structure is superior to a separated structure.

Key-Words: - Overlapping decomposition, Water distribution networks, Adaptive control, Convergence rate, Reliability

1 Introduction

Along with sufficient maintenance of urban infrastructure, the need for water distribution using appropriate pressure aimed at reducing leakage is strongly required. If water pressure in a distribution pipeline is too high, leakage from places such as pipe joints will occur easily. However, if the pressure is too low, the water supply may be cut off at the demand end due to insufficient pressure even when a faucet is open. Therefore, a problem exists in that it is necessary to maintain an appropriate pressure by opening and closing multiple valves at the operation end in the distribution pipe network in response to temporal variations in demand.

In order to avoid this difficulty, we propose that a method of decentralized control be adopted, in which the system in question is decomposed and an adaptive controller is placed in each subsystem. In the present paper, changes in measured variables in response to changes in manipulated variables were first examined as a sensitivity matrix, in order to answer the question of decomposition of the system. Based on the results, a characteristic of the pipe network is found to be that the relationship between manipulated variables and measured variables simultaneously includes disjoint connection areas and tightly coupled relationships. Therefore, it is decided that an overlapping control structure would be constructed as a method that reflects this characteristic. Furthermore, the sensitivity matrix changes according to changes in demand, so that an adaptive control method that carries out pressure-fixation control while successively estimating the sensitivity matrix is proposed, convergence is evaluated mathematically.

In Section 2, an overlapping control structure based on a sensitivity matrix is proposed. Convergence of control is evaluated from a mathematical perspective for a centralized control structure, an overlapping control structure, and a separated control structure.

Section 3 shows a pipe network adaptive control strategy for a control system installed in decentralized controllers based on a linearized adaptive control model that carries out control while adaptively and successively estimating control gains.

2 Overlapping decomposition method

2.1 Formulation of distribution control problem

Pipe network analysis using minimum cost flow calculus is used in the analysis of steady flow in a water distribution network [1].

In an attempt to realize the general mathematical model for pipe networks, a distribution control model is constructed from the viewpoint of conducting distribution control using controllers, as follows. Temporally fluctuating demand is rewritten as $\mathbf{u}_D(k) \in R^m$ by inserting time variable k. In addition, due to temporally fluctuating demand, a pressure change occurs in the pipe network. The objective of distribution control is to fix this pressure change using controllers. The variable that the controller can operate in order to fix the pressure is the resistance coefficient of valves (manipulated variable) $\mathbf{u}_V(k) \in R^l$. When described within the range required to develop the discussion in the present paper, the physical properties of the distribution pipe network can be expressed as a nonlinear pipe network state equation, as follows:

$$\mathbf{f}(\mathbf{x}_C(k),\ \mathbf{u}_V(k),\ \mathbf{u}_D(k)) = \mathbf{0} \tag{1}$$

where $\mathbf{x}_C(k) \in R^l$ is a co-tree flow, l is the number of co-tree pipes, $\mathbf{u}_V(k) \in R^l$ is the resistance coefficient of the valves (manipulated variable), n is the number of valves, $\mathbf{u}_D(k) \in R^m$ is the demand, m is the number of demand ends, and k is the time.

The vector dimension of this function $\mathbf{f}$ is l, which means that function $\mathbf{f}$ consists of l nonlinear algebraic equations.

Information obtained from pressure gauges in the pipe network can be expressed as a measurement system observation equation, as follows:

$$\mathbf{z}(k) = \mathbf{h}(x_C(k),\ \mathbf{u}_V(k),\ \mathbf{u}_D(k)) \tag{2}$$

where $\mathbf{z}(k) \in R^s$ is the measurement information. The vector dimension of Equation (2) is s, which means that s items of information are measured simultaneously.

The general pipe network equations can be rewritten as Equations (1) and (2) from the perspective of distribution control. However, note that Equations (1) and (2) are still nonlinear.

Because the objective of distribution control is pressure fixation, the following mathematical control index is taken:

$$J = (\mathbf{z}_0 - \mathbf{z}(k))^T (\mathbf{z}_0 - \mathbf{z}(k)) \tag{3}$$

where $\mathbf{z}^0 \in R^s$ is the control target value for pressure.

Thus, the manipulated variable that minimizes the control target equation (3) is determined from the pipe network equation (1) and the observation equation (2).

In regard to the control problem, linear control laws are presented next. When deriving control laws for online control, it is permissible to take the demand as constant, taking into account the slowness of change of state of the pipe network due to changes in demand compared to the control cycle. If demand $\mathbf{u}_D$ is assumed to be constant in the pipe network equation (1), then the flow $\mathbf{x}_C$ becomes a function of manipulated variable $\mathbf{u}_V$, and so observation equation (2) also becomes a function of $\mathbf{u}_V$. Therefore, Equation (2) may be written as

$$\mathbf{z}(k) = H(\mathbf{u}_V(k)) \tag{4}$$

If this is linearized by $\mathbf{u}_V(k)$, we have:

$$\Delta\mathbf{z}(k) = \frac{\partial H}{\partial \mathbf{u}_V}(\mathbf{u}_V(k))\Delta\mathbf{u}_V(k) \tag{5}$$

If the above equation is used to find the manipulated variable that minimizes the control target of equation (3):

$$\begin{aligned} &\mathbf{u}(k+1) = \mathbf{u}(k) + \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^{+}(\mathbf{z}_0 - H(\mathbf{u}(k))) \\ &\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^{+} = -\left(\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^T \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))\right)^{-1} \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k)) \end{aligned} \tag{6}$$

where k is the calculation step, and each time instant of the system is taken as one calculation step.

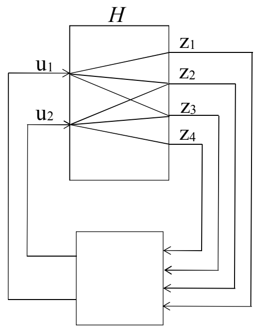
2.2 Basic study of decomposition

Generally, when considering system decomposition, the multiple subsystems obtained by decomposition do not intersect. In other words, full decomposition, in which subsystems do not overlap each other, has often been studied. However, in the decomposition of real systems, it may sometimes be more natural to carry out overlapping decomposition due to the physical properties of the system. Here, due to differences in information structure, overlapping control and separated control are established as decomposition methods, and a comparative evaluation of these methods is conducted for convergence while matching the case of centralized control.

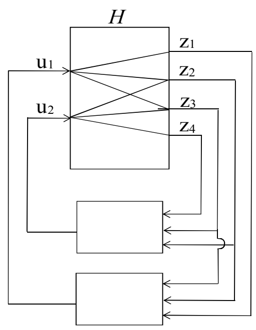
As a basic examination, a distribution system with a simple structure, as shown in Fig.1, is considered:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \mathrm{H}(u_1, u_2) = \begin{bmatrix} h_1(u_1) \\ h_2(u_1, u_2) \\ h_3(u_1, u_2) \\ h_4(u_2) \end{bmatrix} \tag{7}$$

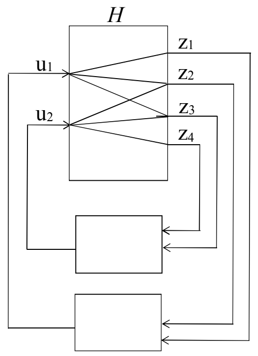
The input-output structure of the control object H illustrated in Fig.1 shows that there is not a mutually disjoint connection. On the other hand, observation point z_1, for example, is not influenced by manipulated variable u_2, which shows that some disjoint connection areas are inherent in the structure. Thus, the characteristics of the pipe network structure are represented by Equation (7) in preparation for the mathematical analysis that will be discussed later.



(a) Fully connected control (Centralized control)



(b) Partially connected control (Overlapping connected control)



(c) Separated control

Fig.1 Scheme of Control

Taking the control target value as z_0, the linear control strategy can generally be expressed as follows:

$$\mathbf{u}(k+1) = \mathbf{u}(k) + Q(\mathbf{u}(k))(\mathbf{z}_0 - H(\mathbf{u}(k))) \qquad (8)$$

$Q(\mathbf{u}(k))$: First derivative at $\mathbf{u}(k)$

First, if control is carried out using one controller, as in the centralized control in Fig.1(a), the control law is the same as Equation (6).

Next, in Fig.1(b), the structure is such that each controller performs control by receiving information from observation points that are influenced by a manipulated variable controlled by the respective controller, and the measurement information is overlapped in this control (referred to as overlapping control). The control laws for each controller can be written as follows:

$$\begin{pmatrix} \frac{\partial h_1}{\partial u_1} \\ \frac{\partial h_2}{\partial u_1} \\ \frac{\partial h_3}{\partial u_1} \end{pmatrix} (u_1(k+1) - u_1(k)) = \begin{bmatrix} z_{01} - h_1(u_1(k)) \\ z_{02} - h_2(\mathbf{u}(k)) \\ z_{03} - h_3(\mathbf{u}(k)) \end{bmatrix} \qquad (9)$$

$$\begin{pmatrix} \frac{\partial h_2}{\partial u_2} \\ \frac{\partial h_3}{\partial u_2} \\ \frac{\partial h_4}{\partial u_2} \end{pmatrix} (u_2(k+1) - u_2(k)) = \begin{bmatrix} z_{02} - h_2(\mathbf{u}(k)) \\ z_{03} - h_3(\mathbf{u}(k)) \\ z_{04} - h_4(u_2(k)) \end{bmatrix} \qquad (10)$$

These laws can be combined and expressed by the following equation:

$$\begin{aligned} &\mathbf{u}(k+1) - \mathbf{u}(k) \\ &= -\begin{pmatrix} \sum_{i=1}^{3} \frac{\partial h_i^2}{\partial u_1} & 0 \\ 0 & \sum_{i=2}^{4} \frac{\partial h_i^2}{\partial u_2} \end{pmatrix}^{-1} \frac{\partial H^T}{\partial \mathbf{u}} (\mathbf{z}_0 - H(\mathbf{u}(k))) \\ &= -\left(\frac{\partial H^T}{\partial \mathbf{u}} \frac{\partial H}{\partial \mathbf{u}} - \begin{pmatrix} 0 & \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} \\ \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} & 0 \end{pmatrix} \right)^{-1} \frac{\partial H^T}{\partial \mathbf{u}} (\mathbf{z}_0 - H(\mathbf{u}(k))) \end{aligned}$$

(11)

Next, respective control laws for separated control, as shown in Fig.1(c), can be expressed as follows:

$$\begin{bmatrix} \frac{\partial h_1}{\partial u_1} \\ \frac{\partial h_2}{\partial u_1} \end{bmatrix} (u_1(k+1) - u_1(k)) = \begin{bmatrix} z_{01} - h_1(u_1(k)) \\ z_{02} - h_2(\mathbf{u}(k)) \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix} \frac{\partial h_3}{\partial u_2} \\ \frac{\partial h_4}{\partial u_2} \end{bmatrix} (u_2(k+1) - u_2(k)) = \begin{bmatrix} z_{03} - h_3(\mathbf{u}(k)) \\ z_{04} - h_4(u_2(k)) \end{bmatrix} \qquad (13)$$

These equations can be combined and rewritten as follows:

$$\begin{aligned} &\mathbf{u}(k+1) - \mathbf{u}(k) \\ &= -\begin{pmatrix} \sum_{i=1}^{2} \frac{\partial h_i^2}{\partial u_1} & 0 \\ 0 & \sum_{i=3}^{4} \frac{\partial h_i^2}{\partial u_2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial h_1}{\partial u_1} & \frac{\partial h_2}{\partial u_1} & 0 & 0 \\ 0 & 0 & \frac{\partial h_3}{\partial u_2} & \frac{\partial h_4}{\partial u_2} \end{pmatrix} (\mathbf{z}_0 - H(\mathbf{u}(k))) \\ &= -\left(\frac{\partial H^T}{\partial H} \frac{\partial H}{\partial \mathbf{u}} - \begin{pmatrix} \frac{\partial h_3^2}{\partial u_1} & \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} \\ \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} & \frac{\partial h_2^2}{\partial u_2} \end{pmatrix} \right)^{-1} \left(\frac{\partial H}{\partial \mathbf{u}} - \begin{pmatrix} 0 & 0 & \frac{\partial h_3}{\partial u_1} & 0 \\ 0 & \frac{\partial h_2}{\partial u_2} & 0 & 0 \end{pmatrix} \right) \\ &+ (\mathbf{z}_0 - H(\mathbf{u}(k))) \end{aligned}$$

(14)

A basic study of control convergence for the above three control structures shall be carried out. Here, it is assumed that

$$\frac{\partial h_2}{\partial u_i}, \frac{\partial h_3}{\partial u_i} \sim 0(\varepsilon) \qquad i = 1,2 \qquad (15)$$

This means that the manipulated variables u_1 and u_2 are loosely coupled to control variables z_1 and z_2.

The linear control strategy equation (8) is rewritten as an equation that represents convergence towards the optimum point $\mathbf{z}_0 = H(\mathbf{u}^*)$.

$$\begin{aligned}&\mathbf{u}(k+1)-\mathbf{u}^*\\&=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))(\mathbf{z}_0-H(\mathbf{u}(k)))-Q(\mathbf{u}(k))(\mathbf{z}_0-H(\mathbf{u}^*))\\&=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))(H(\mathbf{u}^*)-H(\mathbf{u}(k)))\\&=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k))+Q(\mathbf{u}(k))S(\xi)(\mathbf{u}^*-\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k))\end{aligned} \quad (16)$$

where $S(\xi)$ is taken as the second derivative of $H(\mathbf{u})$ with respect to ξ between $\mathbf{u}^*$ and $\mathbf{u}(k)$.

Using this equation, first, in order to examine the convergence of centralized control:

$$Q(\mathbf{u}(k)) = \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^+ \cdot$$

Thus, Equation (16) becomes as follows:

$$\mathbf{u}(k+1)-\mathbf{u}^* = Q(\mathbf{u}(k))S(\xi)(\mathbf{u}^*-\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k)) \quad (17)$$

Taking this absolute value, convergence in the neighborhood of the optimum point can be written as follows:

$$|\mathbf{u}(k+1)-\mathbf{u}^*| \leqq |C| |\mathbf{u}^*-\mathbf{u}(k)|^2 \quad (18)$$

Here, $|C|$ is taken to be $|Q(u(k))S(\xi)| < |C|$.

Therefore, in centralized control, the manipulated variable converges to $\mathbf{u}^*$ by the speed of the square.

In overlapping control, from Equation (11), we obtain

$$\begin{aligned}&Q(\mathbf{u}(k))\\&= -(A-\delta A_P(\varepsilon^2))A^{-1}\frac{\partial H^T}{\partial \mathbf{u}}\\&= -(A^{-1}+A^{-1}\delta A_P(\varepsilon^2)A^{-1})\frac{\partial H}{\partial \mathbf{u}} - O(\varepsilon^4)\end{aligned} \quad (19)$$

Here, convergence can be examined using Equation (16) by writing the following:

$$A = \frac{\partial H^T}{\partial \mathbf{u}}\frac{\partial H}{\partial \mathbf{u}},$$

$$\begin{aligned}&\mathbf{u}(k+1)-\mathbf{u}^*\\&= -\left\{A^{-1}\delta A_P(\varepsilon^2)+O(\varepsilon^4)\frac{\partial H}{\partial u}\right\}(\mathbf{u}^*-\mathbf{u}(k))+C(\mathbf{u}^*-\mathbf{u}(k))^2\end{aligned} \quad (20)$$

Here, C' is taken as the product of $Q(\mathbf{u}(k))$ in Equation (16) and the second derivative of $H(\mathbf{u})$.

Taking the absolute value, convergence of overlapping control is as follows:

$$\begin{aligned}&|\mathbf{u}(k+1)-\mathbf{u}^*|\\&\leqq \left\{\left|A^{-1}\delta A_P(\varepsilon^2)\right|+O(\varepsilon^4)\frac{\partial H}{\partial u}\right\}|\mathbf{u}^*-\mathbf{u}(k)|+|C||\mathbf{u}^*-\mathbf{u}(k)|^2\end{aligned} \quad (21)$$

Furthermore, from Equation (14), separated control can be written as follows:

$$\begin{aligned}&Q(\mathbf{u}(k))\\&= -(A-\delta A_s(\varepsilon^2))^{-1}\left(\frac{\partial H^T}{\partial \mathbf{u}}-\delta B(\varepsilon)\right)\\&= -\left(A^{-1}\frac{\partial H^T}{\partial \mathbf{u}}-A^{-1}\delta A_s(\varepsilon^2)A^{-1}\frac{\partial H^T}{\partial \mathbf{u}}-A^{-1}\delta B(\varepsilon)+O(\varepsilon^3)\right)\end{aligned} \quad (22)$$

Substituting the above expression into Equation (12), we obtain:

$$\begin{aligned}&|\mathbf{u}(k+1)-\mathbf{u}^*|\\&\leqq \left\{\left|A^{-1}\delta A_s(\varepsilon^2)\right|+\left|A^{-1}\delta B(\varepsilon)\frac{\partial H^T}{\partial \mathbf{u}}\right|+O(\varepsilon^3)\frac{\partial H}{\partial \mathbf{u}}\right\}|\mathbf{u}(k)-\mathbf{u}^*|+|C||\mathbf{u}^*-\mathbf{u}(k)|^2\end{aligned} \quad (23)$$

Here, C'' is taken as the product of $Q(u(k))$ in Equation (18) and the second derivative of $H(u)$.

Comparing the convergence of Equations (18), (21), and (23), Equation (18) does not contain the first-order term $|\mathbf{u}^*-\mathbf{u}(k)|$, and the order of the coefficient of $|\mathbf{u}(k)-\mathbf{u}^*|$ is lower in Equation (21) than in Equation (23). Therefore, convergence can be evaluated as follows:

$$\text{Centralized} \succ \text{Overlapping} \succ \text{Separated} \quad (24)$$

3 Strategy for pipe network adaptive control

3.1 Control model for an adaptive controller

It was shown in the previous section that overlapping control, in which controllers are placed in each overlapping decomposed subsystem, is effective in terms of convergence for controllers that do not have the computing power to be able to cover the entire system. However, this is the case when the demand is constant and the sensitivity coefficient for this demand is known. In real problems, the sensitivity coefficient gradually changes along with changes in demand. Therefore, we describe the control model that each controller should possess in this case:

$$S_i : \varDelta\mathbf{z}_i(t+1) = H_i \varDelta\mathbf{u}_i(t) + \mathbf{c}_i \quad (25)$$

where $\mathbf{c}_i$ is an interference term between subsystems, summarized as variables, that are not explained by the manipulated variables covered by the controller in question, when information for manipulated variables covered by other controllers cannot be obtained. As the control method in each controller, the sensitivity coefficient and the interference term are estimated from manipulated variable information and measurement information that is available to the controller in question, and control is performed on this basis.

Next, an adaptive control strategy is derived using the self-tuning regulator (STR) method in which control is performed by estimating process parameters $H_i, \mathbf{c}_i$ in Equation (25) and determining the control parameters.

3.2 Pipe network adaptive control strategy

First, we show a process identification part for estimating, from process input-output, the sensitivity coefficient Hi and $\mathbf{c}i$, which are unknown parameters among the parameters comprising the adaptive control law. In Equation (25), if $t = 1,2,\cdots,k-1$, the following equation is obtained:

$$\begin{bmatrix} \varDelta\tilde{z}_i(1) \\ \varDelta\tilde{z}_i(2) \\ \vdots \\ \varDelta\tilde{z}_i(k) \end{bmatrix} = \begin{bmatrix} 1 & \varDelta\mathbf{u}_i^T(0) \\ 1 & \varDelta\mathbf{u}_i^T(1) \\ \vdots & \vdots \\ 1 & \varDelta\mathbf{u}_i^T(k+1) \end{bmatrix} \begin{bmatrix} \mathbf{c}_i^T \\ H_i^T \end{bmatrix} + \begin{bmatrix} \varepsilon_i^T(1) \\ \varepsilon_i^T(2) \\ \vdots \\ \varepsilon_i^T(k) \end{bmatrix} \quad (26)$$

$$\tilde{Z}_i(k) = Ui(k)\Theta_i^T + E_i(k) \quad (27)$$

where $\varepsilon_i(t)$ is the measurement noise at time t, $\varDelta\tilde{\mathbf{z}}(t)$ is the measurement at time t, $\hat{\Theta}_i(k)$ is the estimated value for Θ_i at time k, is determined so as to minimize the difference between $\hat{Z}_i(k) = U_i(k)\hat{\Theta}_i^T(k)$ and $\tilde{Z}_i(k)$. Specifically, the evaluation function of the square error is set as follows:

$$J_i(k) = \sum_{j=1}^{k} \lambda_i^{k-j} (\varDelta\tilde{\mathbf{z}}_i(j) - \varDelta\hat{\mathbf{z}}_i(j))^T (\varDelta\tilde{\mathbf{z}}_i(j) - \varDelta\hat{\mathbf{z}}_i(j)) \quad (28)$$

Here, the value to be estimated, Θ_i, changes slowly with time, so that a weighting $1 \geqq \lambda_i > 0$ was introduced. Applying the method of least squares to the above equation, if a sequential form is taken for real-time estimation, the following estimation equation can be derived:

$$\hat{\Theta}_i(k+1) = \hat{\Theta}_i(k) + \frac{1}{\varDelta_i(k)} \left\{ \varDelta\tilde{\mathbf{z}}_i(k+1) - \hat{\Theta}_i(k) \begin{bmatrix} 1 \\ \varDelta u_i(k) \end{bmatrix} \right\} \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \varDelta\mathbf{u}_i(k) \end{bmatrix} \right) \quad (29)$$

$$F_i(k+1) = F_i(k) - \frac{1}{\varDelta_i(k)} \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \varDelta\mathbf{u}_i(k) \end{bmatrix} \right) \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \varDelta\mathbf{u}_i(k) \end{bmatrix} \right)^T \quad (30)$$

$$\varDelta_i(k) = 1 + \begin{bmatrix} 1 \\ \varDelta\mathbf{u}_i(k) \end{bmatrix}^T \frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \varDelta\mathbf{u}_i(k) \end{bmatrix} \quad (31)$$

where $\hat{\Theta}_i(k+1) = \lfloor \hat{\mathbf{c}}_i(k+1) \hat{H}_i(k+1) \rfloor$ (32)

Next, we show the control law for the controller unit. The control index of divided subsystem *i* is set as follows:

$$\text{Min.}\, I_i = (\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i})^T P_i (\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i}) + (\mathbf{u}_i(k+1) - \mathbf{u}_i(k))^T W_i (\mathbf{u}_i(k+1) - \mathbf{u}_i(k)) \quad (33)$$

where P_i and W_i are positive definite.

Using the above-mentioned estimated value, the subsystem control model is as follows:

$$\varDelta\mathbf{z}_i(k+1) = \hat{H}_i(k+1)\varDelta\mathbf{u}_i(k+1) + \hat{\mathbf{c}}_i(k+1) \quad (34)$$

Next, $\varDelta\mathbf{u}_i(k+1)$, which minimizes control index, I_i, is found. The decrease in Equation (33), $\varDelta I_i$, can be written as follows:

$$\varDelta I_i = -2(\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i})^T P_i \varDelta\mathbf{z}_i(k+1) - \varDelta\mathbf{z}_i^T(k+1) P_i \varDelta\mathbf{z}_i(k+1) - 3\varDelta\mathbf{u}_i^T(k+1) Q_i \varDelta\mathbf{u}_i(k+1) \quad (35)$$

In addition, if Equation (34) is substituted into Equation (35) and partial differentiation with respect to $\varDelta\mathbf{u}_i(k+1)$ is carried out, setting $\frac{\partial \varDelta I_i}{\partial \varDelta\mathbf{u}_i(k+1)} = 0$, it is possible to find $\varDelta\mathbf{u}_i(k+1)$, which minimizes I_i. In other words,

$$-(\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i})\hat{H}_i^T(k+1)P_i - \hat{H}_i^T(k+1)P_i\hat{H}_i(k+1)\varDelta\mathbf{u}_i(k+1) - \hat{H}_i^T(k+1)P_i\hat{\mathbf{c}}_i(k+1) - 3W_i\varDelta\mathbf{u}_i^T(k+1) = \mathbf{0} \quad (36)$$

$$\varDelta\mathbf{u}_i(k+1) = -(\hat{H}_i^T(k+1)P_i\hat{H}_i(k+1) + 3W_i)^{-1}\hat{H}_i^T(k+1)P_i^T(\tilde{z}_i(k+1) - \mathbf{z}_{0i} + \hat{\mathbf{c}}_i(k+1)) \quad (37)$$

$$\mathbf{u}_i(k+1)=\hat{G}(k+1)(\mathbf{z}_{0i}-\widetilde{\mathbf{z}}_i(k+1)-\hat{\mathbf{c}}_i(k+1))+\mathbf{u}_i(k) \quad (38)$$

where

$$\hat{G}(k+1)=(\hat{H}_i^T(k+1)P_i\hat{H}_i(k+1)+3W_i)^{-1}\hat{H}_i^T(k+1)P_i^T \quad (39)$$

Equation (38) is the control law of the controller unit, and $\hat{G}_i(k+1)$ is the control gain.

Above is the control strategy for an adaptive controller placed in a decomposed subsystem. Next, this method is applied to a large-scale pipe network and verified through simulation.

4 Conclusion

We proposed an overlapping adaptive method in which overlapping decomposition of the system is carried out and adaptive controllers are placed in each decomposed subsystem. Overlapping decomposition was shown to be the best method for decomposing the system in order to implement the proposed method. The pipe network in question has a structure in which the sensitivity of measured variables z to manipulated variables u includes strong nonlinear relationships and disjoint connections. In other words, the input-output structure of the control object H is a nonlinear tightly coupled system that includes partial disjoint connections. By representing the characteristics of this pipe network structure as shown in Equation (7), a mathematical analysis of convergence was prepared. In addition, as shown in Figs.1(a), 1(b), and 1(c), the information structures of centralized control, overlapping control, and separated control were defined. In particular, in the information structure of overlapping control, measurement information is overlapped and measured by multiple controllers. The same linear control strategy was used to study convergence to the optimum point $z_0 = H(u^*)$ in these three control structures. The results showed that, even though the linear control strategy was the same, due to differences in measurement information, the control structures can be ranked in order from high to low convergence as centralized control, overlapping control, and separated control. In other words, it was shown that overlapping control has an advantage over separated control from the perspective of convergence.

Next, in regard to the control method installed in the decentralized controllers, a pipe network adaptive control strategy in which control is carried out by adaptively successively estimating control gains based on a linearized adaptive control model was demonstrated. Control gain G_i, which should be estimated by controller i, is obtained by successive estimation of target process parameter H_i covered by controller i. However, it is not possible to obtain information for manipulated variables covered by other controllers, and so it is necessary to estimate an interference term c_i between subsystems that are not explained by the manipulated variables covered by the controller in question. Therefore, controller i estimates process parameter H_i from measurement information and manipulated variable information that is available to controller i (in other words, controller i estimates control gain G_i), and controller i also estimates the interference term c_i. This is the control strategy of the adaptive controllers in charge of the decomposed subsystems.

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