

# DETERMINING THE EMISSION COEFFICIENT RADIAL DISTRIBUTION FROM A PLASMA SOURCE USING MATHCAD PROGRAM

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*Abstract:* - The purpose of this paper is to determine the emission coefficient radial distribution from the measured intensity distribution emitted by an extended source of radiation, particularly an application to plasma spectroscopy (He-Ne plasma). The plasma was produced inside a gas tube by a DC electrical discharge. The source was assumed to be optically thin and axially symmetrical. This problem is solved by inverting Abel's integral equation. A smoothing procedure is made on the experimental curve in order to attenuate the random errors before computing the derivative. Abel's integral equation is frequently applied in the study of extended radiation sources with cylindrical symmetry. A measurement of the transverse distribution  $I(y)$  of the intensity emitted perpendicularly to the source axis allows the calculation of the emission coefficient radial distribution,  $F(r)$ .  $F(r)$  can be deduced from  $I(y)$  by the inverse formula known as Abel's integral equation.  $I(r)$  and  $F(r)$  need not necessarily be the transverse intensity and the local emission coefficient respectively; they can also represent other physical quantities such as the optical thickness and the absorption coefficient of cylindrical absorbing plasma, and the optical path and the refractive index of cylindrical plasmas. The integral is calculated using a polynomial of second degree for the approximation of  $dI(y)/dy$  in a small interval on the right of the discontinuity point; the other part is calculated using an approximate numerical method made with a Mathcad program.

*Key-Words:* - Radial distribution, Plasma source, Numerical method, Mathcad

## 1 Introduction

The foundation of the modern theory of spectral line broadening in plasmas, based on contributions of several scientists, had been laid out by the 1960s [1]. For over a decade, there appeared a large number of successful applications of the theory to various experiments [2]. With a large number of applications in many areas of plasma physics, applied to plasmas with particle density ranging from several atoms per cubic centimeter to that of solid state and temperature from close to absolute zero to billions Kelvin, the theory of line broadening has been in perpetual development and remains at the heart of plasma spectroscopy [3]. The 1960s were also the years of a rapid progress in development of computers. It was then that Moore's law – the prediction of exponential growth of the computational power with time – was foreseen [4] for the next decade and, somewhat surprisingly, still holds true [5]. It is only natural, then, that one clear

trend in the development and applications of line-broadening calculations is a significant increase in the computational results, in particular, using computer simulations [6]. Evidently, this is not specific to the subject of line broadening, as other fields of science in general show clear signs of intrusion of computers in what used to be a sovereign patrimony of theoreticians [7].

The emission coefficient radial distribution from the measured intensity distribution emitted by a plasma source is one of the most interesting and complex problems in the field of plasma investigation. This problem is solved by inverting Abel's integral equation [8].

## 2 Problem Formulation

Abel's integral equation is frequently applied in the study of extended radiation sources with cylindrical symmetry. A measurement of the

transverse distribution  $I(y)$  of the intensity emitted perpendicularly to the source axis allows the calculation of the emission coefficient radial distribution  $F(r)$ . If the source is optically thin, the intensity  $I(y)$  is connected to the emission coefficient (Fig. 4) by the formula:

$$I(y) = \int_{-x}^x F(r) dx \quad (1)$$

Putting  $r = \sqrt{x^2 + y^2}$  and  $x = \sqrt{R^2 - y^2}$  we can write

$$I(y) = 2 \int_y^R \frac{r \cdot F(r)}{\sqrt{r^2 - y^2}} \cdot dy \quad (2)$$

$F(r)$  can be deduced from  $I(y)$  by the inverse formula:

$$F(r) = -\frac{1}{\pi} \int_r^R \frac{dI(y)}{dy} \cdot \frac{1}{\sqrt{y^2 - r^2}} dy \quad (3)$$

known as Abel's integral equation.

$I(y)$  and  $F(r)$  need not necessarily be the transverse intensity and the local emission coefficient respectively; they can also represent other physical quantities such as the optical thickness and the absorption coefficient of a cylindrical absorbing plasma, and the optical path and the refractive index of cylindrical plasmas.

Figure 1 illustrates the basic idea of the section of the one spray to plasma with the cylindrical symmetry.

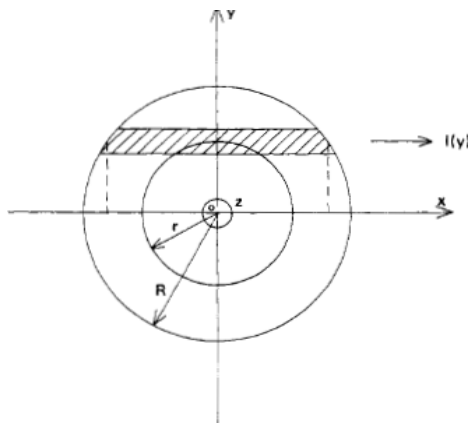


Fig. 1 Section of the one spray to plasma with the cylindrical symmetry.

The plasma source was assumed to be optically thin and axially symmetrical (fig. 2). The plasma was produced inside a gas tube by a DC electrical discharge. The gas in the tube is electrically neutral,

and as long as no external energy is applied, most of the molecules are in the ground state.

When the high electrical voltage is applied, electrons are released from the cathode and accelerated toward the anode. On their way, these electrons collide with the gas molecules and transfer energy to them. Thus, the gas molecules are raised to excited state.

Higher voltage is required to start the electrical discharge in the tube than to keep the discharge. Thus, a preliminary high voltage pulse is applied for initial discharge, and then the voltage is lowered to its operating value.

The ballast resistor is used to limit the current in the tube after discharge is achieved.

The gas mixture is 85-90% Helium gas, and 10-15% Neon gas, a ratio of 1:10.

The gas pressure is 0.01 Atmosphere ( $\approx 10$  [torr]). At the end of the tube, the electrodes are attached to a high voltage power supply (DC).

For the data acquisition we used two types of spectrometric equipment:

- High-resolution Spectrometer HR4000 UV/VIS: Spectral range 245 nm 690 nm, resolution 0,3 nm; Grating H1, installed, select 425-445 nm; Installed optical bench entrance aperture, 5 micron width;
- High-resolution Spectrometer HR4000 NIR: spectral range 685 nm 1100 nm, resolution 0,3 nm; Grating H1, installed, select 425-445 nm; Installed optical bench entrance aperture, 5 micron width; Cross-platform Spectroscopy Operating Software.



Fig. 2 Plasma shape

For the taking over it used optical fibers: QP450-2-XSR: 455 $\mu$ m Fiber, extreme solarization-resistant, 2m length for High-resolution Spectrometer HR4000 UV/VIS and QP400-2-VIS: 400 $\mu$ m Premium Fiber, VIS/NIR, 2m length for High-resolution Spectrometer HR4000 NIR.

The following figure 3 showing the montage of the HR 4000 Spectrometer.

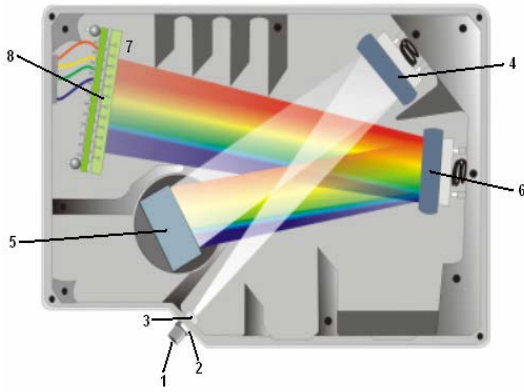


Fig. 3 Optical device of HR 4000 Spectrometer; 1- SMA Connector; 2- Slit; 3- Filter; 4- Collimating Mirror; 5- Grating; 6- Focusing Mirror; 7- L2 Detector Collection Lens; 8- CCD Detector

The spatial resolution is shown in Figure 4; for  $y = 0$ ,  $dy = dz = 0.4mm$  and  $dx = 35mm$ .

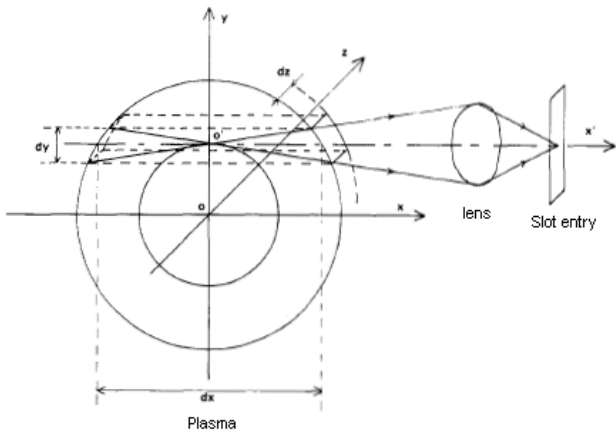


Fig. 4 The optical montage and the spatial resolution of the plasma

The plot containing the distribution of the intensity,  $I(y)$ , for three wave's lengths different is presented in Figure 5.

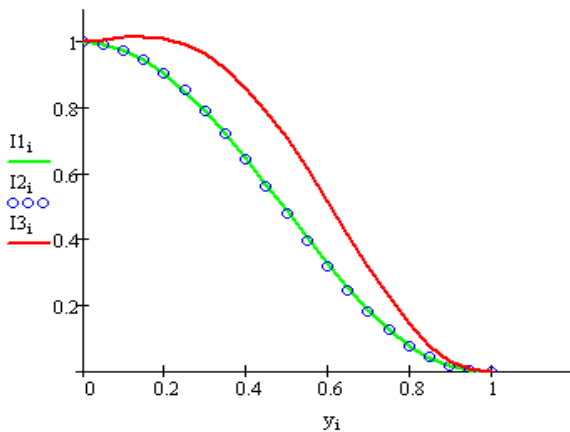


Fig. 5 The distribution of intensity for three wave's lengths different

The values for the distribution of the normalized intensity,  $I(y)$ , for three wave's lengths different are found in the table below:

Table 1: Distribution of normalized intensity for different wave length

y	I1(y) for $\lambda = 587.39[nm]$	I2(y) for $\lambda = 667.71[nm]$	I3(y) for $\lambda = 702.51[nm]$
0.	1.	1.	1.
0.05	0.9938121	0.9928906	1.0062758
0.1	0.9752485	0.9746492	1.0140824
0.15	0.9447778	0.9452760	1.0168376
0.2	0.9030564	0.9045837	1.0107914
0.25	0.8445247	0.8529467	0.9934946
0.3	0.7899869	0.7912067	0.9632634
0.35	0.7213576	0.7209542	0.9192561
0.4	0.6467279	0.6440599	0.8614725
0.45	0.567973	0.5630496	0.7904485
0.5	0.4871554	0.4803555	0.7079443
0.55	0.4063379	0.3985033	0.6160263
0.6	0.3276767	0.3199252	0.5178325
0.65	0.2534221	0.2468662	0.4170366
0.7	0.1857304	0.1811974	0.3179244
0.75	0.1265704	0.1243218	0.2251645
0.8	0.0776299	0.0776427	0.1435788
0.85	0.0405963	0.0416277	0.0776825
0.9	0.0157510	0.0167446	0.0312261
0.95	0.0030002	0.0026193	0.0061228
1.	0.	0.	0.

### 3 Problem Solution

This problem is solved by inverting Abel's integral equation. A smoothing procedure is made on the experimental curve in order to attenuate the random errors before computing the derivative.

Our method consists in computing  $I(y)$  derivative and integrating the formula (3) using the tow function made in Mathcad program. There is a discontinuity at  $y = r$  of the integrand of the equation (3) which causes the principal computational difficulties when the integral is calculated analytically.

For the determination of  $F(r)$  we have to take into consideration the errors give by the following factors:

1. Measurements made for  $I(y)$  which mislead errors in the calculation for  $dI(y)/dy$ . In order to diminish these errors we use the method of approximation with cubic spline interpolations.
2. The integrand of formula (3) have 2 singularities:
  - for  $r = 0$  the integral is convergent if approximation of  $dI(y)/dy$  is considered a polynomial of second degree with the form

$a \cdot y^2 + b \cdot y$  (if the free term is not zero the integral is divergent).

- for  $y = r$ , this representing the hardship in the calculation of  $F(r)$ . For overcome this difficulty the expression of  $F(r)$  gets this form:

$$F(r) = -\frac{1}{\pi} \left[ \frac{dI(y)}{dy} \cdot \ln\left(y + \sqrt{y^2 - r^2}\right) \Big|_r^R - \int_r^R \frac{d^2I(y)}{dy^2} \cdot \ln\left(y + \sqrt{y^2 - r^2}\right) dy \right] \quad (4)$$

The first part of the  $F(r)$  can be easily calculated in Mathcad, and the second integral is calculated using an approximate numerical method give by the function  $sol(r,R,v)$  made in the same program (Fig. 6).

```
S:=cspline(y, Y); f(x):=interp(S,y, Y,x);
d(x) := d/dx f(x); v := ∫ (0.6y^2 + 2y) / sqrt(y^2 - r^2) dy
sol(r,R,v) :=
n ← length(r)
y ← r
for i ∈ 2..n - 1
    F1 ← d(F) · ln[R + sqrt(R^2 - (r_i)^2)] - d(r_i) · ln(r_i)
    F2 ← ∫_r_i^R (d^2/dy^2 f(y)) · ln[y + sqrt(y^2 - (r_i)^2)] dy
    F_i ← -1/π (F1 - F2)
F_1 ← v
F_n ← 0
F
```

Fig. 6 The function  $sol(r,R,v)$  made in Mathcad program

A comparison has been performed using theoretical curves taken from the paper of Becher and Drawing [9]. The error and the standard deviation are calculated:

$$error = \sqrt{\frac{\sum_{i=1}^N (F(r_i)_{computed} - F(r_i)_{exact})^2}{N}} \quad (5)$$

$$N = length(r)$$

The test using the function  $F(r)=(R^2-r^2)^2$  and the error formula (5) give the standard deviation for the emission coefficient radial distribution,  $F1(r)$ ,  $F2(r)$  and  $F3(r)$ , respectively: 0.0002, 0.006 and 0.002.

Table 2: Distribution of radial intensity for different wavelengths.

F1(r), Radial distribution for $\lambda = 587.39[nm]$	F2(r), Radial distribution for $\lambda = 667.71[nm]$	F3(r), Radial distribution for $\lambda = 702.51[nm]$
0.9147523	0.976853	0.6974521
0.8930498	0.8989501	0.7233099
0.8463472	0.8155697	0.7677070
0.7421716	0.7355681	0.7788591
0.654281	0.6564681	0.7651649
0.5914480	0.5785038	0.7332288
0.4989751	0.5023404	0.6881532
0.4421234	0.4287733	0.6320176
0.3683610	0.3593891	0.5680724
0.3076279	0.2946344	0.4990842
0.2491980	0.2358085	0.4268970
0.1972479	0.1829499	0.3545373
0.1507051	0.1370362	0.2837645
0.1106482	0.0976156	0.2171498
0.0765571	0.0651735	0.1564124
0.0490978	0.0391599	0.1039254
0.0278312	0.0195296	0.0605388
0.0128134	0.0058284	0.0283179
0.0035677	0.0024852	0.0072610
0.0008283	0.0014647	0.0027764
0.	0.	0.

The plot containing the distribution of the emission coefficient radial distribution,  $F(r)$ , for three different wavelengths were obtained in Mathcad and presented below.

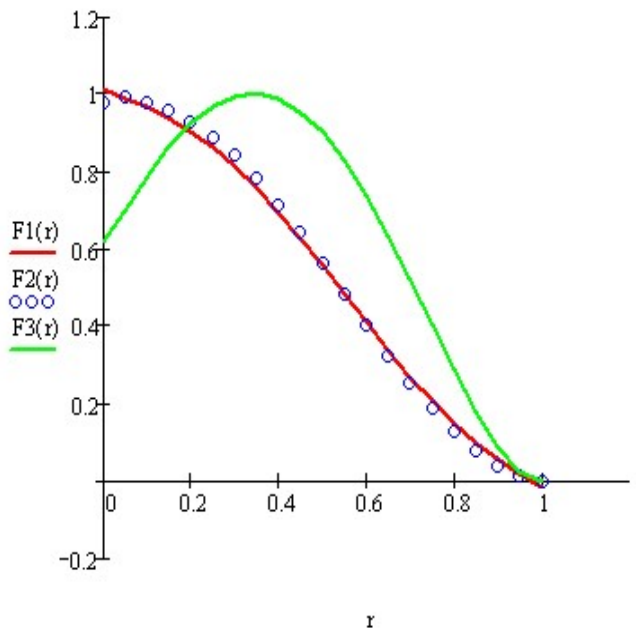


Fig. 7 The emission coefficient radial distribution  $F(r)$ , for three different wavelengths

## 4 Conclusion

The emission coefficient radial distribution was determined from the measured intensity distribution emitted by an extended source of radiation, particularly a plasma source. The source was assumed to be optically thin and axially symmetrical. This problem was solved by inverting Abel's integral equation. A smoothing procedure is made on the experimental curve in order to attenuate the random errors before computing the derivative. The integral is calculated using a polynomial of second degree for the approximation of  $dI(y)/dy$  in a small interval on the right of the discontinuity point; the other part is calculated using an approximate numerical method made with a Mathcad program. In this study the model parameters were assessed based on an analysis of field experimental data with good results.

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