Detection of Changes in Time Series of Economic Growth and Military Expenditure

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Abstract: The article is focused on detection of changes in time series of economic growth and military expenditure in France, Germany, Italy and Portugal in the period from 1950s to 2009. Data were selected from the SIPRI and OECD databases. Detecting the changes in these individual time series authors intend to answer the question whether these changes (either in levels or linear trend) occur simultaneously. This possible common changes can indicate the link between economic development and the level of military expenditure.

Key–Words: economic growth, military expenditure, change point detection, basis pursuit, ℓ_1-trend filtering

1 Introduction

Military expenditure of the NATO member states represent almost 64% of global military expenditure, out of which expenditure of the U.S.A. forms almost 70% of NATO military expenditure. The expenditure level depends especially on the economic standing, security environment and political circumstances of individual countries. Since the fall of the Bipolar World, total global military expenditure have been affected prevailingly by the 2001 terrorist attacks and subsequent war on terrorism. The authors in [11] admit that military expenditure can be determined by five factors, which can be stated as follows: the influence of external conflicts, the requirements of internal security, the domestic bureaucratic and budgetary factors, the influence of the armed forces themselves and the role of the major factors such as military coups, regimes and arms sales. From 2001 to 2010, the absolute levels of military expenditure of the 28 NATO member states fluctuated between 651 trillion (2002) and 1.004 trillion (2010). The structure of NATO military expenditure since 2001 has been influenced especially by a significant increase in military expenditure of the U.S.A. that has risen by almost 80% in the course of the war on terrorism. The trend towards increasing military expenditure is apparent also in enlarging NATO that has been enlarged three times recently. The Czech Republic, Hungary and Poland became new member states in the first phase in 1999 and Lithuania, Latvia, Estonia, Romania, Bulgaria, Slovenia and Slovakia in 2004. In the course of the last NATO enlargement when Albania and Croatia are acceding, it is possible to detect downward trends in the development of military expenditure, especially thanks to improving safety environment in the Balkans. The current developing countries suffering from deficit in public finances intensify pressure for cutting military expenditure, which is noticeable in the NATO countries as well where only a small group of member states fulfills the recommended 2% of GDP investment in military expenditure (see table 1).

2001	USA, Bulgaria, Croatia, Greece, Italy, Romania, Turkey, UK
2002	USA, Bulgaria, Croatia, Czech Republic, Greece, Italy, Portugal, Romania, Turkey, UK
2003	USA, Bulgaria, Croatia, Czech Republic, Greece, Italy, Romania, Turkey, UK
2004	USA, Bulgaria, Greece, Italy, Portugal, Romania, Turkey, UK
2005	USA, Bulgaria, Greece, Czech Republic, Greece, Portugal, Romania, Turkey, UK
2006	USA, Bulgaria, Greece, Portugal, Turkey, UK
2007	USA, Bulgaria, Estonia, Greece, Turkey, UK
2008	USA, Albania, Bulgaria, Estonia, Greece, Turkey, UK
2009	USA, Albania, Bulgaria, Estoni,a Portugal, Greece, Turkey, UK

Table 1: The NATO countries fulfilling the recommended 2% of GDP investment in military expenditure

The purpose of this paper is to identify the link, empirically established in [8], between economic

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growth and military expenditure in the selected NATO states in the extended period from 1950s to 2009 reflecting the economic development as factors influencing the level of military expenditure of the NATO states. To identify the existence link between economic situation and level of military expenditure authors employ selected methods of change point detection (statistical approach, basis pursuit approach and ℓ_1-trend filtering).

The paper follows from the study [8] where the authors theoretically examine possible positive and negative aspects of the effect of military expenditure on selected economies and give a positive example of possible multiplication effect contributing to GDP and employment growth. On the other hand they also see the slowing effect in the form of possible inflation and public finance reduction that cannot be invested in other industries, etc. Taking the example of 15 selected EU states and realistic data, the authors simultaneously attempt to identify the link between military expenditure as a percentage of GDP and economic growth of the given country in the analyzed period 1961–2000. Using the causality test they prove that a link between economic growth and military expenditure has been identified in six countries (Germany, Italy, the Netherlands, Spain, Sweden and the UK) and come to the conclusion that economic development of the selected EU states determines their military expenditure, i.e. the economic standing of the European countries is a one from factors influencing the level of their military expenditure.

2 Change point detection

Let us assume the model with p change points

$$Y_t = \begin{cases} \mu + \epsilon_t & t = 1, 2, \dots, c_1 \\ \mu + \delta_1 + \epsilon_t & t = c_1 + 1, \dots, c_2, \\ \cdots & \cdots \\ \mu + \delta_p + \epsilon_t & t = c_p + 1, \dots, T, \end{cases} \tag{1}$$

where $\mu, \delta_1, \dots, \delta_p \neq 0, t_0 \leq c_1 < \dots < c_p < T - t_0$ are unknown parameters and ϵ_t are independent identically distributed random variables with zero mean and variance σ^2.

Statistical methods

Consider the model (1) with only one change point c. Assuming σ^2 given, the unknown parameters c, μ and δ may be estimated by the least-squares method. The least-squares estimators $\hat{c}$, $\hat{\mu}$ and $\hat{\delta}$ of the parameters c, μ and δ are defined as solutions of the minimization problem

$$\min_{k \in \{1,\dots,T-1\}} \left\{ \sum_{t=1}^{k} (Y_t - \mu)^2 + \sum_{t=k+1}^{T} (Y_t - \mu - \delta)^2 \right\}. \tag{2}$$

In other words, the unknown parameters are estimated in such a way that the sum of squares of residuals is minimal. The estimates of the parameters μ and δ are (see [1], [3])

$$\hat{\mu} = \overline{Y}_{\hat{c}} \quad \text{and} \quad \hat{\delta} = \overline{Y}^0_{\hat{c}} - \overline{Y}_{\hat{c}},$$

where $\hat{c}$ is a solution of the maximization problem

$$\hat{c} = \underset{k \in \{1,\dots,T-1\}}{\arg\max} \left\{ \sqrt{\frac{T}{k(T-k)}} \cdot |S_k| \right\}, \tag{3}$$

where $S_k = \sum_{t=1}^{k} (Y_t - \overline{Y}_T)$, $\overline{Y}_T = \frac{1}{T} \sum_{t=1}^{T} Y_t$, $\overline{Y}_{\hat{c}} = \frac{1}{\hat{c}} \sum_{t=1}^{\hat{c}} Y_t$ and $\overline{Y}^0_{\hat{c}} = \frac{1}{T-\hat{c}} \sum_{t=\hat{c}+1}^{T} Y_t$.

In the case of multiple change points, we can use the described statistical method of one change point detection via the following procedure. At first, find $\hat{c}^{(1)}$ by solving (3). Secondly, divide observations into two groups $Y_1, \dots, Y_{\hat{c}^{(1)}}$ and $Y_{\hat{c}^{(1)}+1}, \dots Y_T$ and find the estimator in each group. The whole procedure is repeated until a "constant mean is obtained". This procedure is called "binary segmentation".

Basis pursuit approach

We briefly describe the method based on basis pursuit algorithm (BPA) for the detection of the change point in the sample path $\{y_t\}$ in one-dimensional stochastic process $\{Y_t\}$. We assume a deterministic functional model on a bounded interval $\mathcal{I}$ described by the dictionary $G = \{G_j\}_{j \in J}$ with atoms $G_j \in L^2(\mathcal{I})$ and with additive white noise e on a suitable finite discrete mesh $\mathcal{T} \subset \mathcal{I}$:

$$Y_t = x_t + e_t, \; t \in \mathcal{T},$$

where $x \in \text{sp}(\{G_j\}_{j \in J})$, $\{e_t\}_{t \in \mathcal{T}} \sim WN(0, \sigma^2)$, $\sigma > 0$, and J is a big finite indexing set. Smoothed function $\hat{x} = \sum_{j \in J} \hat{\xi}_j G_j =: \mathbf{G}\hat{\xi}$ minimizes on $\mathcal{T}$ ℓ_1-penalized optimality measure $\frac{1}{2}\|\mathbf{y} - \mathbf{G}\xi\|^2$ as follows:

$$\hat{\xi} = \underset{\xi \in \ell^2(J)}{\arg\min} \frac{1}{2} \|\mathbf{y} - \mathbf{G}\xi\|^2 + \lambda \|\xi\|_1,$$

where $\|\xi\|_1 := \sum_{j \in J} \|G_j\|_2 \xi_j$ and a smoothing parameter $\lambda = \sigma\sqrt{2 \, \ln{(\text{card}\, J)}}$. Such approaches are also known as basis pursuit denoising (BPDN). Solution of this minimization problem with λ close to zero

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may not be sparse enough: we are searching small $F \subset J$ such that $\hat{x} \approx \sum_{j \in F} \hat{\xi}_j G_j$ is a good approximation. The procedure of BPDN is described in [10].

We build our dictionary from heaviside-shaped atoms on $L^2(\mathbb{R})$ derived from a fixed 'mother function' via shifting and scaling following the analogy with the construction of wavelet bases (see [10]). We define Heaviside atom by the formula

$$G_{a,b}(t) = \begin{cases} 1 & t - a > b/2, \\ 2(t-a)/b & |t-a| \le b/2, b > 0, \\ 0 & t = a, b = 0, \\ -1 & \text{otherwise.} \end{cases}$$

Some examples of Heaviside functions are displayed in figure 1. The shift parameters of the significant atoms (the atoms contained in solution by BPDN) indicates possible change points in the sample path of the process.

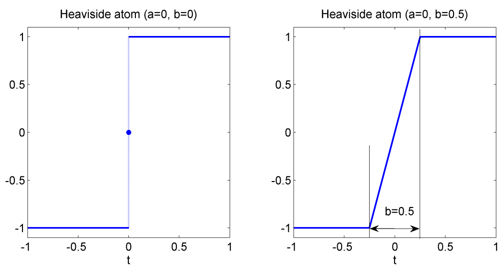

Figure 1: Heaviside atoms with parameters $a = 0, b = 0$ and $a = 0, b = 0.5$

ℓ_1-trend filtering

In [7] the authors propose a variation on Hodrick-Prescott filtering, which they call ℓ_1-trend filtering. The trend is estimated as the minimizer of the objective function

$$\frac{1}{2}\sum_{t=1}^{T}(y_t - x_t)^2 + \chi \sum_{t=2}^{T-1} |x_{t-1} - 2x_t + x_{t+1}|, \quad (4)$$

which can be written in the matrix form as

$$\frac{1}{2}\|\mathbf{y} - \mathbf{x}\|_2^2 + \chi\|\mathbf{D}\mathbf{x}\|_1,$$

where $\chi \ge 0$ is a smoothing parameter, $\mathbf{x} = (x_1, \ldots, x_T)' \in \mathbb{R}^T$, $\mathbf{y} = (y_1, \ldots, y_n)' \in \mathbb{R}^T$ and $\mathbf{D} \in \mathbb{R}^{(T-2)\times T}$ is the matrix

$$\mathbf{D} = \begin{pmatrix} 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \end{pmatrix}.$$

The ℓ_1 trend estimate is piecewise linear in t. The points where the slope of the estimated trend is changed can be interpreted as abrupt changes (change points) in the process. The argument appearing in the second term of (4), $x_{t-1} - 2x_t + x_{t+1}$, is the second difference of $\{x_t\}$. It is zero when and only when three points x_{t-1}, $2x_t$ and x_{t+1} are on the line. This method can be used for change point detection in the linear regression.

3 Data sources and their characteristics

To analyze the link between economic development and level of military expenditure, time series of military expenditure expressed as a percentage of GDP from the SIPRI database were used. The SIPRI definition of military expenditure includes all current and capital expenditure on the following activities: the armed forces (including peace-keeping forces), the civil administrations of the military sector (defence ministries and other government agencies engaged in defence activities), paramilitary forces (non-regular armed forces which are judged to be trained, equipped and available for military operations), military space activities. Such expenditure should include the following components: personnel, operations and maintenance, arms procurement, military research and development (R&D), military construction, military aid (in the military expenditure of the donor country). The economic growth data (the growth rate in per cent) were extracted from the OECD database.

	Average	Std. Deviation	Max	Min
France	4.35	1.66	9.1	2.3
Germany	3.00	1.10	5.2	1.3
Italy	2.50	0.71	4.5	1.7
Portugal	3.85	1.74	7.4	1.9

Table 2: The basic characteristics of military expenditure

To assess the existence of the link between the two variables, four economies (France, Germany, Italy and Portugal) have been selected as a suitable economy characterized by periods of growth of military expenditure as well as downturn of military expenditure as percentage of GDP. The selected temporal series describe the period from 1950s to 2009 (see table 2).

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4 Numerical results

To detect possible changes in time series of economic growth two methods were used: the basis pursuit approach (BPDN) and the binary segmentation method. The linear trend had been subtracted before analysis was performed. The ℓ_1-trend filtering method was applied to the time series of military expenditure in selected countries. (The smoothing parameter χ was set to 1.) All results obtained are summarized in tables 3, 4, 5 and 6 and displayed in figures 2, 3, 4 and 5.

Economic growth	Estimated change points
BPDN	1973 1958 1990 1996 1985
BPDN(sorted)	1958 1973 1985 1990 1996
Binary seg.	1953 1973 1985 1958 1997
Binary seg. (sorted)	1953 1958 1973 1985 1997
Military expenditure	
ℓ_1-trend filtering	1953 1972 1986 2000

Table 3: The estimated change points in France

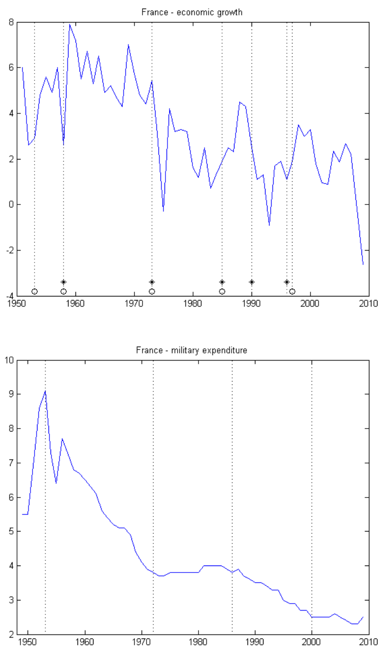

Figure 2: The estimates of changes in time series of economic growth and military expenditure in France (BPDN – asterisk, binary segmentation – circle)

5 Conclusion

The purpose of this paper was to identify the link, empirically established in [9], between economic growth

Economic growth	Estimated change points
BPDN	1960 1991 1987 1973 1956
BPDN(sorted)	1956 1960 1973 1987 1991
Binary seg.	1960 1987 1991 1956 1973
Binary seg. (sorted)	1956 1960 1973 1987 1991
Military expenditure	
ℓ_1-trend filtering	1958 1963 1970 1984 1987 1995

Table 4: The estimated change points in Germany

Figure 3: The estimates of changes in time series of economic growth and military expenditure in Germany (BPDN – asterisk, binary segmentation – circle)

and military expenditure in the case of France, Germany, Italy and Portugal in the period from 1950s to 2009. To prove the existence of the link between economic situation and level of military expenditure authors employ selected methods of change point detection. Detecting the changes in these individual time series they intended to answer the question whether these changes (either in levels or linear trend) occur simultaneously. Based on the estimated change points it can not be said that all the changes in economic growth reflect changes in military spending. However, it is possible to find some periods when these changes occur simultaneously in the both indicators, for example, in France (in the period 1972–1973), in Germany (1956–1958), in Italy (1959–1962) and in Portugal (1958–1959). Adding these change points in models describing military expenditure depending on other macroeconomic indicators can improve their

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Economic growth	Estimated change points
BPDN	1975 1962 1959 1980
BPDN(sorted)	1959 1962 1975 1980
Binary seg.	1958 1962 1993 1965
Binary seg. (sorted)	1958 1962 1965 1993
Military expenditure	
ℓ_1-trend filtering	1952 1960 1978 1988

Table 5: The estimated change points in Italy

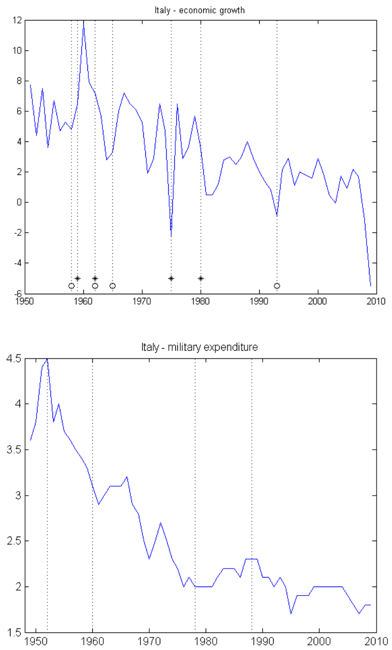

Figure 4: The estimates of changes in time series of economic growth and military expenditure in Italy (BPDN – asterisk, binary segmentation – circle)

Economic growth	Estimated change points
BPDN	1973 1958 1980 1990 1985
BPDN(sorted)	1958 1973 1980 1985 1990
Binary seg.	1958 1973 1969 1984 2001
Binary seg. (sorted)	1958 1969 1973 1984 2001
Military expenditure	
ℓ_1-trend filtering	1959 1962 1968 1971 1974 1977 2000

Table 6: The estimated change points in Portugal

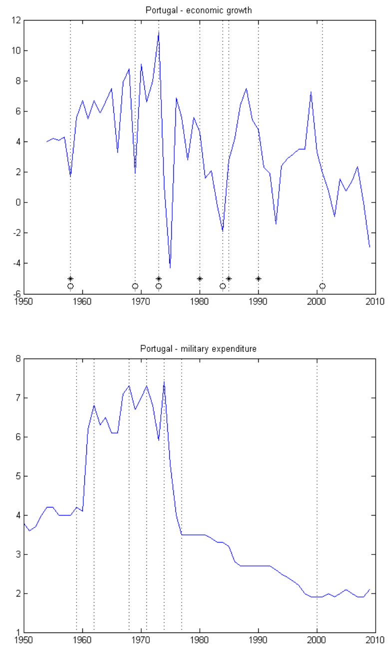

Figure 5: The estimates of changes in time series of economic growth and military expenditure in Portugal (BPDN – asterisk, binary segmentation – circle)

accuracy. The methods described in this article will then be applied to the remaining members of NATO.

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