Second Order Statistics for SSC Receiver in the Presence of Hoyt Fading

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Abstract: - The second order statistics such as level crossing rate (LCR) and average outage duration (AOD) for switch and stay combining (SSC) receiver are analyzed in this paper. The presence of Hoyt fading at the branch's inputs is observed. The obtained expressions are calculated and shown graphically for different fading parameters and values of threshold decision.

Key-Words: - Average outage duration (AOD), Level crossing rate (LCR), switch and stay combining (SSC), Hoyt fading

1 Introduction

The signal propagation through wireless communications channels has great importance in research interest [1]. The random fluctuations of the signal envelope and phase in a radio channel are caused with two propagation phenomena: multipath scattering - fast fading and shadowing -slow fading. The fast fading is characterized by the reception of an infinity number of waves from reflection, diffraction and scattering of a transmitted signal in a propagation environment. The deterministic analysis of the received signal is very difficult because the characterization of these phenomena is very complex [2]. Many authors in the literature show that the use of stochastic approaches is a good option that corresponds good to experimental data in various communication applications, such as satellite propagation channels [3].

Hoyt distribution allows the modeling of a propagation channel without a dominant component over the scattered waves (a situation of non-line of sight). Now, the sum of various waves that arrive at the receiver results in two complexes Gaussian processes with in-phase and quadrature Gaussian distributed variables with different standard deviations and means equal to zero [4]. It involves the range of the fading from the one-sided Gaussian to the Rayleigh distribution

For reducing fading effects and influence of shadow effects various techniques are used in wireless communication systems. These are: diversity reception, dynamic channel allocation and power control. The main purpose of diversity techniques is enhancing the reliability of transmission and increasing of channel capacity without increasing transmission power and bandwidth.

One of the simplest diversity combining techniques is switch and stay combining (SSC). The second order statistics for SSC receivers has not been studied enough in the literature.

Exact expressions for the level crossing rate and average fade duration of M-branch equal-gain and maximal-ratio combining systems in a Hoyt fading environment are presented in [5]. The expressions are applied to unbalanced, nonidentical, independent diversity channels and have been validated by specializing the general results to some particular cases whose solutions are known.

In this paper [6], the second order statistics of the Nakagami-Hoyt fading channel model (Nakagami-q model) is considered. Expressions for the level crossing rate (LCR) and the average duration of fades (ADF) are derived. It is shown that the obtained analytical quantities best fit the corresponding measurement data for an equivalent mobile satellite channel in the case of an environment with heavy shadowing.

A study on the statistical properties of double Hoyt fading channels has been presented in [7]. Analytical expressions for the mean value, variance, PDF, LCR, and ADF of the double Hoyt processes have been derived.

The closed-form expressions for the average level crossing rate (LCR) and average outage duration (AOD) of switch and stay combining (SSC) in the presence of Rayleigh, Nakagami and Rician fading are presented in [8].

These authors are determined the average level crossing rate (LCR) and average outage duration (AOD) of switch and stay combining (SSC) in the presence of Rician, Nakagami and α-μ fading in [9] - [12]. The influence of slow log-normal fading to the second order statistics of SSC receiver is determined in [13].

This paper is organized as follows. First Section, Introduction, presents an abstract about Hoyt distribution and techniques for reducing fading effects on system performances. After is Section 2 which introduces the system model. Section 3 is focused on system performances derivation. For mathematical expressions for the second order statistics: Level Crossing Rate (LCR) and Average fade Duration (ADF) in the presence of Hoyt fading, derived in previous section, graphical presentation are given in Section 4. Finally, Section 5 presents the conclusions and the future work proposals. Also, we present the Acknowledgment.

2 System Model

The model of dual SSC combiner, analyzed in this paper, is shown in Fig. 1. The signals at the combiner input are r_1 and r_2, and the combiner output signal is r.

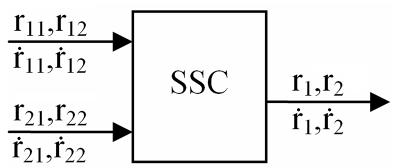

Fig.1. Model of the SSC combiner with two inputs

The combiner first examines the signal from first input with probability P1, and it examines first the signal from second input with probability P2. If combiner examines first the signal from first input and if its value is above the threshold, r_T, SSC receiver will this signal put in the unit for decision. If the value of the signal from first input is below the threshold r_T, SSC combiner forwards the signal from the other input to the unit for decision, regardless of it is greater or less then the threshold.

If the SSC combiner first examines the signal from the second combiner branch, the algorithm of decision is similar to the previous case.

The analysis is given for channels with Hoyt fading distribution for non-identical, independent diversity channels, what happens when the distance between the antennas is large and there is no correlation between the signals.

3 System Performances

The probability density functions (PDFs) of the signals r_1 and r_2 (combiner input channels) in the presence of Hoyt fading, are [4], [6]:

$$p_{r_i}(r_i) = \frac{r_i}{\sigma_{i1}\sigma_{i2}} \exp\left(-\frac{r_i^2}{4}\left(\frac{1}{\sigma_{i1}^{\;2}} + \frac{1}{\sigma_{i2}^{\;2}} \right) \right) \cdot$$

$$\cdot I_0\left(\frac{r_i^2}{4}\left(\frac{1}{\sigma_{i2}^{\;2}} - \frac{1}{\sigma_{i1}^{\;2}} \right) \right) \qquad (1)$$

where $I_0(.)$ designates the zero order Bessel function of the first kind, σ_{i1}^2 and σ_{i2}^2 are variances of uncorrelated zero mean low pass Gaussian processes in Hoyt model. For $\sigma_{i1}=\sigma_{i2}$, (1) reduces to the Rayleigh distribution.

The cumulative probability densities (CDFs) are defined by

$$F_{r_i}(r_T) = \int_0^{r_T} p_{r_i}(x)\,dx \qquad (2)$$

where r_T is the threshold decision.

In the presence of Hoyt fading, CDFs are:

ISBN: 978-1-61804-138-8

$$F_{r_i}(r_T)=\int_0^{r_T}\frac{r_i}{\sigma_{i1}\sigma_{i2}}\exp\left(-\frac{r_i^2}{4}\left(\frac{1}{\sigma_{i1}^2}+\frac{1}{\sigma_{i2}^2}\right)\right)\cdot$$

$$\cdot I_0\left(\frac{r_i^2}{4}\left(\frac{1}{\sigma_{i2}^2}-\frac{1}{\sigma_{i1}^2}\right)\right)dr_i \quad (3)$$

The probabilities P_1 and P_2 are [1]

$$P_i=\frac{F_{r_j}(r_T)}{F_{r_i}(r_T)+F_{r_j}(r_T)},\qquad i\neq j,\qquad i,j=1,2 \quad (4)$$

The joint probability densities of the combiner input signals, r_1 and r_2, and their derivatives $\dot{r}_1$ and $\dot{r}_2$, in the presence of Hoyt fading, are [6]

$$p_{r_i\dot{r}_i}(r_i,\dot{r}_i)=\frac{r_i}{(2\pi)^{3/2}\sigma_{i1}\sigma_{i2}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r_i^2}{2\sigma_{i1}^2\sigma_{i2}^2}\left(\sigma_{i1}^2\cos^2(\theta)+\sigma_{i2}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot e^{-\frac{\dot{r}_i^2}{2\left(\beta_{i2}+(\beta_{i1}-\beta_{i2})\cos^2(\theta)\right)}}\frac{1}{\sqrt{\beta_{i2}+(\beta_{i1}-\beta_{i2})\cos^2(\theta)}}d\theta \quad (5)$$

where β_{ij} are variances of derivatives.

The expression for the joint probability density function of the SSC combiner output signal and its derivative will be determined first for the case when signal is less then threshold, $r<r_T$:

$$p_{r\dot{r}}(r\dot{r})=P_1\cdot F_{r_1}(r_T)\cdot p_{r_2\dot{r}_2}(r\dot{r})+P_2\cdot F_{r_2}(r_T)\cdot p_{r_1\dot{r}_1}(r\dot{r}) \quad (6)$$

and then for the case if signal is above the threshold, $r\geq r_T$:

$$p_{r\dot{r}}(r\dot{r})=P_1\cdot p_{r_1\dot{r}_1}(r\dot{r})+P_1\cdot F_{r_1}(r_T)\cdot p_{r_2\dot{r}_2}(r\dot{r})+$$

$$+P_2\cdot p_{r_2\dot{r}_2}(r\dot{r})+P_2\cdot F_{r_2}(r_T)\cdot p_{r_1\dot{r}_1}(r\dot{r}) \quad (7)$$

where P_i, $p_{r_i\dot{r}_i}(r\dot{r})$ and $F_{r_i}(r_T)$ are defined in (4), (6) and (2). The joint PDF is in the form of single integral.

The level crossing rate is [1]

$$N(r)=\int_0^{\infty}\dot{r}\,p_{r\dot{r}}(r,\dot{r})d\dot{r}\,. \quad (8)$$

For the signal below the threshold, $r<r_T$, it is:

$$N(r)=P_1F_{r_1}(r_T)\frac{r}{(2\pi)^{3/2}\sigma_{21}\sigma_{22}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{21}^2\sigma_{22}^2}\left(\sigma_{21}^2\cos^2(\theta)+\sigma_{22}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{22}+(\beta_{21}-\beta_{22})\cos^2(\theta)}d\theta+$$

$$+P_2F_{r_2}(r_T)\frac{r}{(2\pi)^{3/2}\sigma_{11}\sigma_{12}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{11}^2\sigma_{12}^2}\left(\sigma_{11}^2\cos^2(\theta)+\sigma_{12}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{12}+(\beta_{11}-\beta_{12})\cos^2(\theta)}d\theta \quad (9)$$

For the cases that signal is bigger then threshold, $r\geq r_T$, the level crossing rate is:

$$N(r)=P_1\frac{r}{(2\pi)^{3/2}\sigma_{11}\sigma_{12}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{11}^2\sigma_{12}^2}\left(\sigma_{11}^2\cos^2(\theta)+\sigma_{12}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{12}+(\beta_{11}-\beta_{12})\cos^2(\theta)}d\theta$$

$$+P_1F_{r_1}(r_T)\frac{r}{(2\pi)^{3/2}\sigma_{21}\sigma_{22}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{21}^2\sigma_{22}^2}\left(\sigma_{21}^2\cos^2(\theta)+\sigma_{22}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{22}+(\beta_{21}-\beta_{22})\cos^2(\theta)}d\theta+$$

$$+P_2\frac{r}{(2\pi)^{3/2}\sigma_{21}\sigma_{22}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{21}^2\sigma_{22}^2}\left(\sigma_{21}^2\cos^2(\theta)+\sigma_{22}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{22}+(\beta_{21}-\beta_{22})\cos^2(\theta)}d\theta+$$

$$+P_2F_{r_2}(r_T)\frac{r}{(2\pi)^{3/2}\sigma_{11}\sigma_{12}}\cdot$$

$$\cdot\int_0^{2\pi}e^{-\frac{r^2}{2\sigma_{11}^2\sigma_{12}^2}\left(\sigma_{11}^2\cos^2(\theta)+\sigma_{12}^2\sin^2(\theta)\right)}\cdot$$

$$\cdot\sqrt{\beta_{12}+(\beta_{11}-\beta_{12})\cos^2(\theta)}d\theta \quad (10)$$

ISBN: 978-1-61804-138-8

The average time of fade duration can be obtained from the expression [1]:

$$T(r)=\frac{P_{out}(r)}{N(r)} \qquad (11)$$

where $P_{out}(r)$ is outage probability given by [1]

$$P_{out}(r)=\int_0^r p(r)\,dr \qquad (12)$$

Probability density function (PDF) at the output of SSC combiner is, for $r<r_T$

$$p_r(r)=P_1\cdot F_{r_1}(r_T)\cdot p_{r_2}(r)+P_2\cdot F_{r_2}(r_T)\cdot p_{r_1}(r) \qquad (13)$$

for $r\geq r_T$

$$p_r(r)=P_1\cdot p_{r_1}(r)+P_1\cdot F_{r_1}(r_T)\cdot p_{r_2}(r)+$$
$$+P_2\cdot p_{r_2}(r)+P_2\cdot F_{r_2}(r_T)\cdot p_{r_1}(r) \qquad (14)$$

P_{out} can be obtained, putting (3) and (13)-(14) in (12). It is, for $r<r_T$:

$$P_{out}(r)=P_1F_{r_1}(r_T)F_{r_2}(r)+P_2F_{r_2}(r_T)F_{r_1}(r) \qquad (15)$$

and for $r\geq r_T$

$$P_{out}(r)=P_1\left(F_{r_1}(r)-F_{r_1}(r_T)\right)+P_1\left(F_{r_2}(r)-F_{r_2}(r_T)\right)+$$
$$+P_1F_{r_1}(r_T)F_{r_2}(r)+P_2F_{r_2}(r_T)F_{r_1}(r)\,. \qquad (16)$$

4 Numerical Results

It is assumed that both branches at the input have the same channel parameters. r_T is the optimal decision threshold equal to first moment of PDF of the combiner input signals [6]:

$$r_T=\frac{2\sqrt{\pi}}{(1+q_i^{\,2})^2}q_i^{\,2}\Omega F\left(\frac{3}{4},\frac{5}{4};1;\left(\frac{1-q_i^{\,2}}{1+q_i^{\,2}}\right)^2\right) \qquad (18)$$

where $q_i=\sigma_{i2}/\sigma_{i1}$ is Hoyt fading parameter and $\Omega=\sigma_{i1}^{\,2}+\sigma_{i2}^{\,2}$. $F(.)$ denotes the hypergeometric function [14, Eq. (9.100)].

The level crossing rate $N(r)$ and the average outage duration $T(r)$ curves versus signal value are presented in the next few figures.

The level crossing rate curves are plotted in Figs. 2 and 4, and the average outage duration curves can be seen from Figs. 3 and 5.

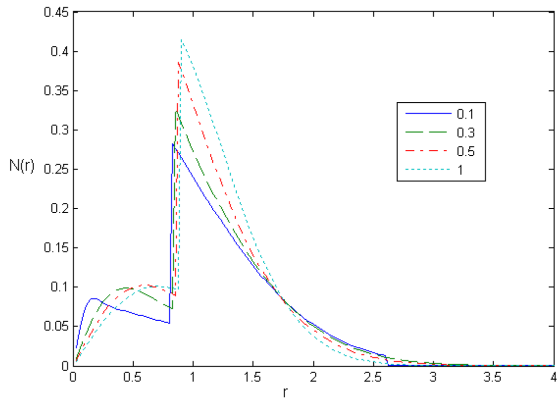

Fig.2. Level crossing rate $N(r)$ for $\Omega_i=1$, $\beta_{i,j}=1$ and varying q_i

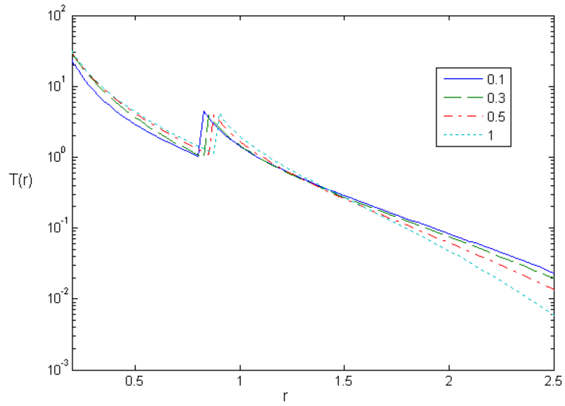

Fig.3. Average outage duration $T(r)$ for $\Omega_i=1$, $\beta_{i,j}=1$ and varying q_i

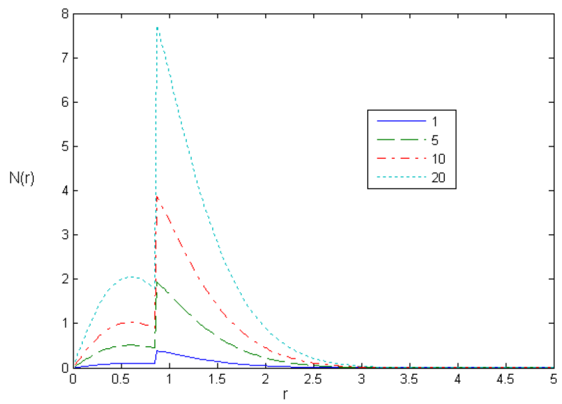

Fig.4. Level crossing rate $N(r)$ for $q_i=1$, $\Omega_i=1$ and varying $\beta_{i,j}$

The influence of varying Hoyt fading parameter q_i is given in Figs. 2. and 3. where, for $q_i=1$, Hoyt fading leads to Rayleigh PDF.

The effect of varying variances of derivatives is depicted in Figs. 4. and 5. where is adopted that all of them are equal.

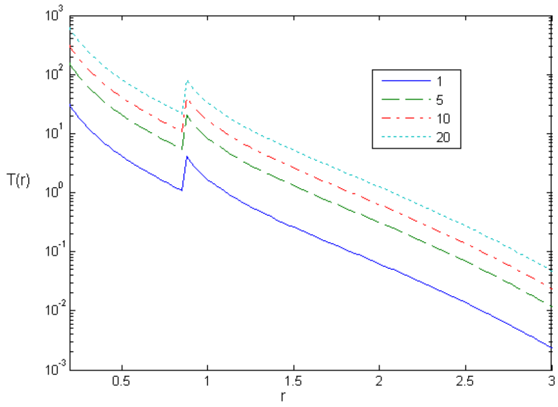

Fig.5. Average outage duration $T(r)$ for $q_i = 1$, $\Omega_i = 1$ and varying $\beta_{i,j}$

It can be noticed that in all cases represented curves have the same shape, but there is discontinuities on the level crossing rate and average outage duration curves when signal value is equal to threshold. Numerical values of threshold determine the discontinuity moment appearance. Larger rise of fade duration corresponds to larger threshold values.

5 Conclusion

In this paper the level crossing rate and the fade duration of the SSC combiner output signal are determined in the presence of Hoyt fading. The results are shown graphically for different parameter and threshold decision values. The influence of threshold numerical values to determining of the discontinuity moment appearance is pointed out.

Acknowledgment

This work has been funded by the Serbian Ministry for Science under the projects III-44006 and TR-33035.

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ISBN: 978-1-61804-138-8