A Simple Approach for Hybrid Transmissions Efficiency

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Abstract: - In this paper a novel fast method for the calculation of the efficiency of shunted Continuously Variable Transmissions is shown. The method is based on the results of a previous research in which it is suggested that the ratio given the power driven through the variable drive (CVT) branch divided by the power driven by the overall drive (CVU) can be determined by kinematic considerations, under the assumption of negligible power losses. Starting from this result, simple formulas of the efficiency of the CVU with non-negligible power losses in the CVT are derived for the case of single stage shunted CVTs with internal power re-circulation. A demonstration is given with application to a case of which results are known from the literature. The method is a powerful tool for the calculation of the efficiency also in more complicated CVUs for hybrid vehicles applications, like the Compound-Split Transmission.

Key-Words: - CVT, Hybrid Transmissions, Efficiency

1 Introduction

A way to improve fuel economy and performance of motor vehicles is to adopt hybrid architectures [1,2]. Toyota Hybrid Synergy Drive System (THS) is a leading integrated electro-mechanical hybrid transmission system mass-produced and commercialized since 1997; it is currently the most popular hybrid system in the market place. Hybrid power systems [3,4] may include one or more motor/generators operatively connected to the transmission i.e., in addition to the engine, or to receive extra mechanical power from the transmission, i.e. from the engine and final drive. In such systems, the motor/generators may be selectively operated to provide extra mechanical power to the transmission. The received mechanical power may be converted into electrical power and stored for later use. These types of device are named Electronic-Continuously Variable Transmission (E-CVST). They can be classified into three different systems such as Input split, Output split and Compound split. Power-Split CVT can be classified into three different systems such as Input split, Output split and Compound split. The input split transmissions and output power split transmissions are often arranged with a single planetary gear. Compound split systems comprise two or three planetary gears combined with two motors/generators. Mantriota [5-7] proposed an original Power Split CVT system with two separate phases of operation able to guarantee a power flow without recirculation. Considering the same ratio range, this transmission allows us to obtain better efficiency in comparison to the traditional CVT. Infinitely Variable Transmissions (IVT) are one type of Power Split CVT system which ensure an infinite ratio range coverage by providing even zero transmission ratio with an unmoving output shaft and an input shaft without zero velocity [8-10]. A compact transmission is thence obtained which includes a clutch function. The absence of friction clutch allows the running of vehicle at very low speed without any problem arising from the clutch’s engage and disengage control. IVTs are generally
made up of three elements: a CVT, a Planetary Gear train (PG), and a Fixed Ratio mechanism (FR).

Our research group has recently studied the power flows and efficiency of the Infinitely Variable Transmissions [8]. The powers flowing in the different paths of the IVT were studied in the two possible directions. What emerged was that a series-IVT is more convenient to upgrade efficiency with type I power flow while a parallel-IVT ensures a better efficiency with the type II power flow. The results have shown that efficiency mainly depends on the type of power flow in the IVT. In particular, the type II flow was observed to lead to higher efficiencies with low transmission ratio values while type I flows proved to ensure greater efficiency and higher input powers with elevated transmission ratio values.

In Ref. [11] the fuel consumptions of a mid-class vehicle equipped with four different type of transmission (manual, robotized, CVT and IVT) was compared. The fuel consumptions was calculated in stationary condition, in ECE and in EUDC cycle.

In this paper a simple fast method for the calculation of the efficiency of Power Split Continuously Variable Transmissions is shown. The method is based on the results of a previous research [12-14] in which it is suggested that the ratio given the power driven through the variable drive (CVT) branch divided by the power driven by the overall drive (CVU) can be determined by kinematic considerations, under the assumption of negligible power losses. Simple formulas of the efficiency of the CVU with not-negligible power losses in the CVT are derived for the case of single stage shunted CVTs with internal power re-circulation.

## 2 Derivation of the efficiency of CVU: the case of simple shunted CVT

A Continuously Variable Unit (CVU) made with a shunted CVT has three elements: a planetary gear drive (PG), a Continuously Variable transmission (CVT) and a fixed ratio drive (FR). CVT can be of mechanical or electrical type (e-CVT).

Simple kinematic equations follow which refers to the schematic picture of the shunted arrangement (Figure 1):

\[ \tau_{CVT} = \frac{\omega_5}{\omega_4} \]  

(1)

\[ \omega_3 + \chi \omega_5 = (1 + \chi) \omega_6 \]

\[ \omega_1 = \omega_2 = \omega_3 \]  

(2)

where \( \omega_i \) is the angular velocity of the \( i \)th shaft. The speed ratio of the CVU can be written as a function of the speed ratio of the CVT:

\[ \tau_{CVU} = (1 + \chi) \tau_{FR} - \chi \tau_{CVT} \]  

(3)

CVTs’ speed ratio varies between a minimum value \( \tau_{CVT\ min} \) and a maximum value \( \tau_{CVT\ max} \). The CVU is designed to give a speed ratio which varies in a range which is different from CVT’s, between \( \tau_{CVU\ min} \) and \( \tau_{CVU\ max} \).

If it is assumed that the ratio spread of the CVU (i.e. \( \tau_{CVU\ max}/\tau_{CVU\ min} \)) is larger than the ratio spread of the CVT (i.e. \( \tau_{CVT\ max}/\tau_{CVT\ min} \)), then there is power re-circulation of Type I or II in the CVU.

Imposing that \( \tau_{CVU} \) is an increasing function of \( \tau_{CVT} \) then a simple system of two equations with two unknown quantities (\( \chi \) and \( \tau_{FR} \)) can be solved:

\[ \chi = \frac{\tau_{CVU\ max} - \tau_{CVU\ min}}{\tau_{CVT\ max} - \tau_{CVT\ min}} \]  

(4)

\[ \tau_{FR} = \frac{\tau_{CVU\ max} - \chi \tau_{CVU\ min}}{1 - \chi} \]  

(5)

It has been demonstrated that under these hypotheses there is a power flow of Type I (see Ref. [5, 8]) in the CVU. A power flow of Type II can be obtained if the \( \tau_{CVU} \) is an decreasing function of \( \tau_{CVT} \) and similar results as eqs. (4, 5) can be found. Finally, a power flow of Type III (no power re-circulation or power split) can be obtained only if the ratio spread of the CVU is smaller than CVT’s [15].

The efficiency of the CVU is sensitive to which type of power flow is working, because it affects the ratio of power transmitted through the CVT branch and the overall input power. We address the reader to references [5] for a complete understanding of the argument.

In [12-14] it has been demonstrated that the ratio \( P_{CVT}/P_{in} \) i.e. the power through the CVT branch divided by the input power of the overall CVU can be easily calculated independently on internal arrangement of the CVU, under the assumption of negligible power loss (ideal CVU). In particular, if
the \( \tau_{CVU} \) is known as a function of the \( \tau_{CVT} \), the ratio is simply given by:

\[
\left| \frac{P_{CVT}}{P_{in}} \right| = \left| \frac{d\tau_{CVU}}{d\tau_{CVT}} \right| (\tau_{CVU}) \tag{6}
\]

In this paper we will present how to use eq. (6) to derive analytical formulas of the efficiency of the CVU, assuming that the power loss is not negligible only in the CVT branch (real CVU). This is often the case of well-designed shunted CVT (Input and Output Split), in which the differential gear works close to a kinematic condition characterized by small relative angular velocities of its shafts (and so the efficiency of the PG is close to one), or in general when the efficiency of CVT is very low (for instance electric motor/generator) compared to the other components.

2.1 Output Split in direct operation with power flow of Type I

Let’s start considering a real CVU made of a shunted CVT in Output Split arrangement (Figure 1) with a power flow of Type I (see Refs. [8]) in direct operation. Shafts 1, 2 and 3 are coupled (equal angular velocity). Power loss in PG and fixed ratio drive (FR) is negligible.

![Figure 1](attachment:image1.png)

Figure 1 – Output Split transmission with a power flow of Type I in direct operation.

We compare the real and the ideal CVU given power, torque and angular velocity of shaft 6 and the angular velocity of input shaft 1 for both. There is no difference of torques and angular velocities at shafts 3 and 5, depending only on the PG gear ratio. The power loss in the CVT of the real system can be written as a function of the power at shaft 5:

\[
P_w = -(1 - \eta_{CVT})P_5 \tag{7}
\]

Because there is no difference between real and ideal CVU at shaft 5, and given that \( P_{in} = P_{out} \) in the ideal CVU, then:

\[
\left| \frac{P_{CVT}}{P_{in}} \right|_{ideal} = \left| \frac{P_5}{P_{al,real}} \right| = \left| \frac{d\tau_{CVU}}{d\tau_{CVT}} \right| \left( \frac{\tau_{CVU}}{\chi\tau_{CVT}} \right) = \left( \frac{\chi\tau_{CVU}}{\chi\tau_{CVT} - (1 + \chi)\tau_{FR}} \right) \tag{8}
\]

Finally, the efficiency of the real CVU can be written as a function of the output power, being it equal for real and ideal CVU, and of the power loss in the CVU by means of: \( \eta_{CVU} = -\frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_w} \) (notice that \( P_{out} = P_6 \)). It follows that:

\[
\eta_{CVU} = \frac{1}{1 + (1 - \eta_{CVT})\left| \frac{P_{CVT}}{P_{in}} \right|_{ideal}} \tag{9}
\]

which gives the CVU efficiency as a function of the speed ratio of the CVT (or, equivalently, of the CVU) and, of course, of the efficiency of the CVT. \( \left| \frac{P_{CVT}}{P_{in}} \right|_{ideal} \) is given in the eq. (8).

2.2 Output Split in direct operation with power flow of Type II

Let’s now consider a real CVU made of a shunted CVT in Output Split arrangement (Figure 2) with a power flow of Type II (see Refs. [8]) in direct operation. In this case, given the output power, torque and speed of shaft 6, there is no difference between real and ideal CVU on shafts 3 and 5 for the same reasons of the previous case (negligible power loss in PG and equal gear ratio of PG).

![Figure 2](attachment:image2.png)

Figure 2 – Output Split transmission with a power flow of Type II in direct operation.

However, the power in the branch 5 is the output
power of the CVT and so the power loss in the CVT can be written as:

$$P_w = \frac{1 - \eta_{CVT}}{\eta_{CVT}} P_s$$ (10)

Because the output power is equal for real and ideal CVU, the efficiency can be written as

$$\eta_{CVU} = -\frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_w},$$

which gives:

$$\eta_{CVU} = \frac{1}{1 + \frac{1 - \eta_{CVT}}{\eta_{CVT}} \frac{P_{CVT}}{P_{in} \text{ ideal}}}$$ (11)

2.3 Output Split in reverse operation with power flow of Type I

In the case of a shunted CVT in Output Split arrangement (Figure 3) with a power flow of Type I (see Refs. [8]) in reverse operation, given the input power, torque and speed of shaft 6, there is no difference between real and ideal CVU on shafts 3 and 5 for the same reasons of the previous cases (negligible power loss in PG and equal gear ratio of PG). The power in the branch 5 is the output power of the CVT and so the power loss in the CVT can be written following eq. (10). Moreover, because there is no difference between real and ideal CVU at shaft 5, and given that in the ideal CVU, then eq. (8) is still valid. Finally, the input power is equal for real and ideal CVU and so the efficiency can be written as

$$\eta_{CVU} = -\frac{P_{out}}{P_{in}} = \frac{P_{in} + P_w}{P_{in}},$$

which gives:

$$\eta_{CVU} = 1 - \left(\frac{1 - \eta_{CVT}}{\eta_{CVT}}\right) \frac{P_{CVT}}{P_{in} \text{ ideal}}$$ (12)

2.4 Output Split in reverse operation with power flow of Type II

Let’s now consider a shunted CVT in Output Split arrangement (Figure 4) with a power flow of Type II (see Refs. [8]) in reverse operation. Given the input power, torque and speed of shaft 6, there is no difference between real and ideal CVU on shafts 3 and 5. The power in the branch 5 is the input power of the CVT and so the power loss in the CVT can be written following eq. (7). Finally, the input power is equal for real and ideal CVU and so the efficiency can be written as

$$\eta_{CVU} = -\frac{P_{out}}{P_{in}} = \frac{P_{in} + P_w}{P_{in}},$$

which gives:

$$\eta_{CVU} = 1 - \left(\frac{1 - \eta_{CVT}}{\eta_{CVT}}\right) \frac{P_{CVT}}{P_{in} \text{ ideal}}$$ (13)

2.5 Numerical Results

Numerical results of eqs. (9, 11-13) are shown for comparison with Ref. [16]. The speed ratio of the CVT ranges between [0.4, 2.5] and the speed ratio of the CVU ranges between [0.2, 2.5]; the efficiency of the CVT is $\eta_{CVT} = 0.9$. Figure 5 shows the efficiency of the CVU as a function of the speed.
ratio of the CVU for power flows of Type I and II in direct operation [eqs. (9, 11)]. Results are equal to those of Ref. [16]. It can be shown that with a power flow of Type I the efficiency is low with low $\tau_{CVU}$ whereas it is large, even close to unity, with large $\tau_{CVU}$.

With a power flow of Type II the efficiency is almost constant but always smaller than the efficiency of the CVT. In reverse operation (Figure 6) the situation is qualitatively similar. However, the efficiency of a CVU with a power flow of Type I is even negative with speed ratios close to the lower bound, which means that the CVU is not reversible under such circumstances. This result is in agreement with Ref. [16].

![Figure 6 – The efficiency of the Output Split with a power flow of Type I and II in reverse operation.](image)

**3 Conclusion**

In this paper a simple fast method for the calculation of the efficiency of shunted CVT transmissions (CVU) has been presented. The method is based on the results of Polder (Ref. [12]) which suggest that the ratio of the power driven through the variable drive (CVT) branch and the power driven by the overall drive can be determined by kinematic considerations, under the assumption of negligible power losses.

Starting from this result, we derived simple formulas of the efficiency of the CVU. The method is expected to give good estimation of the actual efficiency in all those cases where the power loss in the CVT unit is dominant, whereas the power loss in other components is negligible. A demonstration is given with application to the case of simple stage shunted CVT with Type I or II power flows, for which results are available from the literature (Ref. [16]).

The method is a powerful tool for the estimation of the efficiency in more complicated CVU for hybrid vehicles applications, like the Compound-Split Transmission.

**References:**


