An Upper Bound of Soft Decode and Forward Relaying Over Rayleigh Fading Channels

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Abstract—In this paper, we investigate a distributed turbo code (DTC) scheme dedicated to L-relay channels operating in the soft-decode-and-forward (soft-DF) mode. At the destination, the received replicas are combined using the maximal ratio combining (MRC). Then, we define the explicit upper bound for error rate assuming Binary phase shift keying (BPSK) transmission for fully interleaved channels with channel state information (CSI). We use the Rayleigh fading channels with independent fading. By means of the analytical given bound and the simulation results, we show the performance of the proposed scheme in terms of bit error rate (BER).

I. INTRODUCTION

Diversity provides an efficient technique for combating multipath fading in mobile radio systems. Diversity techniques seek to generate and operate multiple replicas of the transmitted signal to show low fade correlation. Cooperative diversity in wireless multi-hop networks confirms their ability to achieve spatial diversity gains by exploiting the spatial diversity of the traditional MIMO techniques, without each node necessarily having multiple antennas. Cooperative diversity can offer a more reliable estimate of the transmitted signal and higher data rates by introducing both temporal and spatial correlation into the transmitted signals from different relays. In addition, cooperative diversity has been shown to provide the network with important throughput, reduced energy requirements and improved access coverage [1], [2], [3], [4], [5].

Within the cooperative diversity framework, there are two schemes known as amplify-and-forward (AF) and decode-and-forward (DF) through a multiple relay nodes [2]. Under the AF mode, the relay simply retransmits an amplified copy of the received partner’s signal (corrupted by noise). This approach provides gains over uncoded cooperative diversity in terms of outage probability and can outperform the DF mode in certain network topologies [5]. However, for AF, when the instantaneous CSI is not available to the receivers, satisfying the relay power constraints greatly complicates the demodulation and the analysis [6]. On the other side, the DF mode is based mainly on the fact that each relay decodes the partner’s message followed by retransmission of the estimated message. The relay power constraint can easily be satisfied and DF can be extended to combine coding techniques and might be easier to incorporate into network protocols [7], [8], [9]. In general, DF outperforms AF relaying in cases where the relay can reliably decode the partner’s message.

Many research works, as [10], [11], [12], have studied DTC performances using soft-decode-and-forward (soft-DF) where each relay node decodes the received signal with a soft-input soft-output (SISO) decoder by calculating a log likelihood ratio (LLR) or a posteriori probabilities of the information bits. Then the decoded sequence will be interleaved and re-encoded with a SISO encoder. The twice SISO decoder and encoder employ the BCJR algorithm introduced by Berrou et al. in [13]. The use of Soft-DF, instead of automatic repeat request (ARQ), mitigate error propagation caused by decoding error at the relay nodes and thus will perform the system transmission throughput. In this paper, we will focus on the performance bounding for the case of error-free recovery and the realistic case with errors at the relay nodes. For each case, a bound expression for the pairwise error probability (PEP) over independent Rayleigh fading channels with BPSK modulation is derived. At low signal-to-noise (SNR), analytic evaluation of the DTC performance using soft-DF has proven to be very difficult. Therefore, we investigate the performance of the DTC scheme at low and middle SNR by means of Monte Carlo simulation. We consider L-relay nodes operational with soft-DF and the destination node is equipped with an iterative turbo decoder. A two-state rate $1/2$ RSC encoder at the source and relay nodes is used to study the pairwise error probability and union upper bound for $(1, 5/7, 5/7)$ code structures.

The rest of this paper is organized as follows: the system model is presented in Section II. In Section III, the bound expression for the pairwise error probability is derived. In Section IV, the derived upper bound and the performance of the simulation results are discussed. Finally, Section V summarizes the work.
One. Firstly, the source broadcasts $x,y$ a rate 1 code, the overall code rate is
perfect synchronization is assumed here.

cooperatively constructed a DTC. Therefore, we remark that
combined using MRC. Then, the source and relay nodes have
simultaneously. At the destination, the received replicas are
receiver. We assume that a node cannot transmit and receive
where data is sent from S to D with the assistance of
interleaving gain of turbo code construction and the turbo
Valenti in [7], which forms an extra coding gain due to the
m
be used for cooperation. We denote the source,
$m$ and D, th
is the number of relay nodes that can
be the first parity part generated by the
be the systematic information se-
2
,...,y
, and destination nodes by
$m$ to the channel. Let
y
1
,...,y
2
,\ldots,y
L
is the input bloc size . Let
y
= \{ \pm \sqrt{E_{S,c}} \} and \eta_{c,t} is the complex additive noise
\eta_{c,t} = \eta_{c,t,1} + j\eta_{c,t,Q} . The \eta_{c,t,1} and \eta_{c,t,Q} are realizations
independent Gaussian random variables with zero mean and variance $\sigma_{\text{\eta,t}}^2 = N_0/2$. The complex fading coefficient
is $a_{c,t} = a_{c,t,1} + j a_{c,t,Q}$. The $a_{c,t,1}$ and $a_{c,t,Q}$ are Gaussian
distributed; their variance is normalized to $\sigma_{a_{c,t}}^2 = 1/2$ such
that the magnitude $|a_{c,t}| = \sqrt{a_{c,t,1}^2 + a_{c,t,Q}^2}$ is modeled with
a Rayleigh pdf,

$$p(a_{c,t}) = 2a_{c,t}e^{-a_{c,t}^2}, \ a_{c,t} > 0. \quad (1)$$

In the following analysis, we consider fully interleaved Rayleigh channels, where the fading amplitudes on each antenna are independent in time. During the first transmission period, the transmitted signal energies from the source to both relay and destination nodes are given by:

$$E_{s,SR^m} = R_S E_{b,SR^m}, \ m = 1, \ldots, L \quad (2)$$

and

$$E_{s,SD} = R_S E_{b,SD}. \quad (3)$$

where $R_S = 1/2$ is the code rate generated at the source node, $E_{b,SR^m}$ and $E_{b,SD}$ is the energy per bit from the source to $m$th relay and destination nodes respectively. In the second transmission period, at a sufficiently SNR, the soft-DF mitigates error propagation and all relays will be used for cooperation. Accordingly, the received signal energies at the destination node are given by:

$$E_{s,R^mD} = \frac{1}{L+2} E_{b,R^mD}, \ m = 1, \ldots, L \quad (4)$$

and

$$E_{s,SD} = \frac{1}{L+2} E_{b,SD}. \quad (5)$$

Assuming perfectly known $a_{c}$ at the receiver, the channel log likelihood ratio (LLR) values are calculated as [14]
where $\sigma^2_{\eta^2}$ is Gaussian distributed with variance $\sigma^2_{\eta^2}$ and mean $\eta$.

The choice of the Soft-DF relaying technique keeps the advantages of both AF and DF techniques. It regenerates the signal while keeping soft information and obtaining decoding gain. Ting et al. in [15] have estimated a noise variance of the relay output based on the LLR values which we consider in our work.

The LLR values $\Lambda_{SRm,in}$ at the $m^{th}$ decoder input can be calculated as

$$\Lambda_{SRm,in} = 2\left(\frac{a_{SRm}}{\eta_{SRm}}\right)^2 \left(x_{SRm} + \frac{\eta_{SRm}}{a_{SRm}}\right).$$

The LLR values $\Lambda_{SRm,out}$ at the $m^{th}$ decoder output can be formulated similarly as

$$\Lambda_{SRm,out} = 2\left(\frac{a_{SRm}}{\eta_{SRm}}\right)^2 \left(x_{SRm} + \frac{\eta_{SRm}}{a_{SRm}}\right),$$

approaches Gaussian distribution with variance

$$\frac{(\eta_{SRm}/a_{SRm})^2}{a_{SRm}}$$

and mean zero, then we have

$$\text{var}(\Lambda_{SRm,out}) = 4\left(\frac{a_{SRm}}{\eta_{SRm}}\right)^4 + \frac{(\eta_{SRm}/a_{SRm})^2}{a_{SRm}}.$$

Thus

$$\frac{a_{SRm}}{\eta_{SRm}} = \sqrt{-1 + \sqrt{1 + \text{var}(\Lambda_{SRm,out})}}.$$

Using Monte Carlo simulation, we can estimate $\frac{a_{SRm}}{\eta_{SRm}}$.

After receiving signals from the source via the $m^{th}$ relay, the equivalent noise variance at the destination is done by:

$$\sigma^2_{\eta_{Rm,D,eq}} = \left(\frac{\eta_{SRm}}{a_{SRm}}\right)^2 + \left(\frac{\eta_{SRm}/a_{Rm}}{a_{SRm}/a_{SD}}\right)^2.$$

III. PERFORMANCE BOUNDING

In this section, we evaluate the performance bounding of the proposed scheme for L-relay channels in terms of the bit error rate (BER) at the destination.

A. Transfer Function Based Union Bound

The union upper bound for the maximum likelihood (ML) decoding of a $(K,N)$ block code is an effective way to bound the performance of block codes provided the weight distribution $A(d)$. To evaluate $A(d)$ in case of turbo codes, average upper bound are used to take into account all possible interleavers [16], [17]. The expression of union upper bound on the average value of bit error probability can be written as [17]

$$P_b \leq \sum_{d=d_{min}}^N \sum_{t=1}^K \sum_{i=d_1}^d \sum_{k=1}^K \left(\frac{K}{i}\right) p(d_1 | i) p(d_2 | i) P_2(d),$$

where $d$ is the Hamming weight of the codeword (systematic output with the two parity outputs) i.e., the individual weights of three sequences $(d = i + d_1 + d_2)$, $P_2(d)$ is the error event probability starting from $d_{min} = (i + d_1 + d_2)_{min}$, denote by $p(d_p | i)$ the conditional probability of producing a codeword fragment of weight $d_p$ given a randomly selected input sequence of weight $i$.

$$p(d_p | i) = \frac{t(l,i,d_p)}{\binom{N}{K}},$$

where $t(l,i,d_p)$ is the number of paths of length $l$, input weight $i$, and output weight $d$, starting and ending in all-zero states.

B. Pairwise Error Probability

On the fully interleaved channel with CSI for no-cooperative transmission from $S$ to $D$, the conditional pairwise error probability (CPER) of decoding a codeword $c_0$ into a codeword $c_d$ that differs from $c_0$, in $d$ bit positions indexed by $(1,2,\ldots,d)$ [18] is

$$P(c_0,c_d | a_{SD}) = Q\left(\sqrt{2R_c R_b N_0 \sum_{k=1}^d (a_{SD})^2}\right).$$

where $R_c$ is the code rate, $a_{SD}$ is the channel gains and $Q(\cdot)$ is the complementary error function given by:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt.$$

The pairwise error probability (PEP) can be written as

$$P_2(d) = \int_{a_{SD1}}^{a_{SD2}} \ldots \int_{a_{SDd}} p(a_{SD}) P(c_0,c_d | a_{SD}) da_{SD1} \ldots da_{SDd}$$

and

$$P(a_{SD}) = \prod_{k=1}^d p(a_{SD})^k.$$

The exact evaluation of $P_2(d)$ seems to be complex. To solve this complexity, we examine the option 4 in [18] to upper bound the $Q$ function in the integrand. Here, the complementary error function can be tightly bounded as

$$Q\left(\sqrt{x+y}\right) \leq \frac{1}{2} Q\left(\sqrt{x}\right) e^{-y/2}, \ x, y \geq 0.$$
where $R$ is the turbo code rate and $\delta = \frac{R E_{b,SD}/N_0}{1 + R E_{b,SD}/N_0}$.

C. Distributed Coded Cooperation with Error-Free Relays

At the destination node, the DTC scheme with error-free relays provides two stream inputs $(x', y')$ for turbo decoder. The first one $(x', y')$ is coming from the source node. The second $y'$ is resulting from the MRC. Thus, the conditional pairwise error probability is given by:

$$P \left( c_0, c_d \mid a_{SD}, a_{R1}, ..., a_{RL,D} \right) = Q \left( \sqrt{2 \left( \frac{L}{L+2} \right) N_0} \right) \left( 1 + \frac{E_{b,SD}(1+\delta_L)}{N_0} \right)^{-d_2},$$

where

$$\delta_L = \frac{E_{b,SD}(L+2)N_0}{1 + E_{b,SD} / (L+2) N_0}. \quad (22)$$

D. Distributed Coded Cooperation with Errors at Relays

In this section, we consider the more realistic case. At the destination node, the DTC scheme with errors at relays provides the equivalent noise variance. Thus, the CPEP is given by:

$$P \left( c_0, c_d \mid a_{SD}, a_{R1}, ..., a_{RL,D} \right) = Q \left( \sqrt{2 \left( \frac{L}{L+2} \right) N_0} \right) \left( 1 + \frac{E_{b,SD}(1+\delta_L)}{N_0} \right)^{-d_2},$$

where

$$\delta_L = \frac{E_{b,SD}(L+2)N_0}{1 + E_{b,SD} / (L+2) N_0}. \quad (22)$$

IV. RESULTS AND DISCUSSION

The results are done for BPSK modulation. The different sub-channels between the source, relay and destination are assumed to be independent Rayleigh fading and fully interleaved with ISI. The source and relay nodes each employ a two-state rate $1/2$ RSC encoder with generator polynomial $(1, 5/7)$ and block size $K = 1000$ bits. At the destination, the iterative decoder is based on the BCJR algorithm and parallel concatenated convolutional code (PCCC) decoding principle [13].

In the analysis, we assume different SNRs between the source and relay nodes, which is the most general case. This incorporates different network topologies and distances between the source and relay nodes. However, we assume that through perfect power control, the average SNR for all sub-channels between the source and relay nodes are equal $(\tilde{\gamma}_{SR} = \tilde{\gamma}_{SR} = ... = \tilde{\gamma}_{SR})$, and all sub-channels between the relay and destination nodes are equal $(\tilde{\gamma}_{RD} = \tilde{\gamma}_{RD} = ... = \tilde{\gamma}_{RD})$. Also, we assume that the sub-channels between the source and destination nodes and the sub-channels between the relay and destination nodes have an equal SNR, i.e., $\tilde{\gamma}_{SD} = \tilde{\gamma}_{RD} = E_b/N_0$.

The performance of that proposed DTC scheme for L-relay channels operating in soft-DI Mode are investigated and compared to conventional $(1, 5/7, 5/7)$ PCCC turbo code (i.e., no-cooperative) scheme.

We have plotted the upper bound on average bit error probability of a DTC scheme with $\tilde{\gamma}_{SR} = 4$ dB, $\tilde{\gamma}_{SR} = 8$ dB and $\tilde{\gamma}_{SR} = 12$ dB in Figure 3. Here, we are considering two relay nodes ($L = 2$). When $\tilde{\gamma}_{SR}$ increases, the performance of errors at relays DTC converges slowly to the performance of error-free relays DTC. It should be noted that the soft-DI relaying contributes greater diversity gain to the system.

In Figure 4, the performances of the DTC scheme with errors $\tilde{\gamma}_{SR} = 4$ dB and error-free recovery at relays are shown for various number of relay nodes ($L = 1$ and 2). At $P_b = 10^{-8}$, with multiple soft-DI relaying $\tilde{\gamma}_{SR} = 4$ dB, a total of 0.7 dB gain is achieved when the number of relays $L$ increases from 1 to 2. Also, with error-free recovery at relays, compared to conventional turbo code, the gain increases from 1.7 dB for single relay ($L = 1$) to nearly 2.8 dB for multiple relays ($L = 2$).

Figure 5 shows the Monte Carlo simulation and the performance bounding of a DTC scheme compared to a $(1, 5/7, 5/7)$ PCCC turbo code. In this figure, we consider the DTC scheme with errors $\tilde{\gamma}_{SR} = 8$ dB and error-free recovery at relays for various number of relay nodes ($L = 1$ and 3). Here, the tightness of bounds to simulation curves from a modulo interleaving scheme is questionable for several reasons. These include the union bound, the averaging over all interleavers, and the bounding of $P_{2}(d)$.

V. CONCLUSION

This paper gives the derivation of an expression for the PEP for DTC scheme operating with soft-DI relaying where, we
conclude that increasing the number of relays leads to improve the advantages of both keeping soft information and obtaining additional interleaving gain. This is confirmed by the achieved simulation results using our proposed DTC scheme.

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