PDFs of Time Derivatives at Two Time Moments in the Presence of Nakagami Fading

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Abstract: - The probability density functions (PDFs) of derivatives in two time moments for dual branch Switch and Stay Combiner (SSC) output signals in the presence of Nakagami-*m* fading are determined in this paper. The second order statistics, such as the average level crossing rate and the average fade duration, can be calculated by using obtained closed-form expressions, because of what it is very important to find PDFs in the closed-form expressions.

Key-Words: - Probability Density Function; Nakagami Fading; Switch and Stay Combining; Time Derivative, Two Time Moments

1 Introduction

The received signal in wireless communication systems is exposed to short-term fading, which is the result of multipath propagation, and long-term fading or shadowing, which is the result of large obstacles and deviations in terrain between transmitter and receiver [1].

An efficient method for improvement system's quality of service with using multiple receiver antennas is space diversity [2]. There are several main types of combining techniques [3]. They have different complexities and amount of channel state information available at the receiver. Combining techniques like maximal ratio combining (MRC) [4] and equal gain combining (EGC) [5] require all or some of the amount of the channel state information of received signal. They require separate receiver chain for each branch of the diversity system, which increase its system complexity. One of the least complicated combining methods is selection combining (SC) [6]. Contrary to previous combining techniques, SC combiner processes only one of the diversity branches because of what is much simpler for practical realization. Generally, selection combining, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [7].

SSC is the method of simplifying the system complexity but with loss in performance. In this case, the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of the signal quality of that antenna [8].

Rayleigh, Rician, Nakagami-*m* and Weibull statistical models are the most frequently used to describe the short-term signal variation. The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. The Rician distribution is often used to model propagation paths consisting of one strong direct LOS component and many random lower components. The Nakagami-*m* distribution has gained widespread application in the modeling of physical fading radio channels [9]. The main advantage of usage of Nakagami-*m* fading model is its good fit to empirical fading data. We can model signal fading conditions that range from severe to moderate, light fading or no fading through Nakagami parameter *m*. It includes the one-sided Gaussian distribution and the Rayleigh distribution as special cases for m=0.5 and m=1, respectively.

The authors determined the expressions for the probability density functions and joint probability density functions for SSC combiner output signals at two time instants in the presence of different fading distributions. Then, these expressions are used for design of systems with better performances, such as the bit error rate and the outage probability. Performance analysis of SSC/SC combiner in the presence of Rayleigh and log-normal fading are given in [10] and [11], respectively.

But, the level crossing rate and the average fading duration are also very often used in designing of wireless communication systems as measures for their quality. These second-order statistical measures are related to criterion used to assess the error probability and to determinate equivalent channel's parameters, modeled by a Markov chain with defined number of states [12]. To obtain second order system characteristics we need the expressions for signal derivatives [13].

For this purpose, the probability density functions of derivatives in two time instants for SSC combiner in Rician and Rayleigh fading channels are determined in [14] and [15], and the probability density functions (PDFs) of the signal derivative of dual switch and stay combining (SSC) combiner output signals at two time instants in the presence of Nakagami fading will be derived in this paper.

2 System Model and Problem Formulation

The system model of dual brunch SSC combiner at two time moments is shown in Fig.1. At the first time moment the input signals are r_{11} and r_{21}, but r_{12} and r_{22} at the second time moment. The output signals are r_1 and r_2. The derivatives are $\dot{r}_{11}$ and $\dot{r}_{21}$ at the first time instant, and $\dot{r}_{12}$ and $\dot{r}_{22}$ at the second time instant. The derivatives at the SSC combiner outputs are $\dot{r}_1$ and $\dot{r}_2$.

The first index represents the branch ordinal number and the other time moment observed. At output signal the indices correspond to the time moments considered.

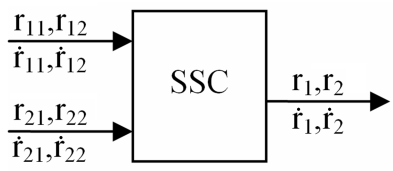

Fig.1. Dual SSC combiner at two time moments

The probabilities that combiner examines first the signal from the first, i.e. second branch are P_1, i.e. P_2. The values of P_1 and P_2 for SSC combiner are obtained in [3].

The four different cases depending on the size of the input signal with respect to the threshold are discussed here:

I $r_1<r_T$, $r_2<r_T$

In the first case all signals are less then threshold r_T, i.e.: $r_{11}<r_T$, $r_{12}<r_T$ $r_{21}<r_T$, and $r_{22}<r_T$. Let combiner considers first the signal r_{11}. Because of $r_{11}<r_T$, then $\dot{r}_1=\dot{r}_{21}$, and because of $r_{22}<r_T$, then $\dot{r}_2=\dot{r}_{12}$. The probability of this advent is P_1. If combiner examines first the signal r_{21}, then $r_{21}<r_T$, $\dot{r}_1=\dot{r}_{11}$, also $r_{21}<r_T$, $\dot{r}_2=\dot{r}_{22}$. The probability of this advent is P_2.

II $r_1\geq r_T$, $r_2<r_T$

The possible combinations for this case are:

$r_{11}\geq r_T$, $r_{12}<r_T$, $r_{22}<r_T$, $\quad \dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{22}$ P_1

$r_{11}<r_T$, $r_{21}\geq r_T$, $r_{22}<r_T$, $r_{12}<r_T$, $\quad \dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{12}$ P_1

$r_{21}\geq r_T$, $r_{22}<r_T$, $r_{12}<r_T$, $\quad \dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{12}$ P_2

$r_{21}<r_T$, $r_{11}\geq r_T$, $r_{12}<r_T$, $r_{22}<r_T$, $\quad \dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{22}$ P_2

III $r_1<r_T$, $r_2\geq r_T$

Now, the possible combinations are:

$r_{11}<r_T$, $r_{21}<r_T$, $r_{22}\geq r_T$, $\quad \dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{22}$ P_1

$r_{11}<r_T$, $r_{21}<r_T$, $r_{22}<r_T$, $r_{12}\geq r_T$, $\quad \dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{12}$ P_1

$r_{21}<r_T$, $r_{11}<r_T$, $r_{12}\geq r_T$, $\quad \dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{12}$ P_2

$r_{21}<r_T$, $r_{11}<r_T$, $r_{12}<r_T$, $r_{22}\geq r_T$, $\quad \dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{22}$ P_2

IV $r_1\geq r_T$, $r_2\geq r_T$

In the last case the possible combinations are:

$r_{11}\geq r_T$, $r_{12}\geq r_T$, $\dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{12}$ P_1

$r_{11}\geq r_T$, $r_{12}<r_T$, $r_{22}\geq r_T$ $\dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{22}$ P_1

$r_{11}<r_T$, $r_{21}\geq r_T$, $r_{22}\geq r_T$, $\dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{22}$ P_1

$r_{11}<r_T$, $r_{21}\geq r_T$, $r_{22}<r_T$, $r_{12}<r_T$ $\dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{12}$ P_1

$r_{21}\geq r_T$, $r_{22}\geq r_T$, $\dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{22}$ P_2

$r_{21}\geq r_T$, $r_{22}<r_T$, $r_{12}\geq r_T$, $\dot{r}_1=\dot{r}_{21}$ $\dot{r}_2=\dot{r}_{12}$ P_2

$r_{21}<r_T$, $r_{11}\geq r_T$, $r_{12}\geq r_T$, $\dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{12}$ P_2

$r_{21}<r_T$, $r_{11}\geq r_T$, $r_{12}<r_T$, $r_{22}\geq r_T$, $\dot{r}_1=\dot{r}_{11}$ $\dot{r}_2=\dot{r}_{22}$ P_2

3 Probability Density Functions of Derivatives

The joint probability density functions of signals and derivatives are:

$r_1<r_T$, $r_2<r_T$

$$p_{r_1r_2\dot{r}_1\dot{r}_2}(r_1,r_2,\dot{r}_1,\dot{r}_2)=P_1\int_0^{r_T}dr_{11}\int_0^{r_T}dr_{22}\,p_{r_{11}r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{11},r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\int_0^{r_T}dr_{12}\,p_{r_{21}r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{21},r_{12},r_1,r_2,\dot{r}_1,\dot{r}_2) \tag{1}$$

$r_1\geq r_T$, $r_2<r_T$

$$p_{r_1r_2\dot{r}_1\dot{r}_2}(r_1,r_2,\dot{r}_1,\dot{r}_2)=P_1\int_0^{r_T}dr_{12}\,p_{r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{12},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{11}\int_0^{r_T}dr_{22}\,p_{r_{11}r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{11},r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{22}\,p_{r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\int_0^{r_T}dr_{12}\,p_{r_{21}r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{21},r_{12},r_1,r,\dot{r}_1,\dot{r}_2) \tag{2}$$

$r_1<r_T$, $r_2\geq r_T$

$$p_{r_1r_2\dot{r}_1\dot{r}_2}(r_1,r_2,\dot{r}_1,\dot{r}_2)=P_1\int_0^{r_T}dr_{11}\,p_{r_{11}r_{21}r_{22}\dot{r}_{21}\dot{r}_{22}}(r_{11},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{11}\int_0^{r_T}dr_{22}\,p_{r_{11}r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{11},r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\,p_{r_{21}r_{11}r_{12}\dot{r}_{11}\dot{r}_{12}}(r_{21},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\int_0^{r_T}dr_{12}\,p_{r_{21}r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{21},r_{12},r_1,r,\dot{r}_1,\dot{r}_2) \tag{3}$$

$r_1\geq r_T$, $r_2\geq r_T$

$$p_{r_1r_2\dot{r}_1\dot{r}_2}(r_1,r_2,\dot{r}_1,\dot{r}_2)=P_1p_{r_{11}r_{12}\dot{r}_{11}\dot{r}_{12}}(r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{12}\,p_{r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{12},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{11}\,p_{r_{11}r_{21}r_{22}\dot{r}_{21}\dot{r}_{22}}(r_{11},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{11}\int_0^{r_T}dr_{22}\,p_{r_{11}r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{11},r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\,p_{r_{21}r_{22}\dot{r}_{21}\dot{r}_{22}}(r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{22}\,p_{r_{22}r_{21}r_{12}\dot{r}_{21}\dot{r}_{12}}(r_{22},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\,p_{r_{21}r_{11}r_{12}\dot{r}_{11}\dot{r}_{12}}(r_{21},r_1,r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\int_0^{r_T}dr_{12}\,p_{r_{21}r_{12}r_{11}r_{22}\dot{r}_{11}\dot{r}_{22}}(r_{21},r_{12},r_1,r,\dot{r}_1,\dot{r}_2) \tag{4}$$

For the case that signal and its derivative are not correlated, after integrating of the whole range of signal values and some mathematical handling, the joint PDF of derivative can be expressed as:

$$p_{\dot{r}_1\dot{r}_2}(\dot{r}_1,\dot{r}_2)=P_1\int_0^{r_T}dr_{11}\int_0^{r_T}dr_{22}\,p_{r_{11}r_{22}\dot{r}_{21}\dot{r}_{12}}(r_{11},r_{22},\dot{r}_1,\dot{r}_2)+$$
$$+P_2\int_0^{r_T}dr_{21}\int_0^{r_T}dr_{12}\,p_{r_{21}r_{12}\dot{r}_{11}\dot{r}_{22}}(r_{21},r_{12},\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{12}\int_{r_T}^{\infty}dr_1\,p_{r_{12}r_{11}\dot{r}_{11}\dot{r}_{22}}(r_{12},r_1,\dot{r}_1,\dot{r}_2)+P_2\int_0^{r_T}dr_{22}\int_{r_T}^{\infty}dr_1\,p_{r_{22}r_{21}\dot{r}_{21}\dot{r}_{12}}(r_{22},r_1,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_0^{r_T}dr_{11}\int_{r_T}^{\infty}dr_2\,p_{r_{11}r_{22}\dot{r}_{21}\dot{r}_{22}}(r_{11},r_2,\dot{r}_1,\dot{r}_2)+P_2\int_0^{r_T}dr_{21}\int_{r_T}^{\infty}dr_2\,p_{r_{21}r_{12}\dot{r}_{11}\dot{r}_{12}}(r_{21},r_2,\dot{r}_1,\dot{r}_2)+$$
$$+P_1\int_{r_T}^{\infty}dr_1\int_{r_T}^{\infty}dr_2\,p_{r_{11}r_{12}\dot{r}_{11}\dot{r}_{12}}(r_1,r_2,\dot{r}_1,\dot{r}_2)+P_2\int_{r_T}^{\infty}dr_1\int_{r_T}^{\infty}dr_2\,p_{r_{21}r_{22}\dot{r}_{21}\dot{r}_{22}}(r_1,r_2,\dot{r}_1,\dot{r}_2) \tag{5}$$

The signal derivative's PDFs can be found from joint PDF based on [14]:

$$p_{\dot{r}_1}(\dot{r}_1)=\int_{-\infty}^{\infty}p_{\dot{r}_1\dot{r}_2}(\dot{r}_1,\dot{r}_2)d\dot{r}_2 \tag{6}$$

$$p_{\dot r_2}(\dot r_2) = \int_{-\infty}^{\infty} p_{\dot r_1 \dot r_2}(\dot r_1, \dot r_2) d\dot r_1 \qquad (7)$$

By replacing (5) in (6) and (7), obtained:

$$p_{\dot r_1}(\dot r_1) = P_1 p_{\dot r_{11}}(\dot r_1) + P_2 p_{\dot r_{21}}(\dot r_1) + \\ + \left(P_2 F_{r_{21}}(r_T) - P_1 F_{r_{11}}(r_T)\right) p_{\dot r_{11}}(\dot r_1) + \\ + \left(P_1 F_{r_{11}}(r_T) - P_2 F_{r_{21}}(r_T)\right) p_{\dot r_{21}}(\dot r_1) \qquad (8)$$

$$p_{\dot r_2}(\dot r_2) = P_1 F_{r_{11}}(r_T) F_{r_{22}}(r_T) p_{\dot r_{12}}(\dot r_2) + P_2 F_{r_{21}}(r_T) F_{r_{12}}(r_T) p_{\dot r_{22}}(\dot r_2) + \\ + P_1 B_1(r_T) p_{\dot r_{22}}(\dot r_2) + P_2 B_2(r_T) p_{\dot r_{12}}(\dot r_2) + \\ + P_1 F_{r_{11}}(r_T)\left(1 - F_{r_{22}}(r_T)\right) p_{\dot r_{22}}(\dot r_2) + P_2 F_{r_{21}}(r_T)\left(1 - F_{r_{12}}(r_T)\right) p_{\dot r_{12}}(\dot r_2) + \\ + P_1 C_1(r_T) p_{\dot r_{12}}(\dot r_2) + P_2 C_2(r_T) p_{\dot r_{22}}(\dot r_2) \qquad (9)$$

where $F_{r_{ij}}(r_T)$ are signal's cumulative distribution functions (CDFs) and $F_{r_{i1}}(r_T) = F_{r_{i2}}(r_T)$, while $B_i(r_T)$ and $C_i(r_T)$ are abreviations obtained after some mathematical manipulations made on integral from [16]:

$$B_i(r_T) = -e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1 - Q_{m_i}\left(r_T\sqrt{\frac{2m_i}{\Omega_i\left(1-\rho_i^2\right)}}, \frac{\rho_i r_T \sqrt{2m_i}}{\sqrt{\Omega_i(1-\rho_i^2)}}\right)\right] + \\ + e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1 - Q_{m_i}\left(\frac{\rho_i r_T \sqrt{2m_i}}{\sqrt{\Omega_i\left(1-\rho_i^2\right)}}, \frac{r_T\sqrt{2m_i}}{\sqrt{\Omega_i\left(1-\rho_i^2\right)}}\right)\right] \qquad (10)$$

$$C_i(r_T) = e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1 - Q_{m_i}\left(r_T\sqrt{\frac{2m_i}{\Omega_i\left(1-\rho_i^2\right)}}, \frac{\rho_i r_T \sqrt{2m_i}}{\sqrt{\Omega_i(1-\rho_i^2)}}\right)\right] + \\ + e^{-\frac{r_T^2}{2\sigma_i^2}} Q_{m_i}\left(\frac{\rho_i r_T \sqrt{2m_i}}{\sqrt{\Omega_i\left(1-\rho_i^2\right)}}, \frac{r_T\sqrt{2m_i}}{\sqrt{\Omega_i\left(1-\rho_i^2\right)}}\right) \qquad (11)$$

where $Q_1()$ is the Marcum Q function [3]:

$$Q_1(\alpha, \beta) = \int_{\beta}^{\infty} x e^{-\left(\frac{x^2+\alpha^2}{2}\right)} I_0(\alpha x) dx \qquad (12)$$

$p_{\dot r_1}(\dot r_1)$ and $p_{\dot r_2}(\dot r_2)$ are obtained in the closed form and are suitable for calculating of the level crossing rate and average fade duration.

The probability density functions of signal derivatives at the combiner input in the presence of Nakagami m fading have a normal distribution with zero mean value [17, 18]:

$$p_{\dot r_i}(\dot r_{i,j}) = \frac{1}{\sqrt{2\pi}\dot\sigma_i} e^{-\frac{\dot r_{i,j}^2}{2\dot\sigma_i^2}}, \quad -\infty < \dot r_{i,j} < \infty \qquad (13)$$

where $i=1,2,\ j=1,2$ and $\dot\sigma_i^2 = \frac{\Omega_i}{m_i}\pi^2 f_m^2$ is variance, while f_m is maximum Doppler shift frequency.

Probability density function of signal derivative at the SSC combiner output at two time instants in the presence of Nakagami m fading is obtained when signal derivative's PDFs from (13) are put in the previously obtained general expressions. The signal's CDF is replaced with [3]:

$$F_{r_i}(r_{i,j}) = \gamma\left(\frac{m_i}{\Omega_i} r_{i,j}^2, m_i\right), \quad r_{i,j} \ge 0 \qquad (14)$$

where $i=1,\ 2,\ j=1,2$.

The final expressions for signal derivative's PDFs at two time instants in the presence of Nakagami-*m* fading are:

$$p_{\dot r_1}(\dot r_1) = P_1 \frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_1^2}{2\dot\sigma_1^2}} + P_2 \frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_1^2}{2\dot\sigma_2^2}} + \\ + \left(P_2\gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right) - P_1\gamma\left(\frac{m_1}{\Omega_1} r_T^2, m_1\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_1^2}{2\dot\sigma_1^2}} + \\ + \left(P_1\gamma\left(\frac{m_1}{\Omega_1} r_T^2, m_1\right) - P_2\gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_1^2}{2\dot\sigma_2^2}} \qquad (15)$$

$$p_{\dot r_2}(\dot r_2) = P_1\gamma\left(\frac{m_1}{\Omega_1} r_T^2, m_1\right)\gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + \\ + P_2\left(1 - e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} + \\ + P_1 B_1(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} + P_2 B_2(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + \\ + P_1\gamma\left(\frac{m_1}{\Omega_1} r_T^2, m_1\right)\left(1 - \gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} + \\ + P_2\gamma\left(\frac{m_2}{\Omega_2} r_T^2, m_2\right)\left(1 - \gamma\left(\frac{m_1}{\Omega_1} r_T^2, m_1\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + \\ + P_1 C_1(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_1} e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + P_2 C_2(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_2} e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} \qquad (16)$$

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The PDFs of signal derivatives are presented in Fig. 2 for different values of parameter $\dot{\sigma}_i$ in the case of channels with identical distributions.

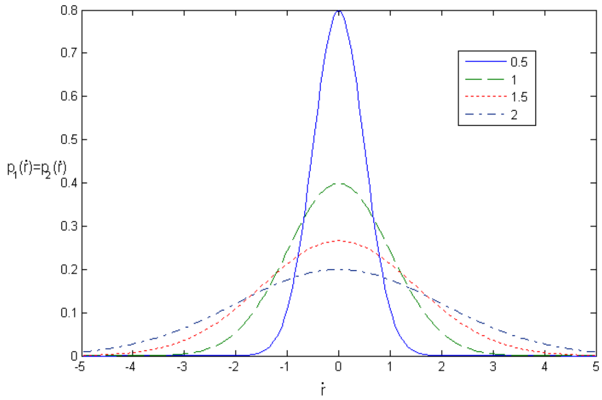

Fig.2. The probability density functions of derivatives at the SSC combiner output at two time instants

4 Conclusion

The expressions for probability density functions (PDFs) of the signal time derivatives in two time moments for dual branch SSC combiner output signals and for the presence of Nakagami-*m* fading at the inputs, are obtained in this paper. The second order characteristics for complex combiners that make the decision based on sampling at two time moments can be calculated by using the closed-form expressions obtained in this paper.

The authors showed at several papers that this approach is justified by calculating the first order characteristics. In the next paper we will determine the average level crossing rate and the average fade duration for these systems.

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