A redundant bi-dimensional inertial navigator in vertical plane

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Abstract: - The paper presents a redundant architecture of a bi-dimensional strap-down inertial navigator in vertical plane. The redundance of the developed system is provided by its inertial measurement unit, containing inertial sensors arrays in a linear redundant configuration. The inertial sensors data in each of the three arrays (two for accelerometers and another one for gyros) are fused by using a statistical method derived from the maximal ratio combining method. Shown are: the navigator inertial measurement unit structure, the fusion algorithm theory, the inertial navigator theory of operation, and the experimental validation of the proposed architecture. In the experimental validation phase the positioning, speed and attitude errors are evaluated and discussed relative to the reference signals provided by an integrated INS/GPS navigator.

Key-Words: - Inertial navigation, data fusion, redundant configuration, experimental validation

1 Introduction

Based on the measurement and numerical integration of moving vehicles angular speed and linear acceleration, the inertial navigation systems (INS) are widely used in many military applications such as aircraft, submarines, spacecraft and guided missiles. It insensitiveness at the natural and artificial perturbations is one of the advantages that placed the inertial navigation ahead the other navigation methods [1]. Currently, both military and special civilian applications assign miniaturization and redundancy requirements that make this area one of the top in current technology [2]. In this way, for a lot of INS applications, the classical inertial measurement units (IMU), oversized and expensive, were replaced with low-cost and low-dimensions ones. Unfortunately, these size and cost related advantages, brought by the miniaturized inertial sensors, were impaired by the obtaining of poor performance in terms of noise density, bias and scale factor stabilities [3]-[5]. Moreover, the spectral overlapping of the inertial sensors noise with the

0Hz-100 Hz frequencies band, considered as a part of the real movement of the monitored vehicle, make impossible theirs outputs classical filtering [6]. As a consequence, the researchers are looking for different methods to remove or limit this noise, many studies being conducted on the statistical filtering methods direction [6]-[8].

Following this trend in the field, a research project was started in Avionics Division, University of Craiova, Romania, which aims to develop some high-precision strap-down inertial navigators, based on the connection and adaptive integration of the nano and micro inertial sensors in low cost networks, with a high degree of redundance. The project researches focus on the new concept in the navigation field related to the obtaining of "miniaturized low-cost, high technology non GPS dependent navigation solution". Until now, we conceived and implemented software some algorithms to statistically fuse the data from miniaturized inertial sensors, organized in redundant networks, in order to obtain high performance and redundant IMUs. In the current phase, the project aims to integrate the sensors data fusion algorithms in different types of inertial navigators (bi- or three-dimensional).

This paper presents such a redundant strap-down inertial navigation system (SDINS) for vertical bi-dimensional monitoring of different vehicles.

2 Data Fusion Algorithm

The here exposed data fusion algorithm is based on the idea of building redundant linear networks of sensors in the same navigator, followed by each sensors network data fusion with a statistical method derived from the maximal ratio combining method; a redundant linear network supposes the mounting of several accelerometers and gyros sensors on each IMU axis. For our bi-dimensional navigator are used two accelerometers arrays on the x and z axes, for vehicle linear displacements monitoring, and another gyros array along the y axis, for vehicle pitch angle monitoring (Fig. 1).

The data fusion algorithm was proposed and presented in detail by the authors in paper at the reference [9]. In the generalized form of the algorithm, each of the three used sensors arrays contains n collinear sensors, which provide independent estimates of the same acceleration or angular speed x applied on respective axis. The n independent measures, obtained from the inertial sensors, are object of the sensors internal errors, and are characterized by the $\sigma_i^2 \; (i=\overline{1,n})$ variances.

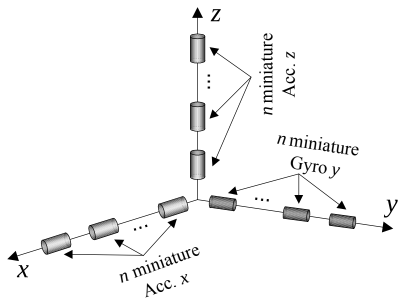

Fig. 1 Redundant IMU structure

The algorithm integrates all of the n acceleration or angular speed sensors fixed on the same axis, being based on the idea in that for the determination of the component of the acceleration or of the angular speed on the considered axis, each sensor should have a weight inverse proportionally with the standard deviation of the last m samples acquired from it ([9]) (Fig. 2).

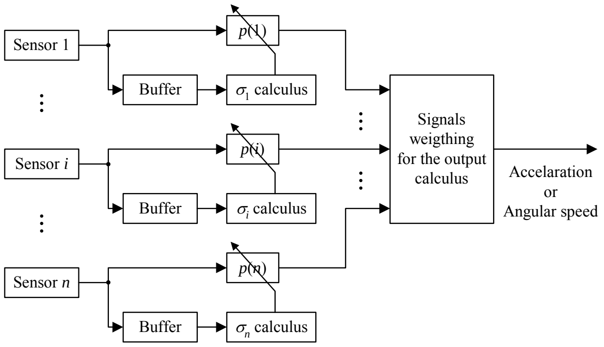

Fig. 2 Basics of the sensors' fusion algorithm

Considering c_{ij} the reading no. j, perturbed, from the i sensor on the input axis, the average of m consecutive samples acquired from the accelerometer i is:

$$\bar{c}_i = \frac{1}{m}\sum_{j=1}^{m} c_{ij}, \tag{1}$$

and the dispersion of the readings:

$$\sigma_i^2 = \frac{1}{m}\sum_{j=1}^{m} (c_{ij} - \bar{c}_i)^2. \tag{2}$$

Each sensor will be given the larger weight $p(i)$ the smaller the standard deviation $\sigma_i = \sqrt{\sigma_i^2}$ is. Considering $c_{i\,(m+1)}$ the reading no. $m+1$ from the sensor i, the quantity read by the integrated system of n sensors after $m+1$ steps of signal acquisitions from the sensors is:

$$c_{m+1} = \left(\sum_{i=1}^{n} p_i \cdot c_{i\,(m+1)}\right) \Big/ \left(\sum_{i=1}^{n} p_i\right). \tag{3}$$

We note the sum of the n accelerometers' weights:

$$\sum_{i=1}^{n} p_i = 1 \qquad (4)$$

and, using the inverse proportionality between weight p_i and the standard deviation σ_i:

$$\sigma_1 \cdot p_1 = \sigma_2 \cdot p_2 = \ldots = \sigma_n \cdot p_n, \qquad (5)$$

it results:

$$p_i = (1/\sigma_i) \cdot \left(1 / \sum_{k=1}^{n} (1/\sigma_k) \right). \qquad (6)$$

So, the output quantity for the sample $m+1$ will be:

$$c_{m+1} = \sum_{i=1}^{n} (c_{i\ (m+1)} / \sigma_i) / \sum_{k=1}^{n} (1/\sigma_k), \qquad (7)$$

$$c_{m+1} = \frac{1}{\displaystyle\sum_{k=1}^{n} \frac{1}{\sqrt{\displaystyle\sum_{j=1}^{m} (c_{kj} - \bar{c}_k)^2}}} \cdot \sum_{i=1}^{n} \frac{c_{i\ (m+1)}}{\sqrt{\displaystyle\sum_{j=1}^{m} (c_{ij} - \bar{c}_i)^2}}. \qquad (8)$$

If one of the sensors breaks down the standard deviation will be null or will have very large values. When implementing the relation (8) into the software one can condition the zero value for the weights of the malfunctioning sensor if its corresponding standard deviation is null. The same weights will be given to it if the standard deviation exceeds a superior limit, set in connection with the type of sensor used in the application, with the navigation problem which is to be solved and with the number m of samples considered for the calculation of the standard deviation.

3 Inertial Navigator Theory and Simulink Implementation

Usually, the solving of a navigation problem by using inertial techniques supposes the determination of the vehicle attitude relative to the navigation frame, followed by the linear acceleration components transformation between the vehicle and navigation frames and theirs numerical integration [6]-[8]. Based on the space-time characteristics of the proposed application, we have chosen a local horizontal frame $\mathrm{O}x_l y_l z_l$ (NED – North East-Down frame) as navigation frame. The vehicle frame (SV) $\mathrm{O}_v x_v y_v z_v$ is rigidly attached to the vehicle, has the origin in the center of gravity of the vehicle and axes oriented after the convention front-right-down.

For our navigator (a bi-dimensional one), only the axes x and z are considered in the monitoring of position and speed. Therefore, we need to know the linear accelerations along the x and z axes, and the angular speed along the y axis. On the other way, the relative angular position of the vehicle frame (SV) and of the navigation frame is given by the vehicle pitch angle (Fig. 3). In Fig. 3 we have: $\vec{r}$ - the position vector of the vehicle in navigation frame, $\vec{v}$ - the vehicle speed relative to the NED frame, $\vec{v}_{xv}, \vec{v}_{zv}$ - components of the vehicle speed $\vec{v}$ in SV frame, $\vec{\omega}_{yv}$ - the component of the vehicle angular speed along the y axis of the SV frame.

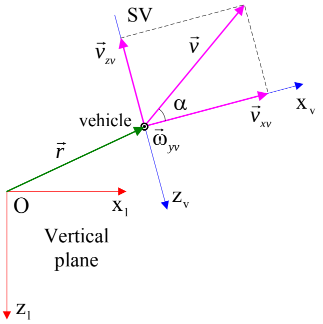

Fig. 3 SV and navigation frames relative position

In inertial navigation problems an important role plays the correction of the gravity acceleration effects on the accelerometric readings. It is very well known that the accelerometers outputs (the components of the specific force $\vec{f}$) are affected by the gravity acceleration, being a resultant between this acceleration and the vehicle kinematic acceleration $\vec{a}$. For our application, due to the navigation in vertical plane, the output $\vec{f}$ of the accelerometers is influenced by the gravitational field. Therefore, in our configuration this correction is needed.

The coordinate transformation between the vehicle frame and navigation frame is realized by using the equations:

$$\begin{aligned} f_{xl} &= f_{xv}\cos\theta + f_{zv}\sin\theta, \\ f_{zl} &= -f_{xv}\sin\theta + f_{zv}\cos\theta, \end{aligned} \qquad (9)$$

with f_{xv}, f_{yv} - the components of the specific force $\vec{f}$ in SV frame (accelerometric readings), f_{xl}, f_{yl} - the components of the specific force $\vec{f}$ in navigation frame, and θ - the pitch angle.

The positioning of the vehicle in navigation frame is made through the double integration of its kinematic acceleration $\vec{a}_c$ components in the same frame. These components are obtained through the correction of f_{xl}, f_{yl} components with the gravity acceleration components in navigation frame (g_{xl}, g_{zl})

$$a_{xl} = f_{xl} + g_{xl},\ a_{zl} = f_{zl} + g_{zl}. \qquad (10)$$

The pitch angle θ value results by numerical integration of the ω_{yv} gyro reading:

$$\theta = \theta_0 + \int_{t_{n-1}}^{t_n} \omega_{yv}\, dt, \qquad (11)$$

with θ_0 - the initial value of the pitch angle.
Implementing in Matlab/Simulink the previous discussed aspects related to the navigation algorithm, the model in Fig. 4 is obtained. To have the vehicle position in terms of latitude, longitude and altitude we used the “Flat Earth to LLA” Matlab/Simulink block. The navigator inputs are the IMU outputs (accelerations in SV along the *x* and *z* axes, and angular speed in SV along the *y* axis), while its outputs are the pitch angle, the vehicle’ position and speed relative to the navigation frame (in North and East directions), and the vehicle position in terms of latitude, longitude and altitude.

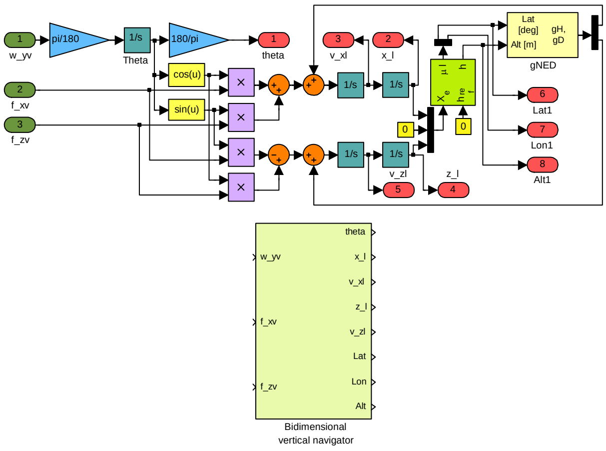

Fig. 4 Navigation algorithm

The navigation algorithm in Fig. 4 uses also the block „gNED” which implements software the gravity model calculating the values of the g_{xl} and g_{zl} components in navigation frame:

$$\begin{aligned} g_{xl} &\cong 0, \\ g_{zl} &\cong 9{,}78 + 0{,}0519 \cdot \sin^2 \phi - 3{,}08 \cdot 10^{-6} \cdot h, \end{aligned} \qquad (12)$$

where *h* is the vehicle altitude expressed in m, and φ is its latitude expressed in deg.
As can be easily observed, the model in Fig. 4 implements only the algorithm processing the data obtained from the IMU. To develop the entire navigator in the redundant proposed variant, between the IMU and this model, the fusion algorithm for each of the three channels should be inserted. The experimental development of our navigator supposed the mounting of a number of four sensors (*n*=4) in each of the three considered sensors arrays. The Matlab/Simulink model obtained for the fusion algorithm (based on equations (1) to (8)) is shown in Fig. 5 [9]. The model inputs are the signals from the sensors in each of the three detection array.

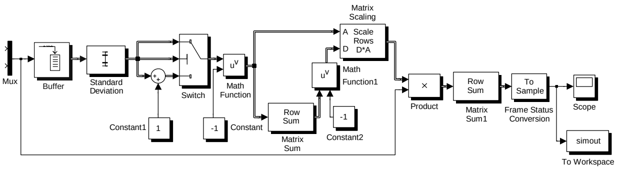

Fig. 5 Simulink model of the fusion algorithm

4 Navigator Experimental Validation

To perform the validation step of the developed redundant navigator, the Matlab/Simulink model in Fig. 6 was used. During the experimental test some data were simultaneously acquired from a redundant IMU, especially realized for this action, and from an integrated navigator INS/GPS, both of these boarded on a testing car which played the role of the monitored vehicle. The INS/GPS system was used as reference system to evaluate the errors of our navigator. The redundant IMU contained eight accelerometers (four disposed along the x axis, and another four along of z axis), and four gyros disposed along the y axis of the SV frame. The collected inertial data were further applied to the inputs of the model in Fig. 6.

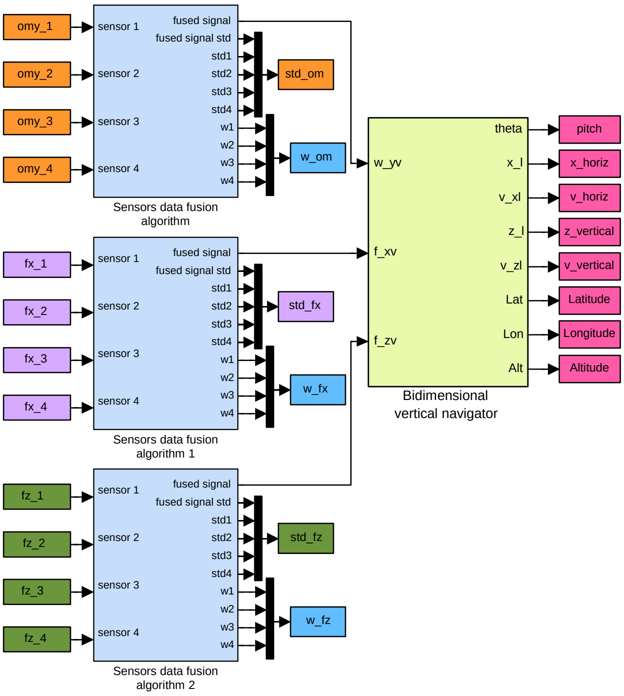

Fig. 6 Matlab/Simulink validation algorithm

The data values from the sensors in the three detection arrays and the fusion results for each array are depicted in Fig. 7. Also, the standard deviations of this data are shown in Fig. 8. Fig. 7 shows an important decrease of the noise through the sensors data fusion, proved otherwise by the reduced values of the standard deviations in comparison with the sensors' standard deviations in Fig. 8.

Fig. 9 presents a comparative study between the solution of navigation of our redundant strap-down INS and of the reference INS/GPS navigator. Also, are shown the results obtained if INS navigation solution is founded by using the data from the fourth sensor in each of the three sensors arrays included in redundant IMU.

The maximum absolute deviations between the redundant INS and reference GPS navigation solutions during the first 120 s are: 0.0797 deg in yaw angle, $9.2867\cdot10^{-4}$ deg in longitude, $4.9268\cdot10^{-4}$ deg in latitude, 92.11 m in horizontal position, 0.46 m in altitude, 1.3921 m/s in horizontal speed, and 0.0108 m/s in vertical speed.

The same deviations, but between the navigation solution in non-redundant configuration (by using the data from the fourth sensor in each of the three sensors arrays) and reference GPS, are: 0.8673 deg in yaw angle, $35.0352\cdot10^{-4}$ deg in longitude, $18.5868\cdot10^{-4}$ deg in latitude, 347.54 m in horizontal position, 8.57 m in altitude, 8.6125 m/s in horizontal speed, and 0.2107 m/s in vertical speed.

A comparative study of the deviations resulted from our system with those resulted from the non-redundant configuration shows that the proposed configuration produces a significant improvement of the positioning precision.

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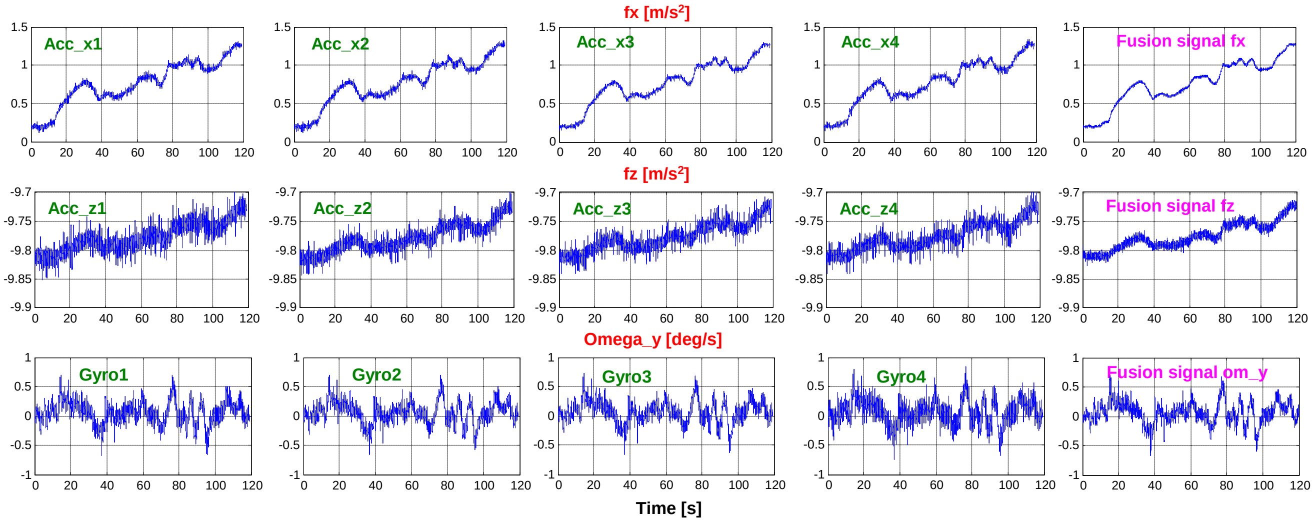

Fig. 7 Acquired data and fusion signals

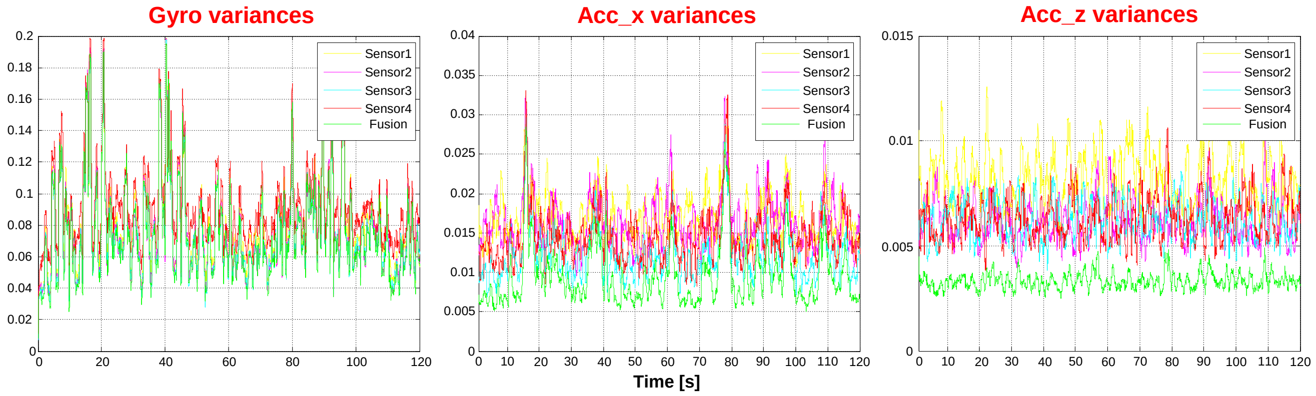

Fig. 8 Standard deviations of sensors data and fusion signals

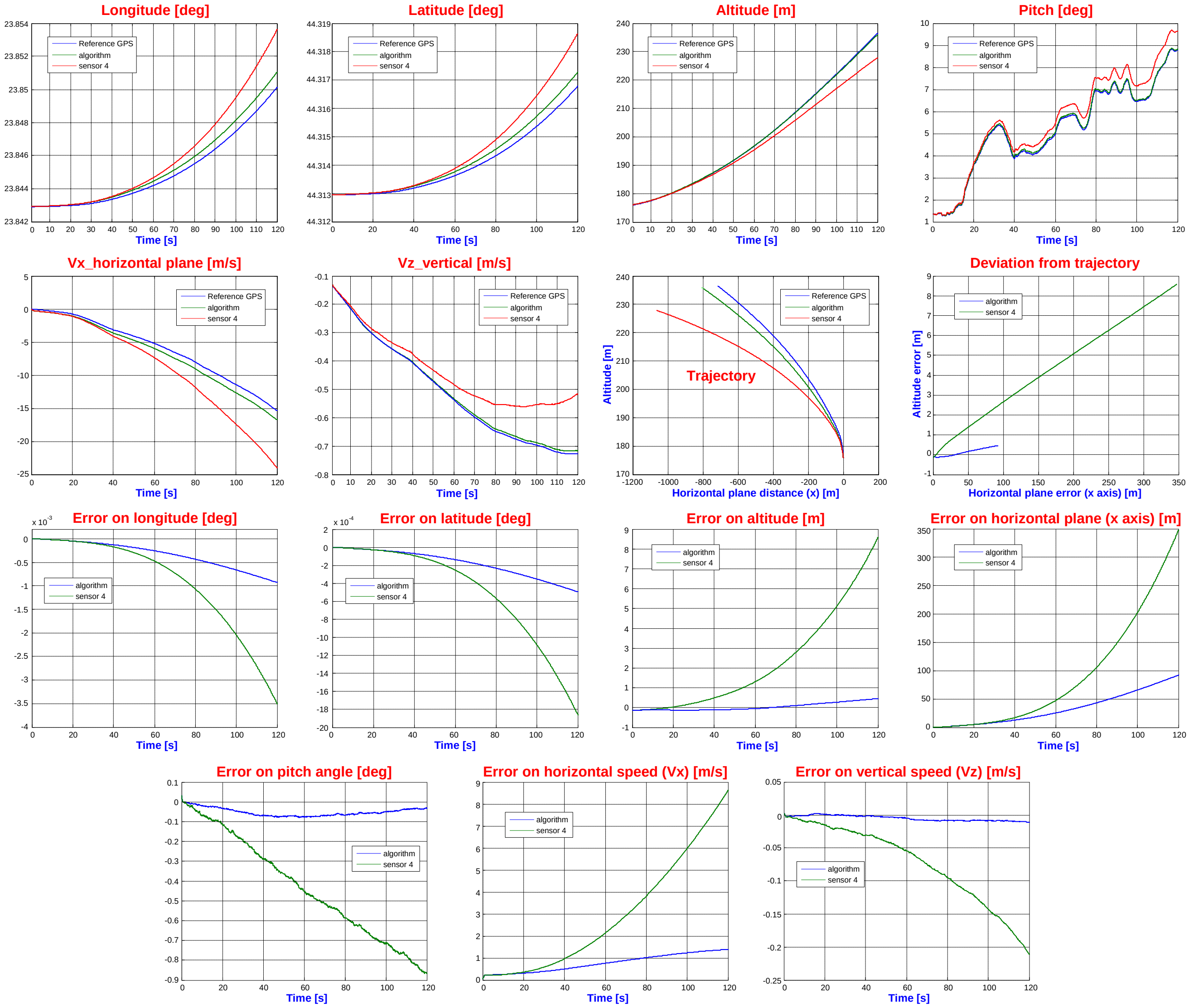

Fig. 9 Navigation solution and errors

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