Abstract: - In this work, multiple model control of a boiler-turbine system is studied. Multiple linear local models are obtained through piecewise linearization. A bank of Kalman filters is constructed using these local models. States and model parameters used for future predictions within the predictive control are updated online through computation of probability density functions. Simulations show that the boiler-turbine coordinated system can be successfully controlled by this methodology.

Key-Words: Power systems, Kalman filtering, Predictive control, Multiple models, Boiler-turbine, Piecewise linearization

1 Introduction

The boiler-turbine system is an essential part of a power plant. The boiler-turbine system exhibits nonlinear, time-varying, coupling behavior. Hence, the control of such system is complex and challenging and a linear model cannot capture the nonlinear dynamics sufficiently. The major control objective of a boiler-turbine system is to keep the output of mechanical energy in balance with the electrical load demand while maintaining the internal variables such as drum steam pressure, temperature and drum water level within the desired ranges. Due to the variable demand for electricity in a grid the power plants are forced to change load frequently in a large magnitude. As a result, the power plants have to operate in multiple operating regimes and the nonlinear behavior becomes more significant during these transitions between operating points. It is essential that the developed dynamic model can capture the dynamics of the system in different operating while keeping the model relatively simple, suitable for the design of feedback controllers.

Numerous modeling and control methodologies have been applied for boiler-turbine system. In [1] multiple model predictive control methodology where the system is modeled by piecewise linear models is applied for control of a boiler turbine unit. Gain scheduling approach allowing the possibility of large changes in operating conditions has been presented in [2]. Fuzzy scheduling model predictive controller where local models are equidistantly distributed in the operating space and the local models parameters are obtained through linearization is discussed in [3]. Neuro-fuzzy network with Controlled Auto-Regressive Integrated Moving Average CARIMA models and interpolation based on the B-splines functions has been successfully applied in [4]. Moon and Lee [5] presented a fuzzy controller that can update the fuzzy rules adaptively by a simple set-point error-checking process. The online learning of Radial Basis Function (RBF) neural network has been tested in [6]. In [7] the dynamic fuzzy model, where local models are obtained using Taylor series around the nominal points, is presented. A two level hierarchical control scheme for boiler-turbine system is implemented in [8]. In this paper, the controlled plant under consideration is a 160 MW boiler-turbine system that was reported in [9]. The state is estimated using the bank of Kalman filters each using the parameters from different operating point.

2 Multiple Model Adaptive Estimation (MMAE) algorithm

The multiple model adaptive estimation (MMAE) method is based on a bank of parallel Kalman filters (KF), each tuned to describe the plant at a different operating point (Fig. 1). The model is assumed to be linear affine and of the form:

\[ x(k+1) = Ax(k) + Bu(k) + E \]
\[ y(k) =Cx(k) + Du(k) + F \]  (1)
Output of each KF is then weighted by its corresponding probability based on the measurement history. Fig. 1 shows a functional block diagram of the MMAE algorithm. Its primary feature is a bank of Kalman filters operating in parallel, using vectors of measurements \( y \) and control commands \( u \) as their input. Each Kalman filter has the same structure based on the linearized description of the process. At every sampling period, each of these Kalman filters is producing an estimate of the state \( \hat{x}_i(k) \) and residual \( r_i(k) \). The idea is that the model with well-behaved residuals contains the parameters that best matches true parameters of the system. Testing the hypothesis which model is the correct one is evaluated in the hypothesis testing block. The initial probability of each hypothesis being correct is distributed evenly:

\[
\alpha_i(0) = 1/M
\]  

(2)

The output prediction is given by the mixture of conditional probability density functions:

\[
p(y(k)|u(k)) = \sum_{i=1}^{M} \alpha_i p_i(y(k)|u(k))
\]

(3)

where \( \alpha_i \) are the probabilities of each model being the correct one and normalized to 1.

\[
\alpha_i = p \ m(t) = i \ \sum_{i=1}^{M} \alpha_i = 1
\]

(4)

The predictive conditional probability density functions \( p_i \) are given by the state-space model as:

\[
p_i(y(k)|u(k)) = p_i(y(k)|x(k)) p_i(x(k)|u(k))
\]

(5)

where \( p_i(x(k)|u(k)) \) is the state-estimate provided by the \( i \)-th Kalman filter. One step of the Kalman filter can be written as:

\[
P(k+1) = A_i P_i(k) A_i + Q_i - K_i C_i P_i(k) C_i + I \ K_i^T
\]

\[
\hat{x}(k+1) = A \hat{x}(k) + B u(k) + E_i +
\]

\[
+ K_i (y - C_i \hat{x}(k) - D_i u(k) - F_i)
\]

(6)

The conditional probability density function for known measurement noise has normal distribution[10] and can be computed using:

\[
p(y(k)|x(k),\sigma_n^2) = N(y,(1+C_i P_i C_i^T)\sigma_n^2)
\]

\[
\Sigma = (1+C_i P_i C_i^T)\sigma_n^2
\]

\[
p(y(k)|x(k),\sigma_n^2) = \frac{1}{(2\pi)^w/2} \exp\left( -\frac{1}{2\Sigma} r_i(k)^T r_i(k) \right)
\]

(7)

\[
r_i = y(k) - y(k)
\]

The estimate of variance can be updated with exponential forgetting with factor \( \rho \) as:

\[
\sigma_n^2(k+1) = \frac{S_n^2(k+1)}{\nu(k+1)}
\]

(8)

![Fig. 1 Multiple Model Adaptive Estimation scheme](image-url)
where variables $S_i^2$ and $\nu(k+1)$ are updated at each step:

$$S_i^2(k+1) = \varphi\left(S_i^2(k) + \frac{r^T r}{1 + C_j P(k+1) C_j} \right)$$

$$\nu(k+1) = \varphi(\nu(k) + 1)$$

### 3 Boiler-Turbine System

The boiler-turbine model used in this paper was first developed by Bell and Astrom and has been popularly adopted in validating various controllers for the boiler-turbine system in simulation. The parameters were estimated from the data collected from the Synvendsk Kraft AB Plant in Malmo, Sweden. The rate power of the plant is 160 MW. The model is a three input, three output, third order nonlinear system (Fig. 2). The inputs are the positions of the valve actuators that control the mass flow rates of fuel ($u_1$ in pu), steam to the turbine ($u_2$ in pu), and water to the drum ($u_3$ in pu). The three major outputs are the electrical power ($y_2$ in MW), drum steam pressure ($y_1$ in kg/cm$^2$), and drum water level ($y_3$ in m). The three state variables are the electrical power ($x_2$ in MW), drum steam pressure ($x_1$ in kg/cm$^2$), and the fluid (steam-water) density ($x_3$ in kg/m$^3$). The model state equations are given as:

$$\dot{x}_i = -0.0018u_1x_1^{9/8} + 0.9u_1 - 0.15u_3$$

$$\dot{x}_2 = 0.073u_2 - 0.016 x_1^{9/8} - 0.1x_2$$

$$\dot{x}_3 = 141u_3 - (1.1u_2 - 0.19)x_1 / 85$$

The outputs of the plant are given:

$$y_1 = x_1$$

$$y_2 = x_2$$

$$y_3 = \frac{0.13073x_1 + 100a_{st} + q_c/9 - 67.975}{20}$$

where $a_{st}$ and $q_c$ are steam quality and evaporation rate (kg/s), respectively.

They are given by

$$a_{st} = \frac{1 - 0.001538x_1 - 0.8x_1 - 25.6}{x_1 1.0394 - 0.0012304x_1}$$

$$q_c = 0.854u_2 - 0.147 x_1 + 45.59u_1 - 2.514u_3 - 2.096$$

Due to actuator limitations, all the control inputs $u_1, u_2, u_3$ are subject to the following constraints:

$$0 < u_i < 1 \quad i=1,2,3$$

$$-0.007 \leq u_1 \leq 0.007$$

$$-0.02 \leq u_2 \leq 0.02$$

$$-0.05 \leq u_3 \leq 0.05$$

### 4 Multiple Linearization of the Process

In this work the global model of the process is obtained through piecewise linearization. The nonlinear system:

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = f(x(t), u(t))$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p, y(t) \in \mathbb{R}^q$ represent the states, input value and output value, respectively. The system can be linearized around the operating point using the Taylor’s series approximation. This results in a series of $M$ local linear models of the form:

$$\delta \dot{x}_i(t) = A_i \delta x_i(t) + B_i \delta u(t)$$

$$\delta y_i(t) = C_i \delta x_i(t) + D_i \delta u(t)$$

where

$$A_i = \left. \frac{\partial f(x,u)}{\partial x} \right|_{x=x^{opt},u^{opt}} \quad B_i = \left. \frac{\partial f(x,u)}{\partial u} \right|_{x=x^{opt},u^{opt}}$$

$$C_i = \left. \frac{\partial g(x,u)}{\partial x} \right|_{x=x^{opt},u^{opt}} \quad D_i = \left. \frac{\partial g(x,u)}{\partial u} \right|_{x=x^{opt},u^{opt}}$$

The above equations can be discretized to obtain a set of linear systems in the form:

$$x_i(k+1) = A_i x_i(k) + B_i u(k) + E_i$$

$$y(k) = C_i x(k) + D_i u(k) + F_i$$

Fig. 2 Boiler-turbine model
Table 1. Operating points of the boiler-turbine dynamics

<table>
<thead>
<tr>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>86.4</td>
<td>97.2</td>
<td>108</td>
<td>119</td>
<td>130</td>
</tr>
<tr>
<td>36.7</td>
<td>50.5</td>
<td>66.7</td>
<td>85.1</td>
<td>105</td>
</tr>
<tr>
<td>469</td>
<td>451</td>
<td>428</td>
<td>398</td>
<td>356</td>
</tr>
<tr>
<td>0.20</td>
<td>0.27</td>
<td>0.34</td>
<td>0.41</td>
<td>0.50</td>
</tr>
<tr>
<td>0.55</td>
<td>0.62</td>
<td>0.69</td>
<td>0.75</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The nonlinear system was linearized at 5 operating points and Table 1 shows the values of inputs and states for these operating points.

5 Predictive Control

The state-space model based predictive control is based on the time-invariant model:

\[ x(k+1) = Ax(k) + Bu(k) \]
\[ y(k) = Cx(k) + Du(k) + F \]

(18)

The model of the process is obtained at every sampling interval and its parameters are used for the entire prediction horizon \( H_p \). The discrete model (17) contains also the affine part that results from linearization around non-zero steady-state:

\[ x(k+1) = Ax(k) + Bu(k) + E \]
\[ y(k) = Cx(k) + Du(k) + F \]

(19)

The \( H_p \)-step ahead output prediction can be deduced:

\[ \hat{Y} = \Phi_{yo} x(k) + \Phi_{yu} U + \Phi_{y0} \]

(20)

where the matrices are given as:

\[ \Phi_{yu} = \begin{bmatrix} CB & D & 0 & 0 & 0 \\ CAB & CB & D & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ CA^{H_p-1}B & CA^{H_p-2}B & \cdots & CB & D \end{bmatrix} \]

(21)

\[ \Phi_{y0} = \begin{bmatrix} CE + F \\ CA + CE + F \\ \vdots & \vdots \\ [CA^{H_p-1}E + \cdots + CE + F] \end{bmatrix} \quad \Phi_{yo} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \end{bmatrix} \]

The computation of a control law of MPC is based on minimization of the following criterion

\[ J_{MPC} = \hat{Y} - W^T Q \hat{Y} - W + DU^T R U \]

(22)

where \( \hat{y}(k+j|k) \) is a \( j \) steps ahead prediction of the system, \( w(k+j) \) is a future reference trajectory and \( Q, R \) are positive definite weighting matrices. The minimization of the criterion can be transformed into a quadratic programming problem:

\[ J_{MPC} = u^T Hu + f_u \]

(23)

where matrix \( H \) and vector \( f_u \) are derived from model parameters given by (20). The quadratic problem is usually solved numerically. As formulated, the nonlinear model predictive controller will exhibit steady-state offset in the presence of plant/model mismatch due to a lack of integral action. To introduce an integral action to remove steady-state error an integrator state must be added to the system:

\[ \hat{V}(k) = \hat{V}(k-1) + (w(k) - y(k)) \]

(24)

System matrices (19) are updated as follows:

\[ \hat{A} = A + \tilde{B}u + \tilde{F} \]
\[ \hat{C} = C + \tilde{D}u + \tilde{F} \]

(25)

In order to minimize the augmented state \( v(k) \) the cost criterion for MPC (22) is transferred to:

\[ J_{MPC} = \hat{X} - W^T Q \hat{X} - W + DU^T R U + \hat{X}^T S \hat{X} \]

(26)

where

\[ \hat{X} = \Phi_{xo} x(k) + \Phi_{uo} U + \Phi_{xo0} \]

(27)
6 Implementation

In this part, multiple-model predictive control is applied to the Bell-Åström boiler-turbine system. Multiple linear models were obtained through linearization in steady-state operating points given in Table 1. The sampling period was set to 1s due to the dynamics of the process. The Kalman filter bank with these local model was constructed with initial condition \( x_0 = [108 \ 66, 65 \ 428]^T \). The initial estimate covariance matrix \( P_0 \) was chosen to be:

\[
P_0 = \begin{bmatrix}
10^4 & 0 & 0 \\
0 & 10^4 & 0 \\
0 & 0 & 10^4 \\
\end{bmatrix}
\]  

Saturation constraints in the manipulated variables are imposed to take into account the minimum/maximum aperture of the valve regulating the flow rates. The prediction horizon was set to 20 samples as a result of using different values and comparing control performances. The weighting matrices \( Q, R, S \) associated with the error from setpoint, control output increment and integrator gain was set to

\[
Q = \begin{bmatrix}
1E3 & 0 & 0 \\
0 & 1E2 & 0 \\
0 & 0 & 1E4 \\
\end{bmatrix}, R = \begin{bmatrix}
1E5 & 0 & 0 \\
0 & 1E5 & 0 \\
0 & 0 & 1E5 \\
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0 & 0.5 \\
\end{bmatrix}
\]  

The simulation represents a frequent load demand change when the power unit is in Automatic Generation Control (AGC) mode [11]. The operating point changes from #3 to #5 and then to #2. Fig. 3 presents the good performance of multiple-model predictive control and Fig.4 shows the output of the predictive controller. The probability that each model describes the plant at current sampling point is depicted on Fig. 5.
7 Conclusion

In the paper, a multiple-model predictive control methodology is applied to a boiler-turbine coordinated system. The correct model at the current sampling point is estimated using the residuals provided by a bank of Kalman filters. The obtained state is in the form of a mixture of states provided by the filters. The parameter of the linearized model and the states are then used within the predictive control approach for prediction of the future plant behavior. The simulation shows that it can be applied to the boiler-turbine coordinated system effectively and the performance can be improved by increase the number of the linear models.

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