Design A STATCOM supplementary Controller for Stability Studies using various state feedback algorithm

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Abstract: - The authors present a state feedback control approach to the Single Infinite bus Machine incorporating a static synchronous compensator (STATCOM). The proposed controllers’ designs are based on a linear time invariant model of the plant and state feedback scheme. First, the linear mathematical model of STATCOM is derived. Then, using polynomial algorithm and pole placement algorithm, a state feedback control law is derived. The proposed control strategy is tested on SMIB system, by digital computer simulations using matlab/simulink program for various types of loads and/or disturbances. Comparison of these results with those methods and without controller establishes the elegance of these control approaches.

Key Words: - STATCOM, State feedback, damping controller, ITAE and Low frequency oscillations.

1 Introduction

Power system oscillations are as a result of lack of sufficient damping torque at the generators rotors. This situation may happen as a result of heavy load in the lines, weak interconnection, high gain excitation systems etc [1]. The oscillation of the generators rotors cause the oscillation of other power system variables such as transmission line active and reactive powers, bus voltage, bus frequency, etc. Depending on the number of generators involved the frequency of the oscillation is usually between 0.1 and 2Hz [2]. There are several types of oscillations: Local mode, Inter area mode, Control mode and Torsion mode [3]. Many devices can be used as damping controllers for power system oscillation such as Statcom devices, power system stabilizers (PSS), HVDC links, Thyristor controller series capacitor, Statcom etc. PSS is applied on selected generators to damp the local mode oscillation and sometimes inter-area mode oscillation but supplementary controller is better for inter-area mode oscillation applied to the FACTS devices [4]. In most cases the design of these controllers are based on a linearised model which, for wide range of operating points and under large disturbances, will not provide satisfactory performance[4-5].

STATCOM is a voltage source converter FACTS device similar with SVC that is usually used for voltage regulation. It can also be used to improve power system stability by injecting reactive power to the network [6-9]. Evaluation study of STATCOM on stability enhancement has been introduced in [10].

Liner quadratic regulator (LQR) provides an optimal control law for a linear system. It’s a control strategy based on minimizing a quadratic performance index [11]. A wide attention has been directed for investigating the use state feedback controller by different authors like [12-14]. In this paper, a state feedback controller has been proposed for STATCOM supplementary controller. Several algorithms have been utilized for different appropriate operating conditions. Two different models have been taking into considerations (1) Single machine infinite bus and (2) three area five machine system.

The remaining sections of this paper are arranged as follows. Section 2 discusses statcom compensating model. The relevant equations for modeling the Statcom current and reactance are derived. In Section 3 an overview of state feedback concepts are discussed. Section 4 details the actual power system model and the results are presented in Section 5. The paper is concluded in Section 6.
The heading of each section should be printed in small, 14pt, left justified, bold, Times New Roman. You must use numbers 1, 2, 3, … for the sections' numbering and not Latin numbering (I, II, III, …)

2. Modeling the Power System with STATCOM

As shown in Fig. 1, the STATCOM is connected to the transmission line through a step-down transformer. The STATCOM consists of a three phase gate turn-off (GTO) – based voltage source converter (VSC) and a DC capacitor. It is a reactive current source with time delay, inductive current generated by STATCOM is assumed positive.

\[
\dot{I}_s = \left( K \left( V_{\text{ref}} - V \right) - I_s \right) / T
\]

Where \( K_r \) is gain of the stabilizing signal.

The voltage difference between the STATCOM bus AC voltage, \( v(t) \) and \( v_n(t) \) produces active and reactive power exchange between the STATCOM and the power system, which can be controlled by adjusting the magnitude \( V_0 \) and the phase \( \psi \) but since the two voltages are in phase the \( \psi \) is assumed to be 0°. The STATCOM is installed to maintain the AC bus voltage \( v_L(t) \) and enhance oscillation damping. The STATCOM control is implemented through the PWM amplitude modulation ratio \( m \) and phase angle \( \psi \) as shown by Fig. 3 [11],

\[
dV_{DC} = \frac{I_{DC}}{C_{DC}} = \frac{mk}{C_{DC}} I_s
\]

From Fig. 2, \( I_2 = I_1 - I_s \)

\[
\tilde{V}_1 = jx_1 \tilde{I}_1 + jx_2 \tilde{I}_2 + \tilde{V}
\]

Substituting eqn. 1 into eqn. 3 gives

\[

\tilde{V}_1 = jx_1 \tilde{I}_1 + jx_2 \tilde{I}_2 + \tilde{V} = j(x_1 + x_2) \tilde{I}_1 - jx_2 \tilde{I}_2 + \tilde{V}
\]

That is

\[
\begin{align*}
I_d &= \frac{E^* + x_1 I_s cos \theta - V sin \delta}{x_1 + x_2 + x_d'} \\
I_q &= \frac{V cos \delta + x_1 I_s sin \theta}{x_1 + x_2 + x_q}
\end{align*}
\]

\[
\tilde{V}_m = jx_1 \tilde{I}_2 + \tilde{V} = jx_2 (\tilde{I}_2 + j\tilde{I}_1) - jx_1 \tilde{I}_1 - jx_2 \tilde{I}_2 = V_{\text{mad}} + jV_{\text{mq}}
\]

Therefore

\[
\begin{align*}
V_{\text{mq}} &= (x_1 + x_2) E^* x_s + x_1 I_s cos \theta x_2 (x_1 + x_2') \\
V_{\text{mad}} &= (x_1 + x_2) V cos \delta + x_1 I_s sin \theta x_2 (x_1 + x_2')
\end{align*}
\]

\[
P = \frac{E^* V_m}{x_1 + x_2'} \sin \theta + \frac{V^2}{2} \frac{x_2' - x_2}{(x_1 + x_2)(x_1 + x_2')^2} \sin 2\theta
\]

The dynamics of the generator and the excitation system are expressed through a fourth order model given as

\[
\Delta \dot{\delta} = \omega_s \Delta \omega
\]

\[
\Delta \dot{\omega} = \left( -\Delta P_e + D \Delta \omega \right) / M
\]

\[
\Delta \dot{E}_q' = \left( -\Delta E_q' + (x_d' - x_q') \Delta I_d + \Delta E_{dq} \right) / T_{\omega o}
\]

\[
\Delta \dot{E}_{\mu} = \frac{K_u}{T_s} \Delta V_s - \frac{1}{T_3} \Delta E_{\mu}
\]

\[
\dot{I}_s = (K_u \Delta u - \Delta I_s) / T
\]

Let the input of STATCOM controller to be \( K_u \Delta u \) and \( K_{\omega o} \) are the gains of voltage and damping control loop, respectively, and \( V_{\text{ref}} \) is the reference voltage of the
STATCOM regulator while Kr Gain of the stabilizing signal. Therefore
\[ \Delta u = -K_r K_s \Delta \omega + K_r K_s \Delta \omega - K_r K_s \Delta \omega' - K_r K_s \Delta \omega'' \] (17)

Where \( K_r \), \( K_s \) are linearization constants derived in Appendix A. The above linearising procedure yields the following linearised power system model

\[
\begin{bmatrix}
\Delta \omega \\
\Delta \omega' \\
\Delta \omega'' \\
\Delta \omega'''
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
-K_r & -D & -K_s & -K_s \\
K_r & K_s & K_s & K_s \\
K_r & K_s & K_s & K_s
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta \omega' \\
\Delta \omega'' \\
\Delta \omega'''
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta \omega' \\
\Delta \omega'' \\
\Delta \omega'''
\end{bmatrix}
\] (18)

3. State Feedback Controller

3.1 Pole placement with state feedback

Assume that the single-input system dynamics are given by
\[ \dot{x}(t) = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t) + Du(t). \] (19)

Assume a control law of the form:
\[ u(t) = Fx(t) \] or \[ u(t) = r(t)Fx(t) \] (20)

Is called state feedback and brings system to the form [15]
\[ x(t) = (A - BF)x(t) + Br(t), \]
\[ y(t) = (C - DF)x(t) + Dr(t) \] (21)

Closed loop transfer function

\[ \frac{r}{y} = \frac{(S-I-A_F)^{-1}B}{C} \]

Fig 3

We have:
\[ s(sI-A)^{-1}B(r-Fx) \Leftrightarrow sx-Ax = Br-BFx \] (22)

Therefore, remembering that \( y = Cx \), the closed loop transfer function is
\[ s_f(s) = C(sI-A_f)^{-1}B \text{ where } A_f = A-BF. \] (23)

Closed loop characteristic polynomial (assuming realization is minimal):
\[ \chi_{cl}(s) = \text{det}(sI-A_f) \] (24)

In brief

Let \( (A,B) \) be controllable, then for an arbitrary polynomial
\[ \chi_{cl}(s) = s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1s + \alpha_0 \] (25)

There exists state feedback gain \( K \) so that \( \chi_{cl}(s) \) is closed loop characteristic polynomial, i.e.,
\[ \chi_{cl} = \text{det}(sI-A_F) \] (26)

The matrix \( A_f = (A - BK) \) dictates not only the stability of the closed loop system, but also the response of the rotor angle or speed output. Thus, the controller gain matrix, \( K \), must be designed such that we meet the stability and performance criteria. This is done first by means of placing the closed loop poles at desired locations in the left half of the complex plane. \( K \) can be determined using the following: (i) Equating of coefficients (ii) Pole placement algorithm

**Polynomial Placement**

It is evident that with higher order systems, arbitrary pole placement can become tedious, at best. One method for placing poles is to employ the polynomial approximation like ITAE and Butterworth pattern discussed by Tewari [15]. Though Butterworth pattern is well suited for higher order systems, but this pattern proved very successful for this second order system. In this pattern, the poles are placed on a circle centered at the origin with a radius, \( R \) corresponding to a settling time of one second.

Another approach is to select the poles to match the nth polynomial that was designed to minimize the ITAE “integral of the time multiplied by the absolute value of the error”
\[ J_{ITAE} = \int_0^\infty t|e(t)|dt \] in response to a step function.

**Pole placement algorithm**

Actually it can be found as \( K = f_v^T \) where the elements of \( F \) are selected and the process of finding \( V \) is as follows [16]:

Let the characteristics equation of the system matrix be
\[ s^n + a_1s^{n-1} + a_2s^{n-2} + \ldots + a_n \]

For closed loop purpose, it is required to move the eigenvalue of the uncompensated system characteristics equation as soon as eigenvalue of the closed system
\[ \gamma_1, \gamma_2, \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n \] as soon as \( \gamma_1, \gamma_2, \gamma_3, \gamma_1, \gamma_2, \gamma_3, \ldots \) are specified the desired closed loop characteristics equation is found from

\[ P(s) = \prod_{i=1}^{n}(s-\gamma_i)(s-\gamma_2) \ldots (s-\gamma_n) \] (27)

The difference between the open-loop Xtics polynomial and that of the desired closed loop Xtics polynomial is \( D(s) \).

\[ D(s) = P(s) - A(s) \] (28)

Then we defined a vector \( d \) whose element the coefficient are of

\[ d = [(p-a_1), (p-a_2), \ldots, (p-a_n)] \] (29)

Select a vector \( F \) such that the element of the matrix

\[ Q = \begin{bmatrix} BF; ABF; A^2 BF; \ldots; A^{n-1} BF \end{bmatrix} \] (31)

Is completely state controllable, then the vector \( V \) can be found as

\[ V = Q^{-1}X^{-1}d^T \] (32)

Where \( X \) is a Toeplitz matrix defined as

\[
X = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
a_1 & 1 & 0 & 0 \\
a_2 & a_1 & 1 & 0 \\
& \ddots & \ddots & \ddots \\
& & \ddots & \ddots \\
& & & a_{n-1} & a_{n-2} & 1
\end{bmatrix}
\] (33)

4. Results and Discussion

The effectiveness of the proposed methods was tested on single machine infinite bus system. To assess the effectiveness of the proposed methods, two different loading conditions are considered which name case 1 and case 2. The results for the systems are presented in what follows:

Case 1 (normal loading): Operating points of \( \text{SIMB}= \text{Pe}=1.0 \text{pu} \) at unity pf.; \( \text{Vt}=1.0 \text{ p.u.} \)

Case 2 (heavy loading): Operating points of \( \text{SIMB}= \text{Pe}=1.2 \text{pu} \) at 0.8 pf.; \( \text{Vt}=1.2 \text{ pu.} \)

4.1 Single Machine infinite bus with STATCOM

This section presents a comparison of the responses obtained by the three proposed method based on pole placement techniques, PID controller tuning based on pole-placement technique produce optimum controller functions for linear systems designed for case 1 and case 2 operating points. By assigning three poles of the compensated closed loop system the appropriate values of \( K_P, K_I \) and \( K_D \) has been obtained. The location of the dominant eigenvalues were selected to be \(-1.8561 \pm j8.2953\), corresponding to the damping ratio of 0.2186, the PID controller parameter for the nominal operating point (case 1) is obtained to be: \( K_P=4.95; K_I=296.3 \) and \( K_D=113.23 \). While for pole algorithm described in section 3 the controller parameter are obtained as \([K_1, K_2, K_3, K_4, K_5] = [-0.9680 \ 63.7854 \ -27.6727 \ -0.1683 \ 0.1585]\). For ITAE method the controller parameter under normal condition (case 1) is give as: \([K_1, K_2, K_3, K_4, K_5] = [0.57; 0.69; 263; 0.43; 1.707]\). Figure 4 to figure 6 shows the responses under the two different conditions.
5 Conclusions

In this paper, STATCOM Controller based on state feedback concepts are proposed for damping oscillations and the effectiveness of the proposed control methods are compared within themselves under some disturbances. The controllers are tested on single machine infinite bus System From the results it can be concluded that the state feedback based on ITAE produces no steady state error and acceptable overshoot under some disturbances.

References

[18] Chun, L., Qirong, J., Xiaorong, X. and Zhonghong, W., Rule-based control for STATCOM to increase power system stability, Power System Technology,
APPENDIX

Generator: $H = 3.542$, $D = 0$, $X_d = 1.7572$, $X_q = 1.5845$, $X'_d = 0.4245$, $X'_q = 1.04$, $T'do = 6.66$, $T'qo = 0.44$, $R_a = 0$, $\delta_0 = 44.370$.

Exciter: $K_A = 400$, $T_A = 0.025$ s

Transmission line: $R = 0$, $X_L = 0.8125$, $X_T = 0.1364$, $X_{TH} = 0.13636$, $G = 0$, $B = 0$;