One approach for decorrelation of multispectral images, based on hierarchical adaptive PCA

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Abstract: - In the paper is presented one new approach for efficient decorrelation of multispectral images, based on hierarchical adaptive Principal Component Analysis. The research is aimed at the efficient compression of multispectral images (MS), whose resolution and band number in some application areas are extremely high. Together with this, also grows up the needed volume of the corresponding image databases.

Key-Words: - Image processing, Image segmentation, Image contents analysis, Lossless image compression, Histogram modification, Inverse pyramid decomposition, Lossy image compression.

1 Introduction

The contemporary research in different application areas sets the task of the efficient archiving of MS images. For this, in most cases is necessary to process several images of the same object. MS images are characterized by very high spatial, spectral, and radiometric resolution and, hence, by ever-increasing demands of communication and storage resources. Such demands often exceed the system capacity like, for example, in the downlink from satellite to Earth stations, where the channel bandwidth is often much inferior to the intrinsic data rate of the images, some of which must be discarded altogether. In this situation the high-fidelity image compression is a very appealing alternative. As a matter of fact, there has been intense research activity on this topic [4, 5, 7, 14, 15, 19], focusing, particularly, on transform-coding techniques, due to their good performance and limited computational complexity. Linear transform coding, however, does not take into account the nonlinear dependences existing among different bands, due to the fact that multiple land covers, each with its own interband statistics, are present in a single image. Based on this observation, a class-based coder was proposed in [4] that addresses the problem of interband dependences by segmenting the image into several classes, corresponding as much as possible to the different land covers of the scene. As a consequence, within each class, pixels share the same statistics and exhibit only linear interband dependences, which can be efficiently exploited by conventional transform coding. Satellite-borne sensors have ever higher spatial, spectral and radiometric resolution. With this huge amount of information comes the problem of dealing with large volumes of data. The most critical phase is on-board the satellite, where acquired data easily exceed the capacity of the downlink transmission channel, and often large parts of images must be simply discarded, but similar issues arise in the ground segment, where image archival and dissemination are seriously undermined by the sheer amount of data to be managed. The reasonable approach is to resort to data compression, which allows reducing the data volume by one and even two orders of magnitude without serious effects on the image quality and on their diagnostic value for subsequent automatic processing. To this end, however, is not possible to use the general purpose techniques as they do not exploit the peculiar features of multispectral remote-sensing images, which is why several ad hoc coding schemes have been proposed in recent years. The transform coding is one of the most popular approaches for several reasons. First, transform coding techniques are well established and deeply understood; they provide excellent performances in the compression of images, video and other sources, have a reasonable
complexity and besides, are at the core of the famous standards JPEG and JPEG2000, implemented in widely used and easily available coders. The common approach for coding MS images [3, 8] is to use some decorrelating transforms along the spectral dimension followed by JPEG2000 on the transform bands with a suitable rate allocation among the bands. Less attention has been devoted to techniques based on vector quantization (VQ) because, despite its theoretical optimality, VQ is too computationally demanding to be of any practical use. Nonetheless, when dealing with multiband images, VQ is a natural candidate, because the elementary semantic unit in such images is the spectral response vector (or spectrum, for short) which collects the image intensities for a given location at all spectral bands. The values of a spectrum at different bands are not simply correlated but strongly dependent, because they are completely determined (but for the noise) by the land covers of the imaged cell. This observation has motivated the search for constrained VQ techniques [13], which are suboptimal but simpler than full-search VQ, and show promising performances. MS images require large amounts of storage space, and therefore a lot of attention has recently been focused to compress these images. MS images include both spatial and spectral redundancies. Usually we can use vector quantization, prediction and transform coding to reduce redundancies. For example, hybrids transform/VQ coding scheme is proposed [13]. Instead, Karhunen-Loeve transform (KLT) is used to reduce the spectral redundancies, followed by a two-dimensional (2D) discrete cosine transform (DCT) to reduce the spatial redundancies [5]. A quad-tree technique for determining the transform block size and the quantizer for encoding the transform coefficients was applied across KLT-DCT method [19]. In [13.] and [14] the researchers use a wavelet transform (WT) to reduce the spatial redundancies and KLT to reduce the spectral redundancies, and then encoded using the 3-dimensional (3D) SPIHT algorithm [9]. The state-of-the-art analysis shows that despite of the vast investigations and various techniques used for the efficient compression of MS images, a recognized general method able to solve the main problems is still not created.

One of the most efficient methods for decorrelation and compression of groups of MS images is based on the KLT, also known as transform of Hotelling, or PCA [1 - 4, 6, 11, 12]. For its implementation the pixels of same spatial position in a group of N MS images compose an N-dimensional vector. The basic difficulty of the PCA implementation is related to the large size of the covariance matrix. For the calculation of its eigenvectors is necessary to calculate the roots of a polynomial of nth degree (characteristic equation) and to solve a linear system of N equations. For large values of N, the computational complexity of the algorithm for calculation of the transform matrix is significantly increased.

One of the possible approaches for reduction of the computational complexity of PCA for N-dimensional group of MS images is based on the so-called “hierarchical adaptive PCA” (HAPCA). Unlike the famous hierarchical PCA (HPCA) [24], this transform is not related to the image sub-blocks, but to the whole image from one MS group. For this, HPCA is implemented through dividing the MS images into groups of length, corresponding to their correlation range. Each group is divided into subgroups of 3 MS images each, on which is applied adaptive PCA (APCA), of size 3×3 [16]. This transform is performed using equations, which are not based on iterative calculations, and as a result, they have lower computational complexity. To obtain decorrelation for the whole group of MS images is necessary to use APCA of size 3×3, which to be applied in several consecutive stages (hierarchical levels), with rearranging of the obtained intermediate eigen images after each stage. In result is obtained a decorrelated group of eigen MS images, on which could be applied other combined approaches to obtain efficient compression through lossy or lossless coding.

The paper is arranged as follows: in Section 2 is presented the approach for coding MS images through hierarchical APCA, in Section 3 is given one example for APCA with a 3×3 matrix and Section 4 is the Conclusion.

2 Principle for coding MS images through hierarchical APCA

The group of MS images is sub-divided into smaller groups (GOP) of 9 images each, for which is supposed that they are highly correlated. On the other hand, each GOP is further divided into 3 subgroups. As it is shown on Fig. 1, on each sub-group of 3 MS images from the first hierarchical level of HAPCA is applied APCA with matrix of size 3×3. In result are obtained 3 eigen images, colored in yellow, blue and green correspondingly. After that, the eigen images are rearranged so that the first subgroup of 3 eigen images to comprise the first images from each group, the second group of 3 eigen images – the second images from each group, etc. For each GOP of 9 intermediate eigen images in the first
hierarchical level is applied in similar way the next APCA, with a 3×3 matrix, on each sub-group of 3 eigen values. In result are obtained 3 new eigen images (i.e. the eigen images of the group of 3 intermediate eigen images), colored in yellow, blue, and green correspondingly in the second hierarchical level. Then the eigen images are rearranged again so, that the first group of 3 eigen images to contain the first images from each group before the rearrangement; the second group of 3 eigen images – the second image before the rearrangement, etc. At the end of the processing is applied a reduction of the 3 eigen images of lowest energy in the GOP, noted as R1, R2 and R3 correspondingly. Their removal practically does not influence the quality of the restored MS images from the original GOP, obtained through inverse HAPCA with a 3×3 matrix. In result is obtained compression ratio of 3 for the group of MS images, retaining the ability for their high-quality restoration after decompression, because HAPCA is reversible.

For the further compression of the original group of eigen MS images could be used for example, the “branched” inverse pyramid decomposition (BIPD) [17, 18] with nonlinear pre- and post-processing, based on the pixel-by-pixel "Adaptive Histogram Matching" (AHM) transform.

3 Example
Calculation of a sub-group of eigen MS images through APCA with a 3×3 matrix.

From each 3 digital MS images of S pixels each, shown on Fig. 2, are calculated the vectors $\hat{C}_s = [C_{1s}, C_{2s}, C_{3s}]^t$ for $s = 1, 2, \ldots, S$ (on the figure are shown the vectors for the first 4 pixels only, respectively,

- $\hat{C}_1 = [C_{11}, C_{21}, C_{31}]^t$,
- $\hat{C}_2 = [C_{12}, C_{22}, C_{32}]^t$,
- $\hat{C}_3 = [C_{13}, C_{23}, C_{33}]^t$ and
- $\hat{C}_4 = [C_{14}, C_{24}, C_{34}]^t$.

Each vector is then transformed into corresponding vectors $L_s = [L_{1s}, L_{2s}, L_{3s}]^t$ through APCA with the matrix $\Phi$ of size 3×3. Its elements $\Phi_{ij}$ are defined below [3]:

1. The covariance matrix $[K_C]$ of size 3×3 for vectors $\hat{C}_s$ is calculated:

$$[K_C] = \left[ \frac{1}{S} \sum_{s=1}^S \hat{C}_s \hat{C}_s^t - \bar{m}_C \bar{m}_C^t \right] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix},$$

2. The elements of the mean vector $\bar{m}_C$ and of the matrix $[K_C]$ are defined in accordance with relations below:

$$\bar{C}_i = E(C_{is}), \quad \bar{C}_2 = E(C_{2s}), \quad \bar{C}_3 = E(C_{3s}),$$

$$k_{11} = E(C_{1s}) - (\bar{C}_1)^2,$$

$$k_{22} = E(C_{2s}) - (\bar{C}_2)^2,$$

$$k_{33} = E(C_{3s}) - (\bar{C}_3)^2,$$

$$k_{12} = E(C_{1s}C_{2s}) - (\bar{C}_1)(\bar{C}_2),$$

$$k_{23} = E(C_{2s}C_{3s}) - (\bar{C}_2)(\bar{C}_3),$$

$$k_{13} = E(C_{1s}C_{3s}) - (\bar{C}_1)(\bar{C}_3).$$

3. The eigen values $\lambda_1, \lambda_2, \lambda_3$ of the matrix $[K_C]$ are defined in accordance to the solution of the characteristic equation:

$$| \det | k_{ij} - \lambda \delta_{ij} | = \lambda^3 + a \lambda^2 + b \lambda + c = 0,$$

where:

$$\delta_{ij} = \begin{cases} 1, & i = j, \\
0, & i \neq j \end{cases},$$

$$a = -(k_1 + k_2 + k_3),$$

$$b = k_1k_2 + k_1k_3 + k_2k_3 - (k_1^2 + k_2^2 + k_3^2),$$

$$c = k_1k_2^2 + k_1k_3^2 + k_2k_3^2 - (k_1k_3 + 2k_2k_3),$$

Since the matrix $[K_C]$ is symmetric, its eigen values are real numbers. For their calculation could be used the equations of Cardano for “casus irreducibilis” (i.e., the so-called “trigonometric solution”):

$$\lambda_i = \sqrt[3]{\frac{|p|}{3}} \cos \left( \varphi \left( \frac{\pi}{3} \right) \right) - \frac{a_i}{3},$$
\[
\lambda_2 = -2 - \frac{p}{3} \cos \left( \frac{\varphi + \pi}{3} \right) \frac{a}{3}, \quad \lambda_3 = -2 - \frac{p}{3} \cos \left( \frac{\varphi - \pi}{3} \right) \frac{a}{3} \tag{8}
\]

for \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0 \),

\[
q = 2a(3/3)^{3/2} - \frac{(a^3/3) + b < 0}{3}.
\]

\[\varphi = \arccos \left[ -q / 3 \right] \right] \tag{9}\]

The components of vectors \( \tilde{L}_s = [L_{1s}, L_{2s}, L_{3s}]^t \) could be processed in various ways (such as for example: decimation and interpolation, filtration, orthogonal transforms, quantization, etc.). In result are obtained the corresponding vectors \( \tilde{L}_s = \psi(L_s) = [\psi_1(L_{1s}), \psi_2(L_{2s}), \psi_3(L_{3s})]^t \) with components \( L_{1s}, L_{2s}, L_{3s} = \psi_1(L_{1s}), L_{2s} = \psi_2(L_{2s}), L_{3s} = \psi_3(L_{3s}) \) and in result are obtained the decoded vectors \( \tilde{L}_s = [\tilde{L}_{1s}, \tilde{L}_{2s}, \tilde{L}_{3s}]^t \).

Using the inverse AРСА, vectors \( \tilde{L}_s \) are transformed into vectors \( \tilde{C}_s = [\tilde{C}_{1s}, \tilde{C}_{2s}, \tilde{C}_{3s}]^t \):

\[
\begin{bmatrix}
\tilde{C}_{1s} \\
\tilde{C}_{2s} \\
\tilde{C}_{3s}
\end{bmatrix}
= \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{12} & \Phi_{22} & \Phi_{23} \\
\Phi_{13} & \Phi_{23} & \Phi_{33}
\end{bmatrix}
\begin{bmatrix}
\tilde{L}_{1s} \\
\tilde{L}_{2s} \\
\tilde{L}_{3s}
\end{bmatrix}
= \tilde{C}_s
\tag{17}
\]

for \( s = 1, 2, \ldots, S \).

Here the matrix of the inverse AРСА is:

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{12} & \Phi_{22} & \Phi_{23} \\
\Phi_{13} & \Phi_{23} & \Phi_{33}
\end{bmatrix}
= \Phi = [\Phi_1, \Phi_2, \Phi_3]. \tag{18}
\]

For the restoration of vectors \( \tilde{L}_s \) are needed not only the vectors, but also the elements \( \Phi_s \) of the matrix \( [\Phi] \), and the values of \( \tilde{C}_1, \tilde{C}_2, \tilde{C}_3 \) as well. The total number of these elements could be reduced representing the matrix \( [\Phi] \) as the product of matrices \( [\Phi_1(\alpha)], [\Phi_2(\beta)], [\Phi_3(\gamma)] \), and rotation around coordinate axes for each transformed vector in angles \( \alpha, \beta \) and \( \gamma \) correspondingly:

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{12} & \Phi_{22} & \Phi_{23} \\
\Phi_{13} & \Phi_{23} & \Phi_{33}
\end{bmatrix}
= \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{13} & \Phi_{23} & \Phi_{33}
\end{bmatrix}
= \Phi_{1(\alpha)} \Phi_{2(\beta)} \Phi_{3(\gamma)} = [\Phi(\alpha, \beta, \gamma)]
\tag{19}
\]

where:

\[
\begin{bmatrix}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
\cos \beta & 0 & -\sin \beta \\
0 & 1 & 0 \\
\sin \beta & 0 & \cos \beta
\end{bmatrix}; \quad (20)
\]

\[
\begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

In this case the elements of the matrix \([\Phi]\) are represented by the relations:

\[
\begin{align*}
\Phi_{11} &= \cos \alpha \cos \beta \cos \gamma - \sin \alpha \sin \gamma; \\
\Phi_{12} &= -(\cos \alpha \cos \beta \sin \gamma + \sin \alpha \cos \gamma); \\
\Phi_{13} &= -\cos \alpha \sin \beta \Phi_{21} = \sin \alpha \cos \beta \cos \gamma + \cos \alpha \sin \gamma; \\
\Phi_{22} &= -\sin \alpha \cos \beta \sin \gamma + \cos \alpha \cos \gamma; \\
\Phi_{23} &= -\sin \alpha \sin \beta \Phi_{31} = \sin \beta \cos \gamma; \\
\Phi_{32} &= -\sin \beta \sin \gamma \Phi_{33} = \cos \beta.
\end{align*}
\]

The inverse APCA matrix is defined by the relation:

\[
[\Phi]^{-1} = [\Phi_3(-\gamma)][\Phi_2(-\beta)][\Phi_1(-\alpha)]. \quad (22)
\]

Then, for the calculation of the elements of the inverse matrix \([\Phi]^{-1}\) is enough to know the values of the rotation angles \(\alpha, \beta, \gamma\), defined by the relations:

\[
\begin{align*}
\alpha &= -\arcsin \left( \Phi_{21} / \sqrt{1-\Phi_{33}} \right); \\
\beta &= \arccos \left( \Phi_{33} \right); \\
\gamma &= \arccos \left( \Phi_{31} / \sqrt{1-\Phi_{33}^2} \right). \quad (23)
\end{align*}
\]

In result, the number of the needed values for the calculation of the matrix \([\Phi]^{-1}\) is reduced from 9 down to 3, i.e. 3 times reduction. The elements \(L_{1s}, L_{2s}, L_{3s}\) for \(s=1, 2, ..., S\) comprise the pixels of the first, second and third eigen image in the sub-group of MS images \(C_{1s}, C_{2s}, C_{3s}\).

4 Conclusions

The basic qualities of the offered HAPCA for processing a group of MS images are:

1. Lower computational complexity than PCA for the whole GOP of MS images, due to the lower complexity of APCA compared to the case, for which the calculation of the PCA matrix are used numerical methods[16];

2. Efficient coding of GOP of MS images, for which the compression coefficient is \(CR=3\), with retained visual quality of the restored images;

3. HAPCA could be combined with BIPD with non-linear pre- and post-processing, based on HM, or on any kind of similar processing, with which could be obtained further (much higher) compression of the processed images;

4. HAPCA could be also used for high-efficient lossless compression of MS images in the case, when there is no reduction for part of the eigen images;

5. There is also a possibility for further development of the HAPCA algorithms, through: use of Integer PCA for lossless coding of MS images by analogy with [19, 20], compression of video sequences, obtained from stationary TV camera; APCA for a matrix of size \(4 \times 4\) as in [21], but without using numerical methods; compression of multi-view images [18], image fusion [23], face recognition [25], etc.

The investigation will be continued with modelling and experiments. For this will be used large image databases with MS images of various kinds [22]. The so obtained results will be evaluated and compared to other similar algorithms and will be investigated possible new application areas: remote investigation of the earth surface, medical diagnostic, automatic manufacturing control, defence, etc.

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References:


Fig. 1. Block diagram of 2-level Hierarchical adaptive PCA 3×3, for a set of MS images