Streamwise vortices in channel flow with a corrugated surface

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Abstract: Visualization of streamwise vortices in channel flow with a corrugated surface has been conducted using smoke-wire technique. The use of 3 nylon strings with uniform spanwise spacing installed prior to and normal to the leading edge of the corrugated surface has induced low velocity streaks in the close field downstream of each string which subjected to centrifugal instability. It gives rise counter-rotating streamwise vortices with the spanwise wavelength equal to the spanwise spacing of the strings. The visualization study has demonstrated that the appearance of mushroom-like structures could be attributed to the evolution of the most amplified disturbances which has reached finite amplitude. It would preserve longer than those which less amplified. Despite the absence of quantitative data which can help elaborate the phenomenon further, the visualization results have successfully described the formation and the evolution of the streamwise vortices in the channel flow with a corrugated surface. To a certain extent, the current study describes the role of the corrugated surface in the occurrence of streamwise vortices.

Key-Words: smoke-wire, corrugated surface, streamwise vortices, flow instability, flow visualization

1 Introduction

The occurrence of counter-rotating streamwise vortices in the flow increases the heat transfer coefficient due to its spiraling motion of the fluids that induces an increased exchange of hot and cold fluid to and from the heat exchangers [1]. To gain benefit from this phenomenon, the use of corrugated surface as a passive method to enhance heat transfer had been widely used [2]. Thus, a comprehensive understanding on the evolution of the counter-rotating streamwise vortices on the corrugated surface would be essential for devising such methodology.

Based on the study on the occurrence of streamwise vortices in a channel of two corrugated surfaces, Nishimura et al. [3] found that such vortices mostly occurred at values of $2\varepsilon/H$ larger than 1, where $\varepsilon$ is the amplitude of the waviness and $H$ is a channel’s width. Nishimura et al. [3] also observed that for flow in such channels ($\varepsilon = 3.5\text{mm}$ and $\lambda = 28\text{mm}$ at Reynolds number of 168) the flow from the lower wall are arising at a discrete spanwise location that reveals the formation of a pair of streamwise vortices in the shape of mushroom-like structures which similar to Taylor-Görtler vortices. The formation of such vortices is believed to take place through the fusion of transverse vortex remnants in the upstream wave and the transverse vortex in the downstream [4]. At Reynolds number of 300, the centrifugal instability leads to the formation of streamwise vortices along the whole channel [4].

The presence of corrugated wall in a Couette flow forces the flow to change direction resulting in the creation of a centrifugal force field whose magnitude is dependent on the amplitude and the wavelength of the corrugation [5]. Its effect differs from destabilizing over the concave to stabilizing over the convex part of the corrugation [5]. The combination of the stabilizing and the destabilizing effects of the field play an important role in the occurrence of the instability. Regardless of the Reynolds number, the vortices will not be destabilized if the amplitude of the corrugation in the channel is smaller than the critical value. The vortices will also not be observed if the amplitude of
the corrugation exceeded the well-defined range [6]. From linear stability analysis of flow in a channel bounded by wavy walls, Cabal et al. [7] found that the wall waviness gives rise to instability that manifest itself through generation of streamwise vortices.

Floryan [5] has formulated the variation of the global critical stability conditions of the Couette flow over a corrugated wall as a function of the amplitude of the waviness that can be expressed as

\[ \ln(Re_c) = -1.2801 \ln(S) + 2.9539 \]  \hspace{2cm} (1)

In Eq. 1, the Reynolds number \( Re \), and \( S \) are respectively defined as

\[ Re = U \cdot (0.5H)/\nu \]  \hspace{2cm} (2)

and

\[ S = \varepsilon/H \]  \hspace{2cm} (3)

where \( H \) is the height of the channel and \( \nu \) the fluid kinematic viscosity.

The aim of this study is to study qualitatively the occurrence of counter-rotating streamwise vortices in channel flow with a corrugated surface. An instability chart of Floryan [5] was used as a guide for the experimental conditions.

2 Description of experiments

A small blow-down wind tunnel having cross-section of 160mm by 160mm that capable in providing a maximum speed of 35 m/s with a turbulent intensity level of 0.25% and the contraction ratio of 9, was used for the current study.

A commercial available corrugated plate having the dimension (\( l \times w \times t \)) of 312mm x 158mm x 3mm which consists of 4 wave number of corrugation with a wavelength \( \lambda \) of 78mm and amplitude \( \varepsilon \) of 7.5mm was used. At one end of this corrugated plate, a flat plate having dimension of 250mm x 158mm x 3mm was glued and used as a leading edge of the corrugated plate, while at the other end, the flat plate with dimension of 100mm x 158mm x 3mm was glued as the trailing edge of corrugated plate (Fig.1).

Smoke streaks were produced by heating up the stainless steel wire of 0.193mm diameter coated by paraffin oil that was dripped from an oil container located on the top of the test section. The stainless steel wire was inserted through the slit on the top wall of the wind tunnel. It is parallel to the corrugated plate that was inserted vertically in the test section of the wind tunnel. Since the wire Reynolds number \( Re_d \) is of about 50 at the velocity of 4 m/s, thus, it is within the \( Re_d \) range where there is no formation of the vortices in the flow downstream of the smoke-wire.

A DC power supply that provides a current in the range of 1 to 1.3 Amperes with the voltage ranging from 11 to 15 Volt was used to heat the coated wire. The amount of the DC power is dependent on the velocity of the air. At high velocity, the flow would provide the cooling effect to the heated wire that result in the use of higher DC power. A green laser light with the wavelength of 532 nm and the power of 300 mW (milliWatt) was used to create a laser sheet on the cross-section plane.

3 Results and discussion

The study on a natural development of streamwise vortices in a channel flow with corrugated surface was carried out by visualizing the flow at the velocity \( U \) of about 1.5 m/s at the channel’s width of 110 mm that corresponds to \( S \) of 0.068, and Reynolds number \( Re \) of 5300. The result presented in Fig. 2 shows the formation of several vortical structures on the vertical plane normal to the 1\textsuperscript{st} peak of the wavy wall that suggests the occurrence of streamwise vortices.

A sinusoidal pattern in the cross-sectional plane between the 1\textsuperscript{st} peak and 2\textsuperscript{nd} peak as depicted in Fig. 3, could estimate the wavelength of the streamwise vortices.
vortices observed. However, it seems that the spanwise wavelengths of the vortices are not uniform. A typical turbulent flow is evident in the visualization results presented in Figs. 4 and 5 where the streamwise vortices have collapsed completely.

The ability to pre-set the wavelength of Görtler vortices as demonstrated by Mitsudharmadi et al. [8], make it possible to further study the growth of these streamwise vortices. At the current study, the spanwise wavelength of the vortices was pre-set using 3 nylon strings of 0.7mm diameter spaced 28mm from each other which were placed prior to and normal to the leading edge of the corrugated plate.

<table>
<thead>
<tr>
<th>$U$ (m/s)</th>
<th>$Re$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>1252</td>
<td>0.25</td>
</tr>
<tr>
<td>1.8</td>
<td>1734</td>
<td>0.25</td>
</tr>
<tr>
<td>2.2</td>
<td>2119</td>
<td>0.25</td>
</tr>
<tr>
<td>2.9</td>
<td>2794</td>
<td>0.25</td>
</tr>
<tr>
<td>3.2</td>
<td>3083</td>
<td>0.25</td>
</tr>
<tr>
<td>3.4</td>
<td>3276</td>
<td>0.25</td>
</tr>
<tr>
<td>3.6</td>
<td>3468</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The velocity of the flow was varied from 1.3 m/s to 3.6 m/s with a laser-sheet normal to the 1st peak of the corrugation. Since the result presented in Figs 3-5 was conducted at Reynolds number of 5300, the width of the channel was set at 30mm which corresponds to $S = 0.25$ to ensure that the Reynolds number would not exceed 5300 for the flow velocity of 3.6 m/s. The flow conditions of the test are in the unstable region of Floryan instability chart and presented in Table 1.

For a given nominal flow velocity and string diameter, low velocity streaks were formed in the close field downstream of each string. It subsequently gives rise to a spanwise modulation of
streamwise velocity \( U \) with the wavelength equal to the spanwise spacing between the strings.

As illustrated in Fig. 1, before reaching the 1st peak of corrugation, the flow has to pass through the concave surface. Accordingly, the spanwise disturbances induced by the perturbation strings would be subjected to the centrifugal instability. The amplification to these disturbances by centrifugal instability would result in the formation of streamwise vortices with the spanwise spacing equal to the spanwise spacing of the strings.

As presented in Fig. 6, the formation of three peaks with the spanwise spacing equal to the spacing of the strings are evident. The formation of mushroom-like structure is clearly seen downstream of the first perturbation strings (the one on the left) that evidences the occurrence of counter-rotating streamwise vortices. It demonstrates the ejection of low momentum fluid from the wall close to the “mushroom’s stem” which returns back toward the wall in the region of maximum shear and gives shape of a mushroom hat.

While a similar pattern of the mushroom-like structures depicted at \( Re \) 3083, 3276, and 3468 (Fig. 6) may indicates that the disturbances have reached finite amplitude prior to turbulence, the structures at \( Re \) of 1252, 1734, 2119, and 2794 (Fig. 6), may suggest that the disturbances still evolve within the linear region of the primary instability in which the flow is dominated by the appearance of the wavy pattern as indicated by the formation of three peaks along the spanwise.  

![Fig. 8 Evolution of the vortices induced by 3 nylon strings of diameter 0.7 mm at S of 0.25 plotted at the instability chart of Floryan [5].](image)

Furthermore, the occurrence of different structures on the cross-section plane normal to the flow direction at \( Re \) of 3083, 3276, and 3468 (Fig. 6) could imply the different amplification rate experienced by the spanwise disturbances induced by the perturbation strings. The highest amplification rate seems to be experienced by that downstream of the first perturbation string (the left structure on Fig. 6) as indicated by the earlier occurrence of the mushroom-like structure.

A downstream evolution of the vortices at \( Re \) of 3083 (Fig. 7), evidence that the mushroom-like structure aligned with the 1st perturbation string preserves until the 2nd peak of corrugation while the other two structures have collapsed completely. An increased mixing due to the onset of turbulence could be attributed to the collapse of these spanwise structures. This verifies that the disturbance induced by the 1st string is most amplified which results in more prominent counter-rotating streamwise vortices that will preserve longer than those that less amplified.

The results presented in Fig. 7 for some extent may also suggest the role of the corrugated surface in the formation of counter-rotating streamwise vortices. To a certain extent, it could describe the role of the stabilizing and destabilizing effect from the corrugation to the occurrence of the instability which give rise the streamwise vortices.

As shown by Floryan’s instability chart (Fig. 8), the formation of the mushroom-like structures becomes more prominent as the \( Re \) increases at constant value of \( S \). Thus, the current visualization study has clearly demonstrated the evolution of counter-rotating streamwise vortices in unstable region of a channel flow with corrugated surface.

### 4 Conclusion

The formation of the counter-rotating streamwise vortices has been visualized by smoke-wire visualization technique. They are indicated by the formation of vortical structures on the vertical plane normal to the corrugated surface. The spanwise spacing of these vortices has been successfully controlled by placing perturbation strings prior to and normal to the corrugated wall. Using this method the spanwise wavelength of the vortices which are well inside the unstable region of the instability chart of Floryan [5], can be pre-set equal to the spacing between the strings. It enables further study on the evolution of induced spanwise disturbances which are subjected to the centrifugal instability. The most amplification rate from the centrifugal instability to such disturbances would give rise the more prominent counter-rotating streamwise vortices in the form of mushroom-like structures, which are coherent and dominating the
flow. These structures would preserve longer while the structures which are induced by the disturbances that less amplified would collapse earlier due to the increase of mixing prior to turbulence. The results of the current study also demonstrate that the instability chart of Floryan [5] could predict the unstable condition of the channel flow with a corrugated surface in which the streamwise vortices would exist.

Acknowledgement
The authors would like to thank to Temasek Laboratories at National University of Singapore for making this research work possible.

References: