Modeling and Simulation of the Double Tube Heat Exchangers. Case Studies

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Abstract: The double tube heat exchangers are used in industry as they have a simple design, operate at high temperatures and pressures in counter flow. The study of the mathematical model of these heat exchangers has proved the fact that the elaborated models are specifically to the types of exchangers and to the way the system is treated (with distributed parameters or with lumped parameters). The analysis of the double tube heat exchangers imposed the development of a model specifically to this type of exchanger. The elaborated model is based on the hypothesis of treating the heat exchanger as a system with concentrated parameters. The calculus of the output temperatures of the two fluids between which the heat transfer is realized uses data about the exchanger geometry and the values of the input temperatures and flow rates of the two flows. The elaborated model has been validated by comparing the results obtained by numerical simulation with the experimental data obtained by the authors. The model is going to be used in the study of the industrial double tube exchangers in the refineries.

Key-Words: double tube heat exchanger, modeling, simulation, non-linear equation, Newton-Raphson algorithm, laboratory data

1 Introduction
The double tube heat exchangers have a simple design, being made of two concentric tubes, where a fluid flows through the interior tube, while the other one flows through the ring-type space. This type of heat exchanger is used in various industrial domains, such as the food industry or the oil refining industry [1, 2]. The double tube heat exchangers present the following advantages: simple structure, operation in counter-flow, operation at relatively low flow rates, operation at high temperatures and pressures, low costs etc. The disadvantage of using these heat exchangers is related to low values of the global heat transfer coefficient that leads to large heat transfer areas [3, 4]. The mathematical modeling of the heat exchangers is treated extensively in literature [5, 6, 7, 8, 9]. The study of these models proved the fact that the elaborated mathematical models are specific to certain types of heat exchangers or treat the heat exchanger as a system with distributed parameters [5]. Due to this cause, for the industrial operation of these double tube exchangers, the authors had to develop their own model for this type of exchanger.

The elaborated model allows the determination of the output temperatures of the two fluids between which the heat transfer is realized, by using data about the exchanger geometry and the values of the input temperatures and flow rates of the two flows.

2 The mathematical modeling of the heat transfer in the double tube heat exchangers

2.1 The structure of the double tube heat exchanger
In figure 1-a is presented a double heat exchanger, having flows in counter-flow. The heat exchanger is characterized by four input values and two output values, figure 1-b. The input variables are the following: \( T_{h1}, Q_{hot} \) – the input temperature and the flow rate of the hot fluid, \( T_{c1}, Q_{cold} \) – the input temperature and the flow rate of the cold fluid. The output variables are represented by \( T_{h2} \) – the output temperature of the hot fluid and \( T_{c2} \) – the output temperature of the cold fluid.
2.2 The mathematical model of the double tube heat exchanger

For the elaboration of the mathematical model, the authors have considered the following simplifying hypothesis:
- The operation of the exchanger in at steady state regime;
- The heat transfer to the surrounding environment is neglected;
- The heat exchanger is considered a system with lumped parameters;
- The two flows are in a liquid phase and don’t change the phase.

The mathematical model of the heat exchanger has been developed by the authors in the paperwork [10] and contains an equation of the heat balance associated to the two material flows $Q_{hot}$ and $Q_{cold}$ as well as the expression of the transferred heat flow. An expression of the mathematical model is described by the system (1).

For the heat flow transferred in the heat exchanger, the global heat exchange coefficient, $k_{ed}$, has a known expression in literature, the relation (2) [3]

$$k_{ed} = \frac{1}{\alpha_{in} \frac{d_e}{d_i} + \frac{d_e}{d_i} \ln \frac{d_e}{d_i} + \frac{1}{\alpha_{out}}}.$$  \hspace{1cm} (2)

Within it, $\alpha_{in}$ represents the convection coefficient inside the internal tube, $\alpha_{out}$ - the convection coefficient in the ring-type space between the two tubes, $c_{p,hot}$ - the specific heat of the hot fluid, $c_{p,cold}$ – the specific heat of the cold fluid, $A$ – the heat transfer area of the exchanger.

The unknown variables of the system (3), the outlet temperature of the hot fluid $T_{h2}$ and the outlet temperature of the cold fluid, $T_{c2}$, are, at the same time, the output variables of the heat exchanger.

The functions $f_1$ and $f_2$ of the system (3) have the expressions defined by the relations (4) and (5).

$$f_1(T_{h2}, T_{c2}) = Q_{hot} \ c_{p,hot} \ (T_{h1} - T_{h2}) - Q_{cold} \ c_{p,cold} \ (T_{c2} - T_{c1});$$  \hspace{1cm} (4)

$$f_2(T_{h2}, T_{c2}) = Q_{hot} \ c_{p,hot} \ (T_{h1} - T_{h2}) - k_{ed} \ A \ \frac{(T_{h1} - T_{h2}) - (T_{h2} - T_{c1})}{\ln \frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}};$$  \hspace{1cm} (5)
The Jacobean matrix associated to the non-linear equation system (3) is given by

\[
J(X) = \begin{bmatrix}
\frac{\partial f_1}{\partial T_{h2}} & \frac{\partial f_1}{\partial T_{c2}} \\
\frac{\partial f_2}{\partial T_{h2}} & \frac{\partial f_2}{\partial T_{c2}}
\end{bmatrix}.
\]  

(6)

The authors have studied the estimation of the partial derivatives of the Jacobean matrix (6). The relations used to evaluate numerically of the partial derivatives have been tested by the authors, the general form of theses relations being presented in (7) [11].

\[
\frac{\partial f_i}{\partial x_j} = \frac{f_i(x_1^{(k)}, \ldots, x_j^{(k)} + h_j^{(k)}, \ldots, x_n^{(k)}) - f_i(X^{(k-1)})}{h_j^{(k)}},
\]

\[i = 1, \ldots, n; j = 1, \ldots, n.\]  

(7)

3 The adaptation of the mathematical model

The adaptation of the mathematical model of a heat exchanger contains the following operations:

a) Defining the two fluids that make the heat exchange and their position in relation to the structure of the heat exchanger.

b) Determining the approximation functions of the physical properties for the two fluids.

c) Specifying the geometrical data of the heat exchanger.

d) Selecting the criteria relations for the calculus of the convection coefficients of the two fluids in the flowing spaces, within the heat exchanger.

3.1 Defining the fluids that take part in the heat exchange

The mathematical model has been used in the simulation of a double tube heat exchanger, for which both the hot and the cold flow were represented by water.

3.2 The calculus of the physical properties of the two fluids

The relation for determining the heat transfer coefficient by convection uses the following physical properties of the fluid: density, specific heat, dynamic viscosity and heat conductivity. Since all these physical properties depend on temperature, these properties can be approximated trough polynomial functions of the form (8). As in the phase of defining the fluids that take part in the heat exchange, there has been selected the water both as hot and cold fluid, the physical properties will be calculated only for this fluid, the authors using Unisim simulation environment for calculating the water properties at various temperatures [12]. By using the polynomial regression, there have been determined each physical property of water, table 1.

\[
property = a_{0,i} + a_{1,i}T + a_{2,i}T^2 + a_{3,i}T^3 + a_{4,i}T^4.
\]  

(8)

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Property index</th>
<th>(a_{0,i})</th>
<th>(a_{1,i})</th>
<th>(a_{2,i})</th>
<th>(a_{3,i})</th>
<th>(a_{4,i})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, (\rho) [kg/m(^3)]</td>
<td>1</td>
<td>999.9</td>
<td>0.044</td>
<td>-0.007</td>
<td>4e-5</td>
<td>-1e-7</td>
</tr>
<tr>
<td>Heat capacity, (c_p) [J/kg °C]</td>
<td>2</td>
<td>4209.8</td>
<td>-2.1041</td>
<td>0.0328</td>
<td>-0.0001</td>
<td>-</td>
</tr>
<tr>
<td>Dynamic viscosity, (\mu \times 10^4) [kg/m s]</td>
<td>3</td>
<td>17.831</td>
<td>-0.5622</td>
<td>0.0103</td>
<td>-1e-4</td>
<td>4e-7</td>
</tr>
<tr>
<td>Thermal conductivity, (\lambda) [W/m m °C]</td>
<td>4</td>
<td>0.5507</td>
<td>0.0027</td>
<td>-1e-5</td>
<td>5e-9</td>
<td>9e-11</td>
</tr>
</tbody>
</table>
3.3 Specifying the geometry of the heat exchanger

The geometrical construction of the double heat exchanger is illustrated in figure 2. The main geometrical characteristics are: the interior and the exterior diameter of the interior tube, the interior and the exterior diameter of the exterior tube, the length tube.

![Figure 2: The geometrical data of the double tube exchanger](image)

3.4 The selection of the mathematical model for the calculus of the convection coefficients

For the calculus of the convection coefficients there have been used the Reynolds and Prandtl similitude criteria, as well as Nusselt criteria relation. In the specialized literature, there are more criteria relations of the type $Nu = c \cdot Re^a \cdot Pr^b$, applicable for straight tubes with a circular section.

On the other hand, in the case of flowing through a ring-type double tube section, the criteria relations are much more restrictive [6, 7]. The Reynolds similitude criterion for the fluid that flows in the interior tube is calculated with the relation

$$Re_i = \frac{w_i \rho_i d_{i1}}{\mu_i}.$$  \hspace{1cm} (9)

For the fluid that flows in the ring-type space, the Reynolds criterion, $Re_e$, will also use the relation (9), where the interior diameter of the tube $1$, $d_{i1}$, will be replaced with the equivalent hydraulic diameter

$$d_{eh} = d_{i2} - d_{e1}.$$  \hspace{1cm} (10)

The Prandtl similitude criterion for the interior fluid is calculated with the relation

$$Pr_i = \frac{c_{p,i} \mu_i}{\lambda_i},$$  \hspace{1cm} (11)

While for the fluid that circulates in the ring-type space, the Prandtl criterion has the expression

$$Pr_e = \frac{c_{p,e} \mu_e}{\lambda_e},$$  \hspace{1cm} (12)

For the calculus of the Nusselt criterion in the circular section of the interior tube, there have been used criteria relations special to the laminar, intermediate and turbulent flowing regime [2]:

$$Nu_i = \begin{cases} 
1.86 \left( Re_i Pr_i \frac{d_{i1}}{L} \right)^{0.33}, & Re_i < 2300 \\
0.023 Re_i^{0.8} Pr_i^{0.4} \left( 1 - \frac{6 \times 10^5}{Re_i^{1.8}} \right), & 2300 \leq Re_i < 10^4 \\
0.023 Re_i^{0.8} Pr_i^{0.4}, & Re_i \geq 10^4
\end{cases}.$$ \hspace{1cm} (13)
For the ring-type space, the Nusselt criterion has the expressions derived from [3]:

\[
Nu_e = \begin{cases} 
4.05 Re_e^{0.17} Pr_e^{1/3}, & Re_e < 2300 \\
1.86 \left( Re_e Pr_e \frac{d_{eh}}{L} \right)^{1/3} \left( 1 - \frac{6 \times 10^5}{Re_e^{1.8}} \right), & 2300 \leq Re_e < 10^4 \\
0.023 Re_e^{0.8} Pr_e^{0.4}, & Re_e \geq 10^4 
\end{cases}
\]  

(14)

For the calculus of the partial heat transfer coefficient in the ring-type space of the heat exchanger there has been used the relation

\[
\alpha_i = \frac{Nu_i \lambda_i}{d_i} 
\]  

(15)

and for the calculus of the partial heat transfer coefficient in the ring-type space of the heat exchanger there has been used the relation

\[
\alpha_e = \frac{Nu_e \lambda_e}{d_e} 
\]  

(16)

The physical properties of the two fluids were calculated at the medium temperatures of the fluids

\[
T_{m,i} = 0.5 (T_{h,1} + T_{h,2}), 
\]  

(17)

\[
T_{m,e} = 0.5 (T_{c,1} + T_{c,2}).
\]  

(18)

Within the relations (17) and (18), the output temperatures have values estimated by the Newton-Raphson algorithm, the algorithm used to solve the mathematical model of the heat exchanger.

4 The experimental study

The authors have studied the heat transfer for the double tube heat exchanger, using water for the hot and the cold fluid. To measure the temperatures and the flow rate, there are used flow meters and bimetal thermometers. The experimental installation is presented in figure 3. Within the experimental study, the authors used two double tube exchangers, for which they made many experiments, modifying the input variables of the system (figure 1), such as the hot flow rate \( Q_h \), the cold flow rate \( Q_c \) and the input temperature of the hot fluid \( T_{h,1} \).

4.1 Case study 1

The first double tube heat exchanger is made of copper, being characterized by the geometrical dimensions presented in table 2. The experimental laboratory results are presented in table 3.

Using the mathematical model elaborated by the authors, there have been calculated the values of the output variables of the heat exchanger for each
The experimental laboratory results are presented in table 7. By using the same mathematical model, there have been calculated the values of the output variables of the heat exchanger for each experiment. The obtained results are presented comparatively in table 8.

### Table 4 The comparison between the experimental (Exp) and model (Mod) data

<table>
<thead>
<tr>
<th>No.</th>
<th>Exp $T_{h,2}$ [ºC]</th>
<th>Mod $T_{h,2}$ [ºC]</th>
<th>Exp $T_{c,2}$ [ºC]</th>
<th>Mod $T_{c,2}$ [ºC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.4</td>
<td>62.4</td>
<td>24.2</td>
<td>20.8</td>
</tr>
<tr>
<td>2</td>
<td>61.2</td>
<td>62.0</td>
<td>23.0</td>
<td>20.1</td>
</tr>
<tr>
<td>3</td>
<td>61.2</td>
<td>62.0</td>
<td>22.4</td>
<td>19.6</td>
</tr>
<tr>
<td>4</td>
<td>60.8</td>
<td>62.4</td>
<td>22.0</td>
<td>19.0</td>
</tr>
<tr>
<td>5</td>
<td>61.2</td>
<td>62.4</td>
<td>21.4</td>
<td>18.6</td>
</tr>
<tr>
<td>6</td>
<td>61.2</td>
<td>62.4</td>
<td>21.4</td>
<td>18.7</td>
</tr>
<tr>
<td>7</td>
<td>61.3</td>
<td>62.5</td>
<td>20.9</td>
<td>18.3</td>
</tr>
<tr>
<td>8</td>
<td>61.2</td>
<td>62.4</td>
<td>20.9</td>
<td>18.3</td>
</tr>
<tr>
<td>9</td>
<td>61.7</td>
<td>62.9</td>
<td>20.4</td>
<td>17.9</td>
</tr>
<tr>
<td>10</td>
<td>62.3</td>
<td>63.4</td>
<td>20.0</td>
<td>17.7</td>
</tr>
</tbody>
</table>

Based on the experimental results, there have been calculated the statistical parameters associated to the output variables of the mathematical model elaborated by the authors, table 5.

### Table 5 Statistical parameters of the double tube heat exchanger model (Case study 1)

<table>
<thead>
<tr>
<th>Output variables</th>
<th>Statistical parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{h,2}$ [ºC]</td>
<td>Maximum absolute deviation [ºC] 1.2</td>
</tr>
<tr>
<td>$T_{c,2}$ [ºC]</td>
<td>Maximum relative deviation [%] 1.96</td>
</tr>
<tr>
<td></td>
<td>Standard deviation [ºC] 0.34</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2 Case study 2

The second case study also refers to a double tube exchanger, having the geometrical dimensions presented in table 6.

### Table 6 Geometrical data of double tube heat exchanger

<table>
<thead>
<tr>
<th>Geometrical characteristics</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior diameter of the interior tube [m]</td>
<td>$d_{i,1}$</td>
<td>0.012</td>
</tr>
<tr>
<td>Exterior diameter of the interior tube [m]</td>
<td>$d_{e,1}$</td>
<td>0.014</td>
</tr>
<tr>
<td>Interior diameter of the exterior tube [m]</td>
<td>$d_{i,2}$</td>
<td>0.026</td>
</tr>
<tr>
<td>Exterior diameter of the exterior tube [m]</td>
<td>$d_{e,2}$</td>
<td>0.028</td>
</tr>
<tr>
<td>Length tube [m]</td>
<td>$L$</td>
<td>0.935</td>
</tr>
</tbody>
</table>

Based on the experimental data, there have been calculated the statistical parameters associated to the output variables of the mathematical model elaborate by the authors, table 9.

### Table 7 Laboratory experimental data

<table>
<thead>
<tr>
<th>No.</th>
<th>$Q_c$ [l/h]</th>
<th>$Q_h$ [l/h]</th>
<th>$T_{c,1}$ [ºC]</th>
<th>$T_{h,1}$ [ºC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>92</td>
<td>190</td>
<td>11.8</td>
<td>55.3</td>
</tr>
<tr>
<td>2</td>
<td>92</td>
<td>210</td>
<td>11.8</td>
<td>55.3</td>
</tr>
<tr>
<td>3</td>
<td>92</td>
<td>210</td>
<td>11.7</td>
<td>55.3</td>
</tr>
<tr>
<td>4</td>
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<td>230</td>
<td>11.7</td>
<td>55.3</td>
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<tr>
<td>5</td>
<td>100</td>
<td>190</td>
<td>11.6</td>
<td>55.3</td>
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<tr>
<td>6</td>
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<td>220</td>
<td>11.5</td>
<td>55.3</td>
</tr>
<tr>
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<td>184</td>
<td>230</td>
<td>11.0</td>
<td>55.3</td>
</tr>
<tr>
<td>8</td>
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<td>240</td>
<td>10.9</td>
<td>55.3</td>
</tr>
<tr>
<td>9</td>
<td>175</td>
<td>245</td>
<td>10.7</td>
<td>55.3</td>
</tr>
</tbody>
</table>

### Table 8 The comparison between the experimental (Exp) and model (Mod) data

<table>
<thead>
<tr>
<th>No.</th>
<th>Exp $T_{h,2}$ [ºC]</th>
<th>Mod $T_{h,2}$ [ºC]</th>
<th>Exp $T_{c,2}$ [ºC]</th>
<th>Mod $T_{c,2}$ [ºC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.2</td>
<td>50.0</td>
<td>24.5</td>
<td>21.5</td>
</tr>
<tr>
<td>2</td>
<td>49.5</td>
<td>51.0</td>
<td>25.1</td>
<td>21.8</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>6</td>
<td>49.2</td>
<td>50.8</td>
<td>22.0</td>
<td>19.2</td>
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<td>16.9</td>
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<td>48.2</td>
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<td>18.6</td>
<td>16.1</td>
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<tr>
<td>9</td>
<td>47.5</td>
<td>50.8</td>
<td>17.7</td>
<td>17.0</td>
</tr>
</tbody>
</table>

### Table 9 Statistical parameters of the double tube heat exchanger model (Case study 2)

<table>
<thead>
<tr>
<th>Output variables</th>
<th>Statistical parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{h,2}$ [ºC]</td>
<td>Maximum absolute deviation [ºC] 3.3</td>
</tr>
<tr>
<td>$T_{c,2}$ [ºC]</td>
<td>Maximum relative deviation [%] 6.94</td>
</tr>
<tr>
<td></td>
<td>Standard deviation [ºC] 0.64</td>
</tr>
</tbody>
</table>

|                  | 3.5 |
|                  | 14.21 |
|                  | 0.94 |
The two case studies highlighted the following aspects:

− The temperature differences between the experimental values and those calculated with the mathematical model are lower for the hot fluid as compared to those for the cold fluid. In the author’s opinion, this situation is mainly due to the mathematical model.

− The deviations of the mathematical model in relation to the experimental data are also caused by the experimental errors. These are due to the accuracy of the measuring devices, respectively ±5% for the temperature measuring and ±10% for the flows measuring.

− Taking into account the calculated values of the output variables in relation to those which were experimentally established, the authors considered that the elaborated mathematical model has been validated.

4 Conclusion

The industrial operation of the double tube heat exchangers has imposed the development of a mathematical model specific to this type of exchanger. Starting from the studies published in literature and from their own research work, the authors have elaborated a model for the double tube heat exchangers based on the hypothesis of treating the heat exchanger as a system with lumped parameters. The model determines the output temperatures of the two fluids between which the heat transfer is made, by using data regarding exchanger geometry and the values of the input temperatures and flow rates of the of the two flows.

The mathematical model developed by the authors has been validated by comparing the output temperatures of the two flows, calculated with the proposed model, with the experimental values obtained for the same operating conditions. The mathematical model and its solving algorithm is going to be used within the study of the industrial exchangers in the refineries.

References:

[1] Milind V., R., Madhukar S., T., Water-to-water heat transfer in tube–tube heat exchanger: Experimental and analytical study,


