A hybrid Differential Evolution – Analytic Hierarchy Process Method for Multi-objective Optimization

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Abstract: - In the last years, new energy sources were used in order to reduce global warming and energy depletion. Besides that, the need to improve reliability and to reduce the energy not supplied under contingency scenarios has lead to implement Demand Response (DR) programs. This paper approaches a multi-objective optimization problem that aims to find the loading of local generators (DG) and the most appropriate DR strategy applied to some important customers in the network, to simultaneously optimize two or three objective functions. A combined method between differential evolution (DE) and analytic hierarchy process (AHP) is used in order to find the solution which fits the best the decision maker requirements.

Key-Words: - multi-objective optimization, distribution networks, distributed generation

1 Introduction
The use of renewable energy has increased in the last years, in order to deal the issues related with global warming and energy depletion. Thus, generating energy locally using renewable energy sources, such as photovoltaic panels, fuel cell and wind turbines has become a good solution to overcome these issues. This type of generation is called distributed generation (DG) [1].

DG has environmental, economical and technical advantages. The environmental advantages entail providing energy services with zero or almost zero emissions of both air pollutants and greenhouse gases. The economical advantages refer to transmission and distribution costs, electricity price and reduction of fuel consumption. Technical advantages include wide varieties of benefits, such as: peak shaving, line loss reduction, increase system voltage profile and thus power quality.

Also, Demand Response (DR) programs [2] were implemented for improving reliability by reducing the energy not supplied under contingency scenarios and for reducing energy usage during peak power hours. Usually, DR refers to different changes in electricity usage, which are taken at the end-user customers to enhance the efficiency of the distribution system operation.

Considering technical, environmental and economical aspects in order to achieve a more efficient operation of the electricity distribution systems, new optimization techniques were developed and implemented for finding suitable solutions. For single objective optimization problem, in the sense of minimization or maximization, methods such as: genetic algorithm (GA) [3], differential evolution (DE) [4], particle swarm optimization (PSO) [5], can be used in order to find the global solution.

Usually, real world problems require the simultaneous optimization of multiple and often conflicting objective functions (alternatives). To solve multi-objective optimization problems, different methods can be implemented, such as: Non-dominated Sorting Genetic Algorithm 2 (NSGA II) [6], Strength Pareto Evolutionary Algorithm 2 (SPEA2) [7], Pareto Archived Evolution Strategy (PAES) [8], etc. These methods provide not a single solution to the optimization problem, but a set of solutions which makes the best compromise between the objective functions and from this set only one must be chosen depending on the decision maker. Thus, multi criteria decision making methods, such as: Analytic Hierarchy Process (AHP) [9], Weighted Scoring Method (WSM), etc., can help decision makers to choose preference decision over the available alternatives.

This paper approaches a multi-objective optimization problem that aims to find the loading of local generators (DG) and the most appropriate DR strategy applied to some important customers in the network, to simultaneously optimize two or three objective functions, as mentioned bellow. A method based on DE and AHP is used in order to find the solution which best fits the decision maker
requirements. The objective functions are minimization of energy losses, energy production costs and greenhouse gas emissions.

2 Optimization problem
An optimization problem can be usually written as:
\[
\begin{align*}
\text{min } f_i(x) & \quad i = (1,2,...,M); \quad x \in \mathbb{R} \\
\text{subject to: } g_j(x) &= 0, \quad j = (1, 2, ..., J); \\
\text{ } h_k(x) &\leq 0, \quad k = (1, 2, ..., K);
\end{align*}
\]
where \( f_i(x) \) is the objective function needed to be minimized or maximized, in order to find the feasible solution \( x = (x_1, x_2, ..., x_n)^T \); \( M \) is the number of the objective functions.

\( g_j(x) \) - the equality constraints;

\( h_k(x) \) - the inequality constraints.

For single optimization problems, the goal is to obtain one solution, which in most cases is the global optimum.

In the case of multi-objective optimization problems the aim is to find a set of solutions which provide the best compromise between the objective functions. These solutions are called Pareto-optimal solutions and the solutions set is called Pareto front [10]. Choosing a solution from the Pareto front means finding a solution which best suits with the decision maker requirements.

3 Proposed Method
Differential Evolution (DE) algorithm was developed by Storn and Price [4]. It starts by generating randomly a population, to which applies mutation, crossover and selection. These steps are implemented for each generation, to all current individuals. DE has some important advantages: is faster for solving numerical optimization problem and has a fast convergence.

Analytic Hierarchy Process (AHP) method was developed by T.L. Saaty [9] and is a very popular technique to multi-criteria decision making. The method uses a reciprocal decision matrix obtained by pair-wise comparisons. It is necessary to evaluate each alternative and to identify the best one. The prioritization process is made by assigning a number from a scale (Table 1) developed by Saaty.

These pair-wise comparisons are made for all the alternatives considered, until a matrix \( A \) (as in (2)), is created by putting the results of pair-wise comparison.

\[
A = \begin{bmatrix}
A_1 & a_{12} & \cdots & a_{1n} \\
a_{21} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & 1
\end{bmatrix}
\]

where:

\( n \) = number of alternatives to be evaluated
\( A_i \) = the \( n^\text{th} \) alternative;
\( a_{ij} \) = importance of \( i \) alternative according to \( j^\text{th} \) alternative

In this paper the \( A \) matrix will be created for the objective functions considered and for the solutions obtained with respect to each objective function.

The next step is to compute the relative weights (called eigenvector) of the objective functions, which is called the Relative Value Vector (RVV) and of the solutions obtained with respect to each objective function called Option Performance Matrix (OPM). The method used for computing eigenvectors is, as recommended in [9]: the values in each row of the matrix are multiply and then the \( n^\text{th} \) root of that product is taken. The \( n^\text{th} \) roots are then summed and that sum is used to normalize the eigenvectors elements to add to 1.

The following step is to compute the consistency index (CI) as in (3) and the consistency ratio (CR) as in (4), where \( \lambda_{\text{max}} \) is the maximum component of the eigenvector and \( n \) is the size of the pair-wise comparison.

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

\[
CR = \frac{CI}{RI}
\]

where \( RI \) represents random consistency index and its values are presented in Table 2.

The \( CR \) value should be less than or equal to 0.1 or 10\%, so the computed result can be consistent or acceptable.

The final step is to compute the vector called

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
</tr>
<tr>
<td>7</td>
<td>Very strong</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>When compromise is needed</td>
</tr>
</tbody>
</table>

Table 2. Random Consistency Index (RI)

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
n  & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
RI & 0.00 & 0.00 & 0.00 & 0.58 & 0.90 & 1.12 & 1.24 & 1.32 & 1.41 & 1.49 & 1.53 & 1.58 & 1.60 & 1.57 & 1.53 \\
\hline
\end{tabular}
Value For Money (VFM) as in (5) which gives the ability to the solution computed to meet the user needs.

\[ RVV \star OPM = VFM \] (5)

The proposed combined method is described as follows. First, the population \( X \) is generated randomly, and for each individual are computed the objective functions. The RVV and OPM matrix are computed as described above, which are used to obtain the VFM vector. Then, at each generation, DE mutation and crossover operators are applied to each individual from population and the individuals obtained will form the new population \( X_{\text{new}} \). For each population \( X_{\text{new}} \) the VFM vector is computed based on \( RVV_{\text{new}} \) and \( OPM_{\text{new}} \). The next step is to compare each individual from the \( X \) population to each individual from the \( X_{\text{new}} \) population, based on VFM and VFMnew. If a component of VFMnew is higher than the one in VFM then the individual from the \( X \) population is replaced with the one from \( X_{\text{new}} \) population. This steps are applied until the stopping criterion is met.

In this way the solution is improved in order to meet the decision maker needs.

4 Problem Formulation

This paper approaches a multi-objective optimization problem that aims to find the loading of local generators (DG) and the most appropriate DR strategy applied to some important customers in the network, to simultaneously optimize the following objective functions:

- energy losses minimization:

\[ \min \left\{ f_1(x) = \sum_{i=1}^{B} \sum_{b \in B} R_b \cdot I_b^2 \right\} \] (6)

where \( R_b \) is the resistance of branch \( b \) and \( I_b \) is the current flowing through branch \( b \) at hour \( t \).

- time of the optimization problem;
- number of branches in the considered network.

- minimization of production cost for the energy supplied by local generators:

\[ \min \left\{ f_c(x) = \sum_{i=1}^{T} (c_{Dg,i} + c_{FC}(t) + c_{CPP}(t)) \right\} \] (7)

where:

- production costs for the Diesel generator :

\[ c_{Dg,i} = \sum_{i=1}^{N} b_i \cdot P_{Dg,i} \]

where:

- Diesel fuel price; \( b_i \) =2\$C/kWh;

Table 3. Specific greenhouse gas emissions (GHG)

<table>
<thead>
<tr>
<th></th>
<th>Diesel generator</th>
<th>Fuel Cell</th>
<th>Coal Power Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOx</td>
<td>0.0021</td>
<td>0.000004</td>
<td>0.00254</td>
</tr>
<tr>
<td>SO2</td>
<td>0.0002</td>
<td>0.0000226</td>
<td>0.006168</td>
</tr>
<tr>
<td>CO2</td>
<td>0.64954</td>
<td>0.4309124</td>
<td>0.95934</td>
</tr>
</tbody>
</table>

- production costs for Fuel Cells:

\[ c_{FC}(t) = \frac{c_{ng} \cdot P_t}{C_p \cdot \eta_t} \]

where:

- \( c_{ng} \) – the natural gas price; \( c_{ng} \) = 1.5 $/m^3;
- \( P_t \) – electrical power produced at interval \( t \);
- \( \eta_t \) – the cell efficiency at interval \( t \) (\( \eta_t =20-30\% \));
- \( C_p \) – caloric capacity (\( C_p =10.462 \text{kWh/m}^3 \))

Where:

- \( P_{Dg,i} \) – the output power \( (\text{kW}) \) of Diesel generator \( i \), at interval \( t \);

\[ N \] – number of Diesel generators;

- production costs for Fuel Cells:

\[ c_{FC}(t) = \frac{c_{ng} \cdot P_t}{C_p \cdot \eta_t} \]

where:

- \( c_{ng} \) – the natural gas price; \( c_{ng} \) = 1.5 $/m^3;
- \( P_t \) – electrical power produced at interval \( t \);
- \( \eta_t \) – the cell efficiency at interval \( t \) (\( \eta_t =20-30\% \));
- \( C_p \) – caloric capacity (\( C_p =10.462 \text{kWh/m}^3 \))

Where:

- \( P_{Dg,i} \) \( P_{FC}(t) \) \( P_{CPP}(t) \) – the power produced by the Diesel generator, Fuel Cell and the coal power plant at interval \( t \);

Specific greenhouse gas emissions (GHG) considered for this study case are shown in Table 3.

The aim of this paper is to find the suitable solution among a set of solutions obtained with the combined DE-AHP method, which satisfies the best the decision maker needs.

5 Study case

For the study case, a test radial distribution network (Fig. 1) with \( N = 44 \) nodes, \( B = 47 \) branches, \( S = 4 \)
supply nodes and $Q = 7$ open branches was considered [11]. For simplifying computation, the supply nodes are considered as a single slack bus.

Several local generators are considered connected to the network as follows: two wind turbines with a rated constant power of 50 kW, one placed at node 10 and one at node 23; a 50 kW photovoltaic system (PV) at node 17; a Diesel generator at node 16 with capacity of 90 kW; a Fuel Cell (FC) at node 20 with capacity of 70 kW; a coal power plant (CPP) at node 24 with capacity of 80 kW.

During the optimization process the load for Diesel generator, FC and CPP will change for the analysed time interval (4 hours) for finding the best solution which optimizes the objective functions mentioned above, but will be limited in order to prevent reversed power flow (export) from the analyzed system. For the PV system and wind turbines the load will remain the same for this time interval, based on a forecast received.

Different types of consumers are considered for this network: residential, services, industrial and commercial. The industrial consumers are considered to participate in DR programs, and are placed in nodes 10, 21 and 30. This type of customers modifies their consumption at each hour, but keeps the same total consumption for the time interval considered.

The chromosome structure used by the combined DE-AHP method is presented in Fig. 2.

The following judgement was made, regarding all the objective functions (case I) computed as in (6), (7) and (8): the production costs is considered to be far more important than losses and is rated as 5 in (C,L) cell and 1/5 in (L,C) cell. Also, costs are rated as 3 with respect to greenhouse gas emissions, and

<table>
<thead>
<tr>
<th>L</th>
<th>C</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/3</td>
<td>3</td>
</tr>
<tr>
<td>1/3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

greenhouse gas emissions are rated as 3 in the cell (E, L) and 1/3 in (L,E) as shown in Table 4, where: $L$ represents energy losses, $C$ are the production costs and $E$ are the greenhouse gas emissions.

Based on the methodology described in section 3, the RVV vector results as $[0.1047;0.6370;0.2583]$ and the CR as 0.0402, well below the critical limit of 0.1, so consistency exists on the choice made. Thus, minimization of the production costs is the first priority of the optimization problem as results from RVV vector, because it has the biggest value, which is 0.6370.

The combined DE-AHP method was applied for 40 individuals and 100 generations. The solution obtained by running the proposed method is represented in Fig. 3. The results obtained are correct because the optimization seeks to find the solution which favours minimization of energy production costs, based on the decision maker judgement made before.

Different solutions obtained by the proposed DE-AHP method, chosen depending on the VFM value, are presented in Table 5. The highest value of VFM gives the solution which fits the best the decision maker needs, which as it can be seen from Table 5 has the minimal energy production costs. Thus, the solution obtained is correct from the judgement point of view.

The same method was applied for only two objective functions (case II): energy losses and energy production costs minimization. In this case, it was considered that energy losses are more important than production costs, rated as 3 in (L,C)

**Table 4.** Pair-wise comparison for all the objective functions considered to be optimized

<table>
<thead>
<tr>
<th>L</th>
<th>C</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/5</td>
<td>1/3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1/3</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 1. Test network

Fig. 2. Chromosome structure
cell, and 1/3 in the (C,L) cell as in Table 7. The notations are the same as in case I. The RVV vector resulted as [0.6753; 0.3247] and CR as 0.0836, so the judgement is good. The solutions obtained by running the proposed method in case II are represented in Fig. 4. The results obtained are also correct because the optimization seeks to find the solution which favours minimization of energy losses, based on the judgement made before.

Different solutions obtained by the proposed DE-AHP method for the case II are presented in Table 6, chosen depending on the VFM value. The highest value of VFM provides the solution which fits the best the decision maker needs, which as it can be seen from Table 6 has the minimal energy losses.

In case I it can be observed that because the objective function related to the greenhouse gas emissions has more importance than energy losses but less importance than energy production costs, the coal power plant tends to generate only a small

amount of energy, this because of its specific GHG which are the highest from all local generators as in Table 1.

In order to see what results may be obtained when standard NSGA II method is combined with AHP technique, a test was performed. The standard NSGA II method was used, but this time the solutions sorting was not made as usually based on their objective functions, but based on eigenvectors computed for the solutions with respect to the two objective functions: energy losses and energy production costs minimisation.

It was observed that even if this sorting method was used, the Pareto front was obtained. Also, after

Table 5. Example of solution with different VFM value obtained by the proposed method for case I

<table>
<thead>
<tr>
<th>Power generation of Diesel Generator</th>
<th>Power generation of Fuel Cell</th>
<th>Power generation of coal power plant</th>
<th>Consumption for DR1</th>
<th>Consumption for DR2</th>
<th>Consumption for DR3</th>
<th>Energy losses [kWh]</th>
<th>Production cost [€/kWh]</th>
<th>Greenhouse gas emission [kg]</th>
<th>VFM values for the solution obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>71</td>
<td>53</td>
<td>48</td>
<td>55</td>
<td>50</td>
<td>51</td>
<td>27</td>
<td>23</td>
<td>8</td>
</tr>
<tr>
<td>42</td>
<td>31</td>
<td>68</td>
<td>53</td>
<td>44</td>
<td>45</td>
<td>7</td>
<td>13</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>54</td>
<td>37</td>
<td>65</td>
<td>43</td>
<td>46</td>
<td>41</td>
<td>26</td>
<td>5</td>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 6. Example of solution with different VFM value obtained by the proposed method for case II

<table>
<thead>
<tr>
<th>Power generation of Diesel Generator</th>
<th>Power generation of Fuel Cell</th>
<th>Power generation of coal power plant</th>
<th>Consumption for DR1</th>
<th>Consumption for DR2</th>
<th>Consumption for DR3</th>
<th>Energy losses [kWh]</th>
<th>Production cost [€/kWh]</th>
<th>Greenhouse gas emission [kg]</th>
<th>VFM values for the solution obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
<td>21</td>
<td>36</td>
<td>40</td>
<td>47</td>
<td>53</td>
<td>37</td>
<td>24</td>
<td>44</td>
</tr>
<tr>
<td>32</td>
<td>9</td>
<td>23</td>
<td>28</td>
<td>40</td>
<td>49</td>
<td>50</td>
<td>14</td>
<td>16</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>37</td>
<td>21</td>
<td>55</td>
<td>21</td>
<td>45</td>
<td>44</td>
<td>50</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 4. Solution obtained by the DE-AHP method for case II

Fig. 5. Pareto front for standard NSGA II and modified NSGA II with AHP

Table 7. Pair-wise comparison for the two objective functions considered to be optimized

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>1/3</td>
<td>1</td>
</tr>
</tbody>
</table>
computing the objective functions for the Pareto solutions obtained, a Pareto front was achieved. Comparing with the Pareto front obtained using the standard NSGA II method when objective functions are ranked, the modified NSGA II method with the AHP techniques obtain better results at the extreme margins of the Pareto front. Fig. 5 shows the two Pareto fronts obtained with standard NSGA II and modified NSGA II with AHP. Also, it can be observe from Fig. 5 that the Pareto front for the modified method offers a larger area of solutions, widening the choice space for the decision maker.

4 Conclusion
This paper presents a study case regarding multi-objective optimization solved using a combined DE-AHP method.

Different cases were considered for three or two objective functions in order to find the suitable solution based on the decision maker needs. It was shown that this method provides good solution taking into consideration the decision maker judgment.

Also, it was shown that in the case of using standard NSGA II method combined with AHP techniques, in which the non-dominated sort was based on the eigenvectors computed for each solution with respect to the objectives considered, a Pareto front for the solutions obtained was achieved. For the same solutions provided, the objective functions were computed, which also form a Pareto front that provides better results at the edge of the front. Also, the decision space is increased because this method finds a larger area of solutions, providing new choices for the decision maker.

ACKNOWLEDGMENT
This work was developed in the framework of the EURODOC “Doctoral Scholarships for research performance at European level” project, financed by the European Social Found and Romanian Government.

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