A Fuzzy Algorithm for Parameter Estimation of a Superheater System

RAZIDAH ISMAIL & NOOR AINY HARISH
Faculty of Computer & Mathematical Sciences
Universiti Teknologi MARA
40450 Shah Alam, Selangor,
MALAYSIA
razidah@tmsk.uitm.edu.my , ainy@tmsk.uitm.edu.my

Abstract: - The Fuzzy State Space algorithm (FSSA) is the main feature in the development of the Fuzzy State Space Model (FSSM) for solving inverse problems in multivariable dynamic systems. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, which requires excessive computer time and thus can be very costly. In the formulation of the FSSA, the uncertain value parameters of the system to be controlled are represented by triangular fuzzy numbers with their membership function derived from expert knowledge. The optimal combination of the input parameters was extracted using the Modified Optimized Defuzzified Value Theorem. In this paper, this fuzzy algorithm is implemented to the FSSM of a superheater system of a combined cycle power generation plant. The result reveals that the proposed approach is reasonable and effective. To take advantage of the effectiveness and some distinguish features of the FSSA, a graphical user’s interface is developed using Visual Basic programming tool. This computational tool is flexible and can be adaptable to other multivariable systems. Besides providing an efficient computation for estimating the optimal combination of the parameters, it is an innovative tool for industrial applications.

Key-Words: - Fuzzy graph; Graphical Fuzzy State Space Model; Inverse problems; Uncertainty modeling

1 Introduction
Due to the complexity of most practical multivariable control systems, it is necessary to develop a mathematical model of the systems by simplifying and idealizing the processes involved. Control system analysis normally addresses forward problems. However, disturbance in power systems motivate analysis questions that are classed as inverse problems [1]. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, for example [2], [3]. Thus, this led to the development of Fuzzy State Space Model (FSSM) for multivariable system [4]. The Fuzzy State Space algorithm (FSSA) is the main feature in this modeling approach, where the uncertain value parameters of the system to be controlled are represented by triangular fuzzy numbers [5] with their membership function derived from expert knowledge. The effectiveness of this modeling approach was illustrated by implementing it to the state space model of a furnace system. The results demonstrate that the proposed new modeling approach is reasonable and provides an innovative tool for decision-makers [4], [6]. In order to facilitate the implementation of FSSA to other multivariable system, such as superheater, an efficient computational tool together with the user’s interface is developed. Thus, the objective of this paper is to illustrate the implementation of FSSA to estimate the optimal parameters of a superheater system. To facilitate its implementation, a graphical user’s interface (GUI) is developed using Visual Basic programming tool.

This study was carried out in two phases. The first phase involves the development of the state space model of a superheater system, which is based on first principles physical and thermodynamic laws with appropriate simplifications. A simple derivative equation of each energy storage element and the output equations are expressed as a linear combination of any of the state and input variables [7]. In the second phase, the input and the output parameters of the superheater system are estimated using the Fuzzy State Space algorithm embedded in the interface, which is named as Interactive Parameter Estimator (IPE)
2 Fuzzy State Space Model of a Superheater System

Human perception of uncertainties plays the important role in handling parameters in multivariable system. Often, human experts describe the decision parameters of the system through vague and uncertain statements. Since uncertainty may contain useful information, several techniques and approaches that integrate uncertainties in system modeling had been studied by many researchers. Among the recent research is the development of Fuzzy State Space Model, which incorporated the flexibility of fuzzy modeling and crisp state-space representation.

A FSSM of multivariable dynamic system is defined as follows:

\[
S_{FS} : \quad \dot{x}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t)
\]

where \(u\) denotes the fuzzified input vector \([u_1, u_2, ..., u_n]^T\) and \(y\) denotes the fuzzified output vector \([y_1, y_2, ..., y_m]^T\) with initial conditions as \(t_0 = 0\) and \(x_0 = x(t_0) = 0\). The elements of state matrix \(A_{psp}\), input matrix \(B_{pxn}\), and output matrix \(C_{mxp}\) are known to specified accuracy.

The main feature of FSSM is that it provides an inverse Fuzzy State Space algorithm (FSSA) that can be applied to any multivariable dynamic systems for determination of optimal input parameters estimation with respect to wide range of initial constraints or specifications. FSSA has been shown to give good parameter estimation [4] and it is flexible as the input parameter can be described as approximately as desired at the initial stages of the control process. In addition, it provides a fast and simple way for including uncertainties and expert knowledge.

In the construction of the state space model of a superheater system, it is assumed that the system can be represented by a lumped-parameter model. The detail derivation is published in [8]. The state equation and the output equation of the superheater system can be represented by the following:

\[
\begin{bmatrix}
\frac{dT_s}{dt} \\
\frac{d\rho_s}{dt}
\end{bmatrix} = \begin{bmatrix}
A_{psp} \\
B_{pxn}
\end{bmatrix} \begin{bmatrix}
Q_{gs} \\
\omega_s
\end{bmatrix}
\]

where the state matrix is

\[
A = \begin{bmatrix}
\frac{k_k}{MC_{st}} & \frac{k_s}{MC_{st}} & \frac{f_1}{h_{s-ref}} \frac{g_{s-ref}}{g_{s-ref-ref}} \\
\frac{1}{h_{s-ref}} & \frac{1}{h_{s-ref}} & \frac{1}{h_{s-ref}} \\
\frac{\rho_v k_{pf}}{f_s Q_s} & \frac{\rho_v k_{pf}}{f_s Q_s} & \frac{f_1}{h_{s-ref}} \frac{g_{s-ref}}{g_{s-ref-ref}}
\end{bmatrix}
\]

and the input matrix is \(B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\), the state vector, \(\ddot{u}(t) = \begin{bmatrix} T_s \\ \rho_s \end{bmatrix}\)

and the input vector, \(\dddot{u}(t) = \begin{bmatrix} Q_{gs} \\ \omega_s \end{bmatrix}\)

where the output matrix is

\[
C = \begin{bmatrix}
0 & \frac{R_{h_s}}{e_{ps}} & \frac{h_{ref}}{e_{ps}} & \frac{T_{ref} e_{ps}}{e_{ps}} \\
0 & \frac{h_{s-ref} + + e_{ps} T_{ref}}{h_{s-ref} + + e_{ps} T_{ref}} & \frac{h_{s-ref} + e_{ps} T_{ref}}{h_{s-ref} + e_{ps} T_{ref}} & \frac{T_{ref} e_{ps}}{e_{ps}}
\end{bmatrix}
\]

The state space model of the superheater system is given by the state equation, (1) and the output equation, (2) with two state variables, two input and three output parameters. The state variables consists of \(T_s\) (metal tube temperature) and \(\rho_s\) (superheated steam density) whereas the input parameters are \(Q_{gs}\) (heat supplied to the superheater from the furnace model) and \(\omega_s\) (steam mass flow out from the superheater). The output parameters that are involved in the development of state space model of a superheater system are \(p_s\) (superheated steam pressure), \(T_s\) (superheated steam temperature) and \(w_s\) (steam mass flow to drum). The nomenclature for the other symbols is listed in the Appendix. Hence, the input parameters and the output parameters of the superheater system are estimated using the Fuzzy State Space algorithm.
3 Fuzzy State Space Algorithm

In formulating the inverse Fuzzy State Space algorithm, the approach introduced in [9] is modified by considering the state space representation of the system. In his work, he had developed a fuzzy algorithm for optimization of geometrical and electrical parameters of microstrip lines using algebraic equations.

Given an input $g_i$ that takes values in set $I_i$, and let preferences for different values of $g_i$ be expressed by a fuzzy set $F_{li}$ on $I_i$. For each $x \in I_i$, the value $F_{li}(x)$ designates the degree of desirability of using the particular value $x$ within the given set of values $I_i$. Index $i$ is used here to distinguish different input parameters. The fuzzy sets expressing preference for all input parameters are employed for calculating the associated fuzzy sets for performance parameters. The target values of performance parameters are specified by functional requirements. Performance parameters, resulting from calculations with uncertain or vague input parameters, are also represented by fuzzy preference functions. Similarly, each of the output parameter is represented by a range and a preference function.

It is assumed that all the fuzzy sets $F_{li}$ expressing preferences of all input parameters $g_i \in I_i \subset R^r (i \in N)$ are determined, normalised and convex. $I_i$ is a close interval of positive real numbers. $S_g$ is a performance parameter based on the FSSM whereby all input parameters are considered as its variables and can be presented within a fuzzy set $F_{sg}$. The algorithm to determine a fuzzy set $F_{sg}$ that is induced on the output parameters by fuzzy sets $F_{li}$ through $S_g$ has the following steps:

Step 1:
Let $S_g: R^r \rightarrow R$. $S_g$ is the performance parameter such that $r = S_g(g_1, g_2, g_3, ..., g_n)$.

Step 2:
Select appropriate values for $\alpha$-cut such that $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_4 \in (0,1]$ which are equally spaced.

Step 3:
To fuzzify the input, determine all the $\alpha_c$-cuts for all $F_{lp} (p \in N)$.

Step 4:
Generate all $2^n$ combinations of the endpoints of intervals representing $\alpha_c$-cuts for all $F_{lp} (p \in N)$. Each combination is an $n$-tuple $(g_1, g_2, g_3, ..., g_n)$.

Step 5:
Determine $r_j = S_g(g_1, g_2, g_3, ..., g_n)$ for each $n$-tuple, $j \in 1, 2, ..., 2^n$.

Step 6:
Set $F_{sg} = [\min(r_j), \max(r_j)]$ for all $j \in 1, 2, ..., 2^n$.

Step 7:
Determine all the $\alpha_c$-cuts for the preferred output parameter, $O_{pref}$.

Step 8:
Set $[O_{pref} \land F_{sg}]$.

Step 9:
Determine $f^* = \sup[O_{pref} \land F_{sg}]$ and find the $S_g^*$, the $S_g$ value for $f^*$.

Step 10:
Find the endpoints of interval for each input $I_p$ where $p = 1, 2, ..., n$.

Step 11:
Generate all $2^n$ combinations of the endpoints of intervals representing $f^*$-cuts for all $F_{lp} (p \in N)$. Each combination is an $n$-tuple $(g_1^*, g_2^*, g_3^*, ..., g_n^*)$.

Step 12:
Determine $r^* = S_g^*(g_1^*, g_2^*, g_3^*, ..., g_n^*)$ by using Modified Optimized Defuzzified Value Theorem.

The values determined in the final step of the algorithm are the approximate optimal value of input parameters that will produce the desired value of the output parameters as determined by applying the following Modified Optimized Defuzzified Value Theorem [4]:

Let $S_g: R^n \rightarrow R$ where $S_g$ is a performance parameter based on the Fuzzy State Space Model. If $S_g^* = r^* = \max r_j^*$ such that $\mu(r_j^*) = f^*$ for all $(r_j, f^*) \in F_{set}$, then $r^* = S_g^* = \max[\mu(g_1^*, g_2^*, g_3^*, ..., g_n^*)]$.

4 Interactive Parameter Estimator

The Interactive Parameter Estimator (IPE) of a superheater system is an interface to facilitate the implementation of the Fuzzy State Space Algorithm. IPE is a user-friendly interactive stand-alone system which can be adaptable to other multivariable dynamic systems [10]. It is an innovative tool for industrial applications and can also be beneficial as a teaching tool especially in control system modeling. The GUI application is designed with flexibilities in mind. An example of the screens is displayed in Fig. 1.
5 Implementation on a Superheater System

The state space representation of a superheater system is extended to include uncertainties in order to gain a better understanding of the system. According to [11], multivariable systems can be represented in a decomposed form as a set of coupled multiple-input single-output (MISO) systems. Thus, the global modeling problem of the superheater system can be reduced to MISO system. By using the operating data from [2], the following state space model with two input parameters and three output parameters is considered.

\[
\begin{align*}
\frac{d [Q_{gs}]}{dt} &= (-1.585 \times 10^{-5}) T_s + (1.997 \times 10^{-7}) 0 \text{ } Q_{gs} \\
\frac{d [w_s]}{dt} &= (5.048 \times 10^{-10}) T_s + (2.7616 \times 10^{-4}) 0 \text{ } w_s
\end{align*}
\]

(3)

\[
\begin{bmatrix}
Q_{gs} \\
T_s \\
w_s
\end{bmatrix} =
\begin{bmatrix}
0 & 6.559 \times 10^{-3} & 0 \\
0 & 3.184 \times 10^{-2} & 0 \\
2.336 \times 10^{-2} & 7.441 \times 10^{-3} & 0
\end{bmatrix}
\begin{bmatrix}
T_w \\
\rho_s \\
\psi
\end{bmatrix}
\]

(4)

The input parameters and the output parameters of the superheater system as given in eqs. (3) and (4) are estimated using the fuzzy state space algorithmic approach. An interactive approach using IPE is used for the computations involved in this algorithm.

Each of the input parameter of the superheater system is fuzzified. The desired value for each input parameter has a value \( \alpha = 1 \) whereas the extreme values are specified as \( \alpha = 0 \) as shown in Table 1. In this illustration, \( \alpha \)-cuts with increment of 0.2 are used to calculate \( F_{ind} \), the fuzzy values of induced output or performance parameters \( S_{g} \). Combinations of the endpoints of intervals for all input parameters with respect to each particular value of \( \alpha \)-cut are determined and are used to plot the graph of \( F_{Sg} \).

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>( \alpha = 0 )</th>
<th>( \alpha = 1 )</th>
<th>( \alpha = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{gs} )</td>
<td>( 6.0 \times 10^6 )</td>
<td>( 6.2 \times 10^6 )</td>
<td>( 6.4 \times 10^6 )</td>
</tr>
<tr>
<td>( w_s )</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

Similarly, each of the desired output parameter is set to the values published in [2], which are obtained through forward calculations and simulation. The desired value and its domain are shown in Table 2 and are used to calculate the preferred or desired output parameters. \( \alpha \)-cuts with increment of 0.2 are used to calculate \( O_{pref} \), the fuzzy values of preferred or desired output parameters. Combinations of the endpoints of intervals for all output parameters with respect to each particular value of \( \alpha \)-cut are determined and are used to plot the graph of \( O_{pref} \). The calculation is shown in Fig. 2.

<table>
<thead>
<tr>
<th>Output parameters</th>
<th>Domain</th>
<th>Desired value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_s )</td>
<td>( [4 \times 10^6, 5 \times 10^6] )</td>
<td>( 4.5 \times 10^6 )</td>
</tr>
<tr>
<td>( T_s )</td>
<td>( [710, 730] )</td>
<td>720</td>
</tr>
<tr>
<td>( W_v )</td>
<td>( [11, 13] )</td>
<td>12</td>
</tr>
</tbody>
</table>

The intersection of the fuzzy preferred output parameter and the fuzzified performance parameter is determined by superimposing the two graphs in order to obtain the \( f^* \)-value. The fuzzy value obtained by considering each of the output parameter is shown in Fig. 3 - 5.
With the $f^*$-value, the best possible combination of the input parameters to accommodate all the constraints defined is calculated. Each of the eight combinations of the endpoints of interval are determined and processed by the extension principle [12]. The selection of the optimal combination for the input parameters is determined by the Modified Optimized Defuzzified Value Theorem. Since the membership function designates the degree of desirability, the largest fuzzy value $f^* = 0.3849$ is chosen and used in the rest of the algorithm. Table 3 shows the optimized input parameters of the superheater system are $Q_{gs} = 6.02 \times 10^6$ J/kg and $w_s = 12.23$ kg/s. These values differ from the desired values with a difference of 2.98% and 1.92% respectively. Table 4 shows the percentage error for each of the output parameters of the superheater system. It is interesting to note that the calculated values obtained using this algorithm are very close to the desired target values of the system.

### Table 3: Optimized input parameters

<table>
<thead>
<tr>
<th>Parameter $f^*$</th>
<th>Calculated value</th>
<th>Desired value</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{gs}$</td>
<td>$6.02 \times 10^6$</td>
<td>$6.2 \times 10^6$</td>
<td>2.98</td>
</tr>
<tr>
<td>$w_s$</td>
<td>12.23</td>
<td>12</td>
<td>1.92</td>
</tr>
</tbody>
</table>

### Table 4: Calculated output parameters for Superheater System

<table>
<thead>
<tr>
<th>Parameter $f^*$</th>
<th>Calculated value</th>
<th>Desired value</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_s$</td>
<td>4.14</td>
<td>4.5</td>
<td>8.05</td>
</tr>
<tr>
<td>$T_s$</td>
<td>677.44</td>
<td>720</td>
<td>5.91</td>
</tr>
<tr>
<td>$W_v$</td>
<td>11.09</td>
<td>12</td>
<td>7.62</td>
</tr>
</tbody>
</table>

Subsequently, a comparison is made between the optimal input parameters obtained using the inverse Fuzzy State Space algorithm and the result obtained through simulation carried out by [2]. The percentage difference is calculated and tabulated in Table 5. The aim of this comparison is to highlight the difference between inverse modeling by utilizing fuzzy sets and a widely accepted forward modeling based on simulation. With the triangular fuzzy number used in modeling the uncertainty, the obtained result should have the same value as the result in [2] with no uncertainty consideration. It is observed that the values of the input parameters are $Q_{gs}$ (heat supplied to the superheater from the furnace model) and $w_s$ (steam mass flow out from the superheater) differ with a difference of 2.3% and 1.9% respectively. In order to improve the results, the parameters of the fuzzy numbers which are used to model uncertainties in this system, need to be adjusted based on the historical data or human experience. For a better resolution, $\alpha$-cuts with much smaller increment can be used.

### Table 5: Comparison of Optimized Input Parameters

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Ismail’s Ref. [2]</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{gs}$</td>
<td>$6.02 \times 10^6$</td>
<td>2.3</td>
</tr>
<tr>
<td>$w_s$</td>
<td>12.23</td>
<td>1.9</td>
</tr>
</tbody>
</table>

6 Conclusion

The focal element of this paper is the implementation of the fuzzy state space algorithm to the Fuzzy State Space model of a superheater system. The development of Interactive Parameter Estimator (IPE) had provide an efficient computational tool. The results obtained in this application had shown that the performance of the algorithm is reasonable, promising and effective. In fact, this algorithm require significantly fewer computer runs compared to the several hundred computer runs required for forward simulation method. Besides providing an efficient computation for estimating the optimal combination of the parameters, IPE is an innovative tool for decision-makers. IPE can be made available online in an intranet environment and it is hoped that the effort can be further utilized and improved to suit the needs of other similar modeling systems. In general, this new technique for determination of optimal input parameters gives broader and useful information and provides a faster and innovative tool for decision-makers.

In recent studies of complex control system, directed graphs have been introduced to define and interpret the interconnections structure underlying the dynamics of the interacting subsystems [13]. Subsystems were associated with vertices while interconnections with edges of the graph. Thus, this has motivates the extension of the underlying structure of FSSM. The basic concept of graph theory is examined in order to explain some physical interpretation of the multi-connected systems of FSSM [14]. This can serve as a strong theoretical framework and provide an invaluable reference for further research.

Appendix

$w_s$ steam mass flow from the drum to the superheater (kg/s)
$w_t$ steam mass flow out from the superheater (kg/s)
$V_s$ superheater volume (m$^3$)
$\rho_s$ superheated steam density (kg/m$^3$)
$w_a$ attemporator water mass flow (kg/s)
$p_v$ steam drum pressure (Pa)
$\rho_v$ saturated steam density (from the drum model) (kg/m$^3$)
$f_s$ friction coefficient (m$^{-4}$)
$Q_s$ heat supplied to the superheater (from furnace model) (J/s)
$Q_t$ heat transferred to the steam (J/s)
$M_s$ mass of superheater tubes (kg)
$T_{st}$ metal tube temperature (°K)
$k_s$ an experimental coefficient
$T_s$ steam temperature (°K)
$h_s$ specific enthalpy of saturated steam (from drum) (J/kg)
$h_{st}$ specific enthalpy of saturated steam (J/kg)
$h_{at}$ specific enthalpy of atttemporation water (J/kg)
$h_{ev}$ specific enthalpy of evaporation (J/kg)
$h_{ref}$ reference steam enthalpy condition (J/kg)
$T_{ref}$ reference steam temperature condition (°K)
$C_{ps}$ specific heat of steam at constant pressure (J/kg °K)
$R_s$ ideal gas constant
$x_{s1}$ $h_s * \rho_s$
$C_{st}$ heat capacitance of superheater tubes (J/kg °K)

Acknowledgment

The authors would like to thank the Ministry of Higher Education of Malaysia for the financial support through the Fundamental Research Grant Scheme. Their gratitude is also extended to anonymous reviewers for their valuable comments and suggestions.

References:


