Abstract: The development of the efficient technique of the experimental research of the continuous communication channel for amplitude-frequency characteristics (AFC) identification is given. The given technique is Volterra model based and founded on the application of the modified approximating method of identification of the nonlinear dynamic system in frequency domain. The identification is carried out by means of compiling the linear combinations of the responses from researched system to the input test polyharmonic signals with different amplitudes. The developed software-hardware tools implementing the methodology of the identification are used for construction of the informational model of the communication channel in the form of the first and second order AFCs on the basis of the data of the input-output experiment, using harmonic and biharmonic test signals.

Key Words: communication channels, nonlinear dynamic systems, identification, Volterra models, Volterra series, multidimensional transfer functions, multifrequency characteristics, polyharmonic signals

1 Introduction

An accuracy of information transmitted from the source to recipient is the one of the most important demands made to communication systems. In real conditions to fulfill such demands we have to eliminate errors caused by external interference form communication channel (CC) at receiver entrance; internal noise of the receiver; and signal distortion during transmission. In connection with this problems for the last ten years intensively developing field related to the methods of signals optimal reception which take into account characteristics of the hardware and CC [1]. The expendiency of the communication system application depends on how effectively its potential abilities are used.

The aim of this work is an identification of the continuous CC using Volterra model in the frequency domain, i.e. the determination of its multifrequency characteristics on the basis of the data of the input-output experiment, using test polyharmonic signals.

2 Volterra models and identification of dynamical systems in the frequency domain

In general case “input–output” type ratio for nonlinear dynamic system can be presented by Volterra series [2, 3]:

\[
y[n(t)] = \sum_{n=1}^{\infty} y_n[n(t)] = \sum_{n=1}^{\infty} \prod_{\tau=1}^{\infty} w_n(\tau_1, \tau_2, ..., \tau_n) x(t-\tau_n) d\tau_n,
\]

where \(x(t)\) and \(y(t)\) are input and output signals of system respectively; \(w_n(\tau_1, \tau_2, ..., \tau_n)\) – weight function or \(n\)-order Volterra kernel; \(y_n[n(t)]\) – \(n\)-th partial component of object response.

In practice, Volterra series are replaced by polynomial and generally limited to several first members of the series. Identification of nonlinear dynamic system in the form of a Volterra series consists of determination of \(n\)-dimensional weighting functions \(w_n(\tau_1, ..., \tau_n)\) or their Fouriers–images \(W_n(j\omega_1, ..., j\omega_n)\) – \(n\)-dimensional transfer functions, accordingly for system modeling in time or frequency domain [4, 5].

Identification of nonlinear system in frequency domain coming to determination of absolute value and phase of multidimensional transfer function at given frequencies – multidimensional AFC \(|W_n(j\omega_1, j\omega_2, ..., j\omega_n)|\) and phase-frequency
The choice of amplitudes $a_j$ should provide the convergence of series (1) and an minimum error during extraction of a partial component $y_k[x(t)]$ according to (4) defined by reminder of series (1) – members of degree $N+1$ and above. If $x(t)$ – is a test effect with maximum admissible amplitude at which a series (1) converges, amplitudes $a_j$ should be by their absolute values no more than unit: $|a_j| \leq 1$ for $\forall j=1,2,\ldots,n$. [6].

The more $N$ the less an impact of loped off members of the Volterra series and more probes are making.

To define the case of minimal impact of series residue (4) it can be written as:

$$\sum_{j=1}^{N} c_j y[a_j, x(t)] = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} w_{n}(\tau_1, \tau_2, \ldots, \tau_n) \prod_{l=1}^{n} x(t - \tau_l) d\tau_l,$$  

(6)

The last summand in right side (6) is the methodological error during identification of partial constituent $y_k[x(t)]$. The less its absolute value the more accurate the $k$–th term of Volterra series defined after experimental data. $\varepsilon(a_j)$ are the terms of series (1) of $(N+1)$–th order and higher. Using triangle inequality and replacing the function of series reminder $\varepsilon(a_j)$ with its maximum value we can write the last summand of (6) as:

$$\sum_{i=1}^{N} c_j \sum_{n=N+1}^{\infty} y_n[a_j, x(t)] \leq \max_{1 \leq j \leq N} \sum_{i=1}^{N} c_i \left| y_n[a_j, x(t)] \right| \sum_{i=1}^{N} |c_i|,$$  

(7)

To minimize the impact of Volterra series reminder on error of partial constituent identification of the test object response we need to provide the minimal sum of absolute values of the coefficients $c_i$ which are defined of equations system (5):

$$e = \sum_{j=1}^{N} |c_j| = \sum_{j=1}^{N} \left| \sum_{a=1}^{N} a_{j}^a \delta^a_k \right| = \sum_{j=1}^{N} |a^{-1}_j| =$$  

$$= \frac{1}{\det A} \sum_{j=1}^{N} |M_{jk}| = \min_{1 \leq k \leq N},$$  

(8)

where $\delta^a_k$ – Kronecker symbol, $\delta^a_k = 0$ if $n\neq k$ and $\delta^a_k = 1$ if $n=k$; $a^{-1}_{jk}$ – elements of inverse matrix of coefficients $A$:

$$A = \begin{pmatrix} a_1 & a_2 & \ldots & a_N \\ \vdots & \vdots & \ddots & \vdots \\ a_1 & \ldots & \ldots & a_N \\ a_N & a_2 & \ldots & a_N \end{pmatrix},$$  

(9)

where $\det A$ – determinant, $M_{jk}$– matrix $A$ minors.
According to (8) the problem of minimal methodological error supplying using approximation method of identification is come to local minimum of function of several variables defining, in other words the minimal sum of absolute values of coefficients in linear combination of responses of control object.

The optimal values of the amplitudes and corresponding coefficients we defined using different amplitudes exhaustive search procedure and every time solving the (8) for it [6]. The search interval is specified by inequalities (5). The defined optimal amplitudes of test effects for different orders of approximation model \( N = 1, 2, 3 \) and determined Volterra kernels \( 1 \leq m \leq N \) are shown in a table 1 (search interval \([-1, 1]\)).

Table 1. Optimal amplitudes of test effects in interval \([-1, 1]\)

| \( N \) | \( m \) | \( a_i \) | \( c_i \) | \( \min \sum_{i=1}^{N} |c_i| \) |
|---|---|---|---|---|
| 1 | 1 | -1 | -1 | 1 |
| 2 | 1 | -1 | -0.5 | 1 |
| | | 1 | 0.5 | 1 |
| | 2 | -1 | 0.5 | 1 |
| | | 1 | 0.5 | 1 |
| 3 | 1 | -1 | 0.33 | 3 |
| | | -0.5 | -2 | 3 |
| | | 0.5 | 0.67 | 3 |
| 2 | -1 | 0.5 | 1 |
| | | 0 | 0 | 1 |
| | | 1 | 0.5 | 4 |
| 3 | -1 | 0.33 | -2.67 | 4 |
| | | 0.5 | 1 | 4 |

The test polyharmonic effects for identification in the frequency domain representing by signals of such type:

\[
x(t) = \sum_{k=1}^{n} A_k \cos(\omega_k t + \varphi_k),
\]

(10)

where \( n \) – the order of transfer function being estimated; \( A_k, \omega_k \) and \( \varphi_k \) – accordingly amplitude, frequency and a phase of \( k \)-th harmonics. In research, it is supposed every amplitude of \( A_k \) to be equal, and phases \( \varphi_k \) equal to zero.

Thus, the test signal can be written in the complex form:

\[
x(t) = \sum_{k=1}^{n} A_k \cos(\omega_k t + \varphi_k) = \frac{A_n}{2} \sum_{k=1}^{n} \left( e^{j\omega_k t} + e^{-j\omega_k t} \right),
\]

(11)

Then the \( n \)-th partial component in the response of system can be noted in an aspect:

\[
y_n(t) = A_n^2 \sum_{k=1}^{n} C_k e^{j\omega_k t},
\]

(12)

\[
\cdots \sum_{k=1}^{n} W_n \left( j\omega_k, \ldots, j\omega_{k_n} \right) \times \cos \left( \sum_{k=1}^{n} \omega_k t + \arg W_n \left( j\omega_k, \ldots, j\omega_{k_n} \right) \right)
\]

(13)

here \( \cdots \) mean function of extraction of an integer part of number.

The component with frequency \( \omega_1 + \ldots + \omega_n \) is selected from the response to a test signal (10):

\[
A^* W_n(j\omega_1, \ldots, j\omega_n) \cos \left( \sum_{k=1}^{n} \omega_k t + \arg W_n(j\omega_1, \ldots, j\omega_n) \right)
\]

In [7] it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies which provide an inequality of combination frequencies in output signal harmonics.

Described method was tested on a nonlinear test object – amplitude detector described by Riccati equation:

\[
dy(t) + \alpha \cdot y(t) + \beta \cdot y^2(t) = u(t).
\]

(14)

Analytical expressions of AFC and PFC for the first and second order model where received:

\[
| W_1(j\omega) | = \frac{1}{\sqrt{\alpha^2 + \omega^2}},
\]

(15)

\[
\arg W(j\omega) = -\arctg \frac{\omega}{\alpha},
\]

(16)

\[
| W_2(j\omega_1, j\omega_2) | = \frac{\beta}{\sqrt{(\alpha^2 + \omega_1^2)(\alpha^2 + \omega_2^2) \times [\alpha^2 + (\omega_1 + \omega_2)^2]}}
\]

\[
\arg W(j\omega_1, j\omega_2) = -\arctg \frac{2\alpha^2 - \omega_1 \omega_2}{\alpha(\alpha^2 - \omega_1^2) - \alpha(\omega_1 + \omega_2)}.
\]

(17)

(18)

Results (first order AFC and PFC) received after procedure of identification are presented in fig. 1 (approximation order of the model \( N = 4 \)).
Fig. 1. First order AFC and PFC of the test object: analytically calculated values (1), section estimation values with approximation order of the model $N=4$ (2)

Results (second order AFC and PFC) received after procedure of the identification are presented in fig. 2 (approximation order of the model $N=2$ and $N=4$).

Fig. 2. Subdiagonal sections of the second order AFC and PFC of the test object: analytically calculated values (1), section estimation values with approximation order of the model $N=2$ (2), $N=4$ (3)

3 The technique and hardware-software tools of radiofrequency CC identification

Experimental research of an Ultra High Frequency range CC for the purpose of identification of its multifrequency performances, characterizing nonlinear and dynamic properties of the channel are fulfilled. The Volterra model in the form of the second order polynomial is used. Thus physical CC properties are characterized by transfer functions $W_1(j2\pi f)$ and $W_2(j2\pi f_1 j2\pi f_2)$ − by the Fourier-images of weighting functions $w_1(t)$ and $w_2(t_1, t_2)$.

Implementation of the identification method on the IBM PC computer basis has been carried out using the developed software in C++ language with the usage of such classes as CWaveRecorder, CWavePlayer, CWaveReader, CWaveWriter which allow to provide rather convenient interacting with MMAPI Windows. The software allows automating the process of the test signals forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of PC soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the CC responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical S.P.RADIO A/S, RT2048VHF VHF–radio stations (a range of operational frequencies 154,4–163,75 MHz) and IBM PC with Creative SBLive! sound cards were used. Sequentially AFC of the first and second orders were defined. The method of identification with an order of approximation $N=4$ was applied. Structure charts of identification procedure – determinations of the $n$-order AFC of CC are presented accordingly on fig. 3. The general scheme of a hardware–software complex of the CC identification, based on the data of input–output type experiment is presented in fig. 4.

Fig. 3. The structure chart of identification procedure of the $n$-order AFC

Fig. 4. The general scheme of the experiment
The CC received responses \( y[a, x(t)] \) to the test signals \( a, x(t) \), compose a group of the signals, which amount is equal to the used number of experiments \( N \) (\( N=4 \)). In each following group the signals frequency increases by magnitude of chosen step. A cross-correlation was used to define the beginning of each received response. Information about the form of the test signals, amplitudes and corresponding to them coefficients given in [6] were used.

Maximum allowed amplitude in described experiment with use of the sound card was \( A=0.25 \text{V} \) (defined experimentally). The used range of frequencies was defined by the sound card pass band \((20...20000 \text{ Hz})\), and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the main experiment: start frequency \( f_1 =125 \text{ Hz} \); final frequency \( f_2 =3125 \text{ Hz} \); a frequency change step \( \Delta f=125 \text{ Hz} \); to define AFC of the second order determination, an offset on frequency \( \delta f=f_2-f_1 \) was increasingly growing from \( 201 \) to \( 3401 \text{ Hz} \) with step \( 100 \text{ Hz} \).

The weighed sum is formed from received signals – responses of each group. As a result we get partial components of response of the CC \( y_1(t) \) and \( y_2(t) \). For each partial component of response a Fourier transform (the FFT is used) is calculated, and from received spectra only an informative harmonics (which amplitudes represents values of required characteristics of the first and second orders AFC) are taken.

The first order amplitude-frequency characteristic \( |W_1(j2\pi f)| \) is received by extracting the harmonics with frequency \( f \) from the spectrum of the partial response of the CC \( y_1(t) \) to the test signal \( x(t)=A\cos2\pi ft \).

The second order AFC \( |W_2(j2\pi f_1j2\pi f_2(f+\delta f))| \), where \( f_1=f \) at \( f_2=f+\delta f \), was received by extracting the harmonics with summary frequency \( f_1+f_2 \) from the spectrum of the partial response of the CC \( y_2(t) \) to the test signal \( x(t)=A\cos2\pi f_1t+A\cos2\pi f_2t \).

The results received after digital data processing of the data of experiments (‘Coiflet’ wavelet de-noising) for the first order AFC are presented in fig.5 [8].

The results received after digital data processing of the data of experiments (‘Reverse Biorthogonal’ wavelet de-noising) for the second order AFC are presented in fig. 6 and 7.

4 Conclusions
The method of the determinate identification of the nonlinear dynamical systems based on Volterra
models using polyharmonic test signals is analyzed. To differentiate the object response for partial constituents we use the method based on composition of linear responses combination on test signals with different amplitudes.

New values of amplitudes are defined and they are greatly raising the accuracy of the identification in compare with amplitudes and coefficients written in [9].

The modified approximation method of the identification using the methodology written in [10] is applied for the constructing of the informational Volterra model as an APC of the first and second order for UHF band radio channel.

The received result of the researches showing substantial nonlinearity of the CC and that is the reason of signal distortion in the radio channel. This reduces such important parameters of the TCS: exactness of the reconstitution of the signals, bandwidth and noise immunity of the CC. Practical use of this research lies in the further researches for filters construction to compensate distortions in the CC.

References: