An Analysis of Errors in RFID SAW-Tag Systems with Pulse Position Coding

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Abstract: We analyze the code reading error probability (EP) in the radio frequency identification surface acoustic wave (SAW) tags with pulse position coding (PPC) and peak-pulse detection. EP is found in a most general form assuming *M* groups of codes with *N* slots each and allowing individual signal-to-noise ratios (SNRs) in each slot. The basic case of zero signal in all Off-pulses and equal signals in all On-pulses is investigated in detail. We show that if a SAW-tag with PPC is designed such that the spurious responses are attenuated on more than 20 dB below On-pulses, then EP can be achieved at the level of 10^{-8} (one false per 10^8 readings) with SNR $>$ 17 dB for any reasonable M and N.

Key–Words: RFID surface acoustic wave tag, error probability

1 Introduction

The radio frequency identification (RFID) tags designed using surface acoustic wave (SAW) substrates have found industrial applications owing to several attractive features. Passive wireless SAW-tags can be read at a distance of more than 10 m with the reader radiating $<$ 10 mW [1]. They are cheap, robust, can work in harsh environment such as high temperatures or high levels of ionizing radiation, and demand no maintenance. First SAW-tags were invented in 1970s [2] and used in industry in 1980s [3]. Originally, binary coding was implemented [4] similarly to binary amplitude shift keying (BASK) used in communications. Today, SAW tag products employ mostly the pulse position coding (PPC) scheme proposed in [1]: the total time delay is divided into groups of slots with one On-pulse in each group [2, 5]. We find such a design having 2 groups with 4 slots each in [6, 7]. A 5 digit decimal number (10 slots) design is described in [8] and a 16×4 designs was addressed in [2, 11].

It has to be remarked that the longer the code is the larger the probability of false reading must be expected. Since the code reading *error probability* (EP) strongly depends on noise and signal power at the reader, the effective distance to the SAW tag is limited. Note that attempts were made in [12] to specify EP via the threshold and the first results on EP in peak-pulse detection have recently been published in [11].

Below, we show an exact formula for the code reading EP of RFID SAW tags with PPC and peak-pulse detection and sketch some recommendations for designers.

2 RFID SAW-Tag Structure

A structure of the SAW-tag with PPC is sketched in Fig. 1. Equal slots of time duration comparable to $1/B$ (B is the tag frequency bandwidth, in Hz) are allocated for responses and the center of each slot is dashed. Only a unique response (On-pulse) is allowed in each group of slots. The groups are separated by additional “guard” slots. The code-reflector array is designed such that the responses have near equal amplitudes at the reader. The reader time scale is calibrated with two reflectors depicted as “Start” and “End”.

A typical response of a tag manufactured to have 6 groups with 16 slots each is shown in Fig. 2. Note that this 6×16 slot structure allows for about 1.7×10^6 different codes (24 bits). In addition to the main responses one can also watch in Fig. 2 an intensive response preceding “End” and representing the checksum used for error control. Observing Fig. 2, one can note the following specifics: 1) Although On-pulses are supposed to have equal peak-amplitudes, measurement reveals variations. Noise is able to enforce this difference, 2) The noise floor is not uniform and im-

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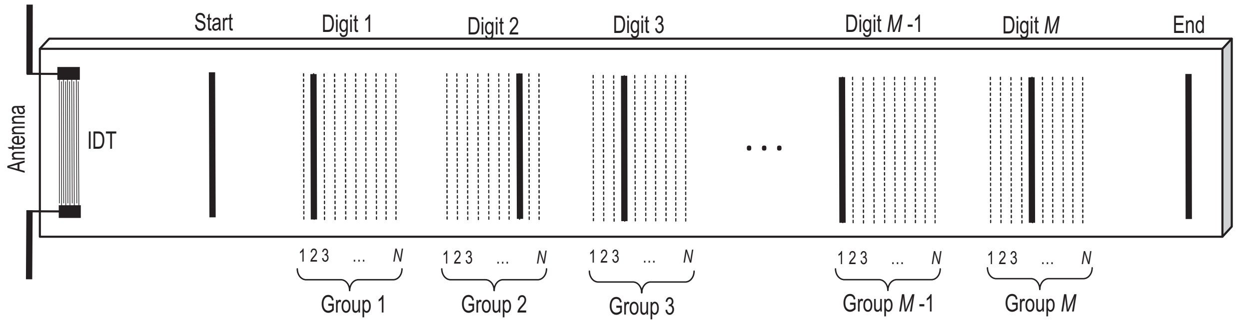

Figure 1: A generalized structure of the SAW-tag with time position coding: each group has N slots with one reflector and coding is organized with M groups.

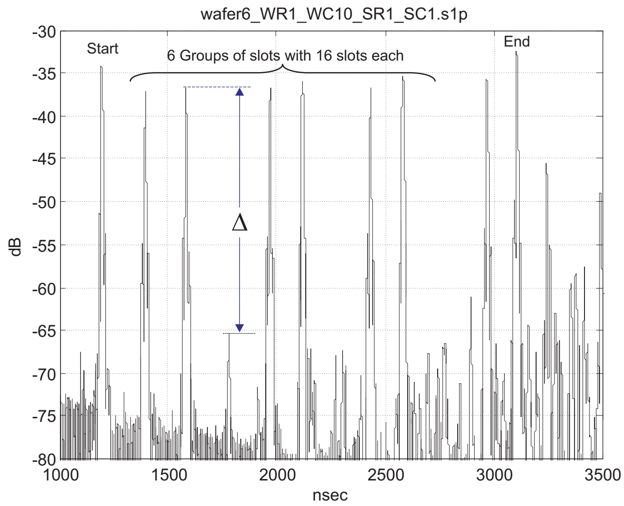

Figure 2: Typical time response of a SAW-tag with PPC manufactured to have 6 groups with 16 slots each. "Start" and "End" pulses are used to calibrate time scale.

pulsive noise may occur in Off-pulses. See, for example, excursions between the 2nd and 3rd as well as 4th and 5th On-pulses.

3 Code Reading Error Probability

The code reading EP can be determined by the ratio of the number of false reading to the total number of readings. In turn, in the mth group of slots, EP can be defined by the ratio of false reading of the On-pulse to the total number of reading.

3.1 Error in a Single Group

Consider the mth group of N slots (Fig. 1). If we suppose that On-pulse appears in the nth slot in the presence of noise then the response picture can be sketched as in Fig. 3. Designer tries to create the system such that signal is always larger than noise. In

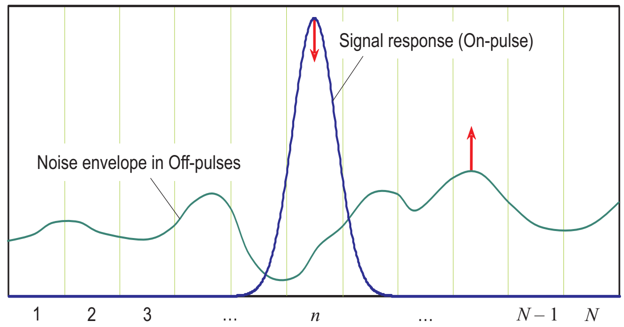

Figure 3: Signal response in the nth slot and noise envelope in the mth group of N slops.

turn, Nature may act in an opposite direction (see arrows in Fig. 3) so that On-pulse can suddenly find itself below noise that leads to false reading.

Let us suppose that the RF On-pulse representing the nth slot in mth group arrives at the reader with the peak power $2S_{n,m}$. Here, the pulse is contaminated by narrowband noise having the variance $\sigma^2_{n,m}$ and becomes noisy with the peak-envelope $V_{n,m} \geqslant 0$ [12, 13]. We thus can introduce the SNR $\gamma_{n,m}$ and normalized peak-envelope $z_{n,m}$ as, respectively,

$$\gamma_{n,m} = \frac{S_{n,m}}{\sigma^2_{n,m}}, \tag{1}$$

$$z_{n,m} = \frac{V_{n,m}}{\sqrt{2\sigma^2_{n,m}}}. \tag{2}$$

Since location of the On-pulse does not affect errors, we replace it to the last Nth slot. Normal operation of the mth group is implied if On-pulse in the Nth slot exceeds Off-pulses in the remaining ones, from 1 to $N-1$. The events when each Off-pulse does not exceed On-pulse are depicted as $\bar{\mathrm{C}}_{1,m}$, $\bar{\mathrm{C}}_{2,m}$, ..., $\bar{\mathrm{C}}_{N-1,m}$. Then the probability of simultaneous occurring of these events can be written as

$$P(\bar{\mathbf{C}}_m|\mathbf{\Upsilon}_m, z_{N,m}), \tag{3}$$

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where $\bar{\mathbf{C}}_m = [\,\bar{\mathrm{C}}_{1,m}, \bar{\mathrm{C}}_{2,m}, \ldots, \bar{\mathrm{C}}_{N-1,m}\,]$ and $\boldsymbol{\Upsilon}_m = [\,\gamma_{1,m}, \gamma_{2,m}, \ldots, \gamma_{N,m}\,]$, under the condition that $z_{N,m}$ and $\boldsymbol{\Upsilon}_m$ are given. The probability that at least one Off-pulse exceeds On-pulse (event $\hat{\mathrm{C}}_m$) is

$$P(\hat{\mathrm{C}}_m|\boldsymbol{\Upsilon}_m, z_{N,m}) = 1 - P(\bar{\mathbf{C}}_m|\boldsymbol{\Upsilon}_m, z_{N,m})\,. \quad (4)$$

Now assign the probability that On-pulse has the envelope $z_{N,m}$ (event D_m), given $\gamma_{N,m}$, as

$$P(z_{N,m}|\gamma_{N,m}) = P(\mathrm{D}_m|\gamma_{N,m})\,. \quad (5)$$

Because $z_{N,m}$ is fully determined, with a small tolerance $\Delta Z_{N,m} \ll 1$, by the probability density function (pdf) $p(z_{N,m}|\gamma_{N,m})$ of the On-pulse envelope, $P(z_{N,m}|\gamma_{N,m})$ can also be represented as

$$\begin{aligned} P(z_{N,m}|\gamma_{N,m}) &= \int_{z_{N,m}}^{z_{N,m}+\Delta z_{N,m}} p(x|\gamma_{N,m})\,dx \\ &\cong p(z_{N,m}|\gamma_{N,m})\Delta z_{N,m}\,. \quad (6) \end{aligned}$$

EP $\mathcal{P}_m$ in the mth group can now be defined by simultaneous occurring of the events $\hat{\mathrm{C}}_m$ and D_m, meaning that at least one Off-pulse exceeds On-pulse,

$$\begin{aligned} \mathcal{P}_{em}(z_{N,m}, \gamma_{N,m}) &= P(\hat{\mathrm{C}}_m \mathrm{D}_m) \quad (7a) \\ &= [1 - P(\bar{\mathbf{C}}_m|\boldsymbol{\Upsilon}_m, z_{N,m})] \\ &\quad \times P(z_{N,m}|\gamma_{N,m}) \quad (7b) \\ &\cong [1 - P(\bar{\mathbf{C}}_m|\boldsymbol{\Upsilon}_m, z_{N,m})] \\ &\quad \times p(z_{N,m}|\gamma_{N,m})\Delta z_{N,m} \quad (7c) \end{aligned}$$

For the mth group, EP can thus be found as

$$\mathcal{P}_{em}(\gamma_{N,m}) = \int_0^\infty [1 - P(\bar{\mathbf{C}}_m|\boldsymbol{\Upsilon}_m, x)]p(x|\gamma_{N,m})\,dx\,. \quad (8)$$

Typically, noise in slots is uncorrelated and thus the events $\bar{\mathrm{C}}_{1,m}$, $\bar{\mathrm{C}}_{2,m}$, ..., $\bar{\mathrm{C}}_{N-1,m}$ are uncorrelated as well. Accepting this, next assigning the opposite events when Off-pulse exceeds On-pulse as $\mathrm{C}_{1,m}$, $\mathrm{C}_{2,m}$, ..., $\mathrm{C}_{N-1,m}$, and using the relationships

$$\begin{aligned} P(\bar{\mathbf{C}}_m|z_{N,m}) &= \prod_{n=1}^{N-1} P(\bar{\mathrm{C}}_{n,m}|\gamma_{n,m}, z_{N,m}) \quad (9a) \\ &= \prod_{n=1}^{N-1} [1 - P(\mathrm{C}_{n,m}|\gamma_{n,m}, z_{N,m})]\,, \quad (9b) \end{aligned}$$

where $\mathbf{C}_m = [\mathrm{C}_{1,m}, \mathrm{C}_{2,m}, \ldots, \mathrm{C}_{N-1,m}]$, we come up with the most general form of EP,

$$\begin{aligned} \mathcal{P}_{em}(\boldsymbol{\Upsilon}_m) &= \int_0^\infty \left\{1 - \prod_{n=1}^{N-1} [1 - P(\mathrm{C}_{n,m}|\gamma_{n,m}, x)]\right\} \\ &\quad \times p(x|\gamma_{N,m})\,dx\,, \quad (10) \end{aligned}$$

suitable for the mth group of slots with the individual SNRs in On-pulse and Off-pulses.

3.2 Total Error

Now, assign the event of normal operation of the mth group as $\bar{\mathrm{A}}_m$, meaning that On-pulse exceeds all Off-pulses. Otherwise, the event is A_m. The probability of successful code reading can be defined by the probability of simultaneous normal operation of all of the groups as $P(\bar{A}_1\bar{A}_2\ldots\bar{A}_M)$. The reader makes a mistake if error occurs at least in one of the groups. Because errors in groups can appear independently, the code reading EP can be defined as

$$\begin{aligned} \mathcal{P}_e &= 1 - P(\bar{A}_1\bar{A}_2\ldots\bar{A}_M) \quad (11a) \\ & 1 - \prod_{m=1}^{M} P(\bar{A}_m) \quad (11b) \\ & 1 - \prod_{m=1}^{M} [1 - P(A_m)]\,. \quad (11c) \end{aligned}$$

Here $P(A_m)$ can be substituted with EP (10) in the mth group and we finally come up with the code reading EP for the SAW-tag (12) (see next page).

4 Gaussian Model

In additive narrowband Gaussian noise environment, the normalized envelope z of On-pulse has the Rice pdf [12]

$$p(z_{n,m}, \gamma_{n,m}) = 2z_{n,m}e^{-z_{n,m}^2-\gamma_{n,m}} I_0(2z_{n,m}\sqrt{\gamma_{n,m}})\,, \quad (13)$$

where $I_0(x)$ is the modified Bessel function of the first kind and zeroth order. This pdf becomes Rayleigh's,

$$p(z_{n,m}) = 2z_{n,m}e^{-z_{n,m}^2}\,, \quad (14)$$

for Off-pulses formed only by noise. However, it follows from Fig. 2 that some of Off-pulses can suffer of residual reflections so that, most generally, (13) should be used to describe responses of both On- and Off-pulses. By (13), the probability $P(\mathrm{C}_{n,m}|\gamma_{n,m}, z_{N,m})$ can be rewritten as

$$P(\mathrm{C}_{n,m}|\gamma_{n,m}, z_{N,m}) = \int_{z_{N,m}}^\infty p(x, \gamma_{n,m})\,dx\,. \quad (15)$$

Substituting in (12) $P(\mathrm{C}_{n,m}|\gamma_{n,m}, x)$ with (15) and $p(z_{n,m}, \gamma_{N,m})$ with (13) gives us EP for the Gaussian noise.

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$$\mathcal{P}_e(\boldsymbol{\Upsilon}_m, M, N) = 1 - \prod_{m=1}^{M} \left\{ 1 - \int_0^\infty \left\{ 1 - \prod_{n=1}^{N-1} [1 - P(\mathrm{C}_{n,m}|\gamma_{n,m}, x)] \right\} p(x|\gamma_{N,m})\, dx \right\} . \tag{12}$$

4.1 Basic and Limiting Cases

The case of zero signal in all Off-pulses and equal signals in all On-Pulses can be said to be basic or ideal for RFID SAW tagging. It implies using (13) for all On-pulses and (14) for Off-pulses with $z = z_{n,m}$ and $\gamma = \gamma_{n,m}$. That gives us

$$p(z, \gamma) = 2ze^{-z^2-\gamma} I_0(2z\sqrt{\gamma}) , \tag{16}$$

$$P(\mathrm{C}_{n,m}|z) = 2\int_z^\infty xe^{-x^2}\, dx = e^{-z^2} \tag{17}$$

and transforms (12) to (18) (see next page) that still has no closed form.

The case of $M = 1$ and $N = 2$ is limiting for PPC. It gives us the EP lower bound

$$\mathcal{P}_e(\gamma, 1, 2) = 2e^{-\gamma} \int_0^\infty xe^{-2x^2} I_0(2x\sqrt{\gamma})\, dx \tag{19}$$

that cannot be crossed by any design of SAW-tags with PPC. Fig. 4 illustrates (19) as a function of SNR. It also shows several limiting EPs computed by (18) for tags having $M > 1$ and $N > 2$. These errors inherently trace above (19). Errors computed by simulation are depicted with circles and bars.

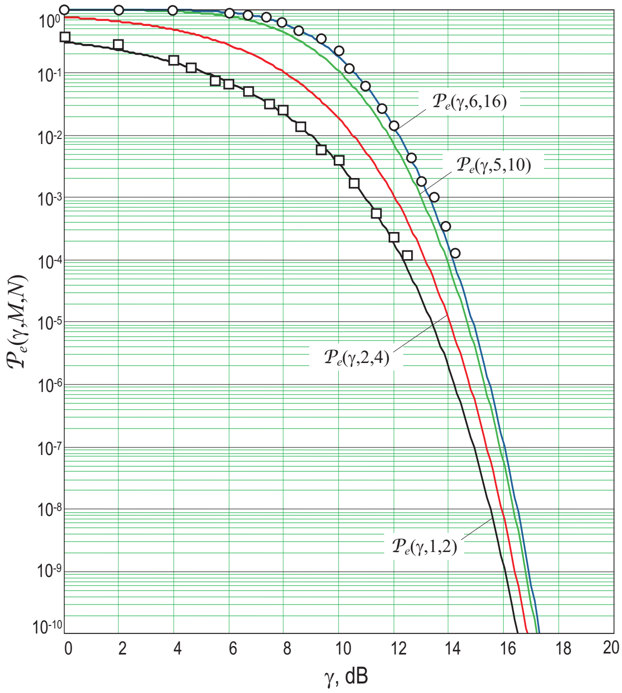

Figure 4: Code reading EP for the SAW-tag with PPC having M groups of N slots each as a function of SNR, $10\log(\gamma)$, dB. Simulation is depicted with ○ and □, provided 2×10^5 readings for each SNR.

5 Effect of Spurious Responses

Measurement shows that the SAW-tag time response is accompanied with a number of residuals neatly seen in Fig. 2. The difference Δ dB between the main and spurious responses fundamentally depends on the SAW-tag design. To evaluate effect of Δ on EP, the general relation (12) is required.

Allowing $M = 2$ and $N = 4$ and supposing that a single spurious response in Off-pulse is attenuated with respect to the On-pulse on Δ dB, we calculate EP for different Δ as shown in Fig. 5. Several important observations can be made observing this plot. When noise dominates, $\gamma < 0$ dB, the EP tends toward unity irrespective of Δ. If $\gamma > 0$ dB, we recognize the following specifics. With $\Delta = 0$ dB, the responses in On- and Off-pulses have equal amplitudes and EP in this isolated case tends with $\gamma \gg 0$ to 0.5. When $\Delta < -20$ dB, effect of the spurious response on EP is negligible. Since practical designs guarantee $\Delta < -20$ dB, we conclude that spurious responses can be neglected and EP specified with the basic curves shown in Fig. 4.

6 Effect of External Impulsive Noise

For $M = 1$ and $N = 2$, we now suppose that On-pulse arrives at the reader with γ and impulsive noise in Off-pulse has SNR γ_{off} as shown in Fig. 6. Here, $\mathcal{P}_e(\gamma, 1, 2)$ represents EP for white Gaussian noise (see Fig. 4). In order to figure out the effect of impulsive noise, EP was calculated for several values of γ_{off} as shown in Fig. 6 that allowed us to make the following conclusion. As long as signals in On- and Off-pulses exist independently, PE of 0.5 occurs when their peak-envelopes are equal. In fact, one can see in Fig. 6 that a vertical line associated with each γ crosses EP calculated for $\gamma = \gamma_{\text{off}}$ at the level of 0.5. Thus, large γ_{off} may cause dramatic increase in EP. For example, $\mathcal{P}_e < 10^{-6}$ can easily be achieved in white Gaussian noise environment with $\gamma > 14$ dB,

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$$\mathcal{P}_e(\gamma, M, N) = 1 - \left\{ 1 - 2e^{-\gamma} \int_0^{\infty} \left[1 - \left(1 - e^{-x^2}\right)^{N-1} \right] x e^{-x^2} I_0(2x\sqrt{\gamma})\, dx \right\}^M . \quad (18)$$

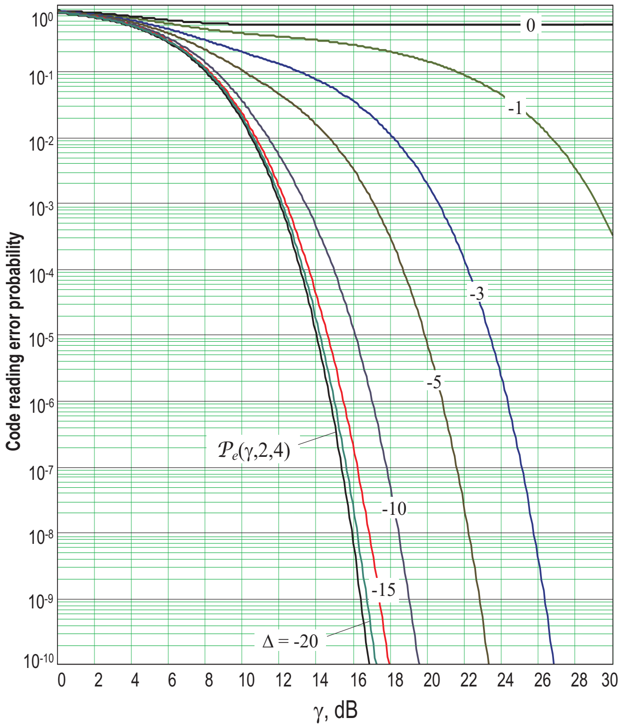

Figure 5: Code reading EP for the SAW-tag with $M = 2$ and $N = 4$. Effect of a spurious response attenuated on Δ dB below On-pulse.

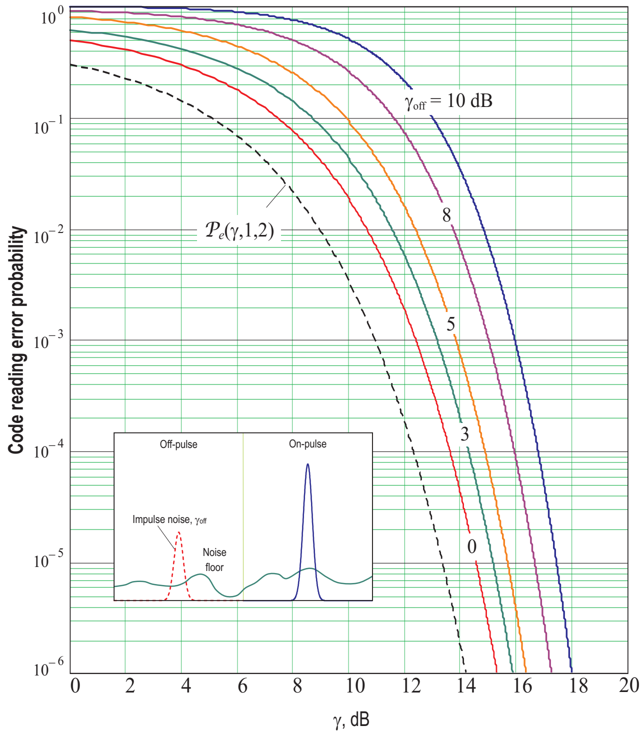

Figure 6: Code reading EP for $M = 1$ and $N = 2$. Effect of impulsive noise with γ_{off} dB in Off-pulse.

whereas impulsive noise with $\gamma_{\text{off}} = 5$ dB is able to elevate it to 10^{-3}.

6.1 Single Impulsive Noise in Tags with $M = 6$ and $N = 16$

EP for the tag with $M = 6$ and $N = 16$ (Fig. 2) and a single noise pulse was calculated as shown in Fig. 7.

Because this tag consists of 90 Off-pulse slots, effect of the external impulsive noise has appeared to be a bit lesser pronounced, although it is still strong. One can deduce that $\mathcal{P}_e = 10^{-7}$ achieved with $\gamma = 16$ dB in white Gaussian noise can be elevated to 10^{-4} if $\gamma_{\text{off}} = 8$ dB. On the other hand, to guarantee $\mathcal{P}_e = 10^{-8}$ with $\gamma_{\text{off}} = 10$ dB, the SNR $\gamma > 19$ dB must be obtained in On-pulse that can be done easily.

One more observation can be pointed out in addition, by comparing Fig. 6 and Fig. 7. By increasing γ_{off}, EPs converge irrespective of the design. In fact, there is almost no difference in EPs in Fig. 6 and Fig. 7 if $\gamma_{\text{off}} = 10$ dB. The latter means that when the impulsive noise substantially dominates the noise floor and becomes comparable with On-pulse, the EP is basically determined by 2 slots with On- and Off-pulses, being virtually poor dependent on the $M \times N$ tag structure. There should also exist an approximate relation for EP implying any reasonable M and N.

7 Conclusions

We have examined the code reading EP for passive wireless remote SAW-tag systems with PPC and peak-pulse detection. In a basic case of zero signal in all Off-pulses and equal signals in On-pulses, EP has a compact form. Its analysis allows producing some recommendations to determine the reader range or transmitted power as functions of SNR and EP for wirelessly remote SAW-tags.

It follows that if a SAW-tag with PPC is designed such that the spurious responses are attenuated on more than 20 dB below On-pulses, then EP of code reading over peak-pulse detection can be achieved at the level of 10^{-8} (one false per 10^8 readings) with SNR > 17 dB for any reasonable M and N.

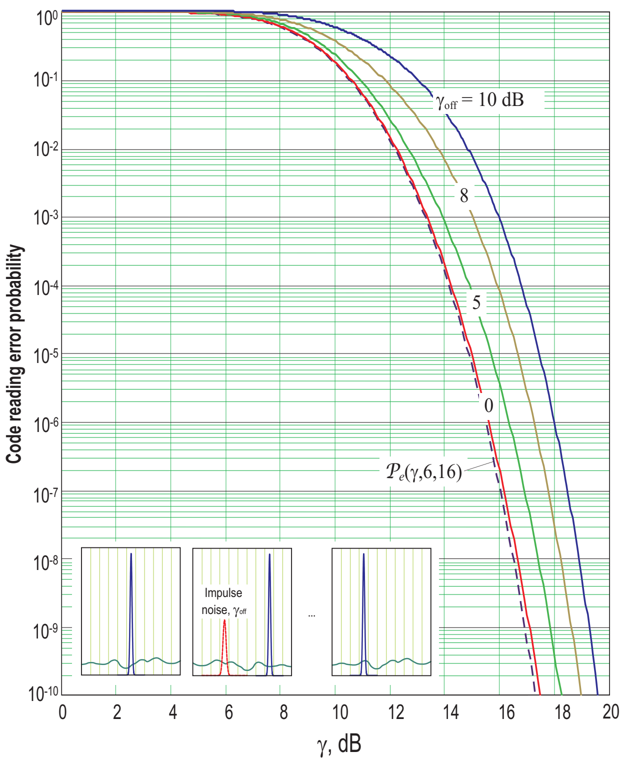

Figure 7: Code reading EP for $M = 6$ and $N = 16$. Effect of impulsive noise with SNR γ_{off} dB in one of Off-pulses.

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