Position Function in mathematics

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Abstract: - The *Position Function* is a new and original mathematical function introduced by the author into the mathematical domain it can be used in numerous fields and domains as signal theory, signal processing, mathematics, physics and many others. The main goal of this function is to give as an output two values which are (1 and -1) whatever is the input and whatever is the coordinate system we are working on, for example the same function can be used in Cartesian coordinate system, in Spherical and Cylindrical coordinate system or any other system. This function is similar to the *sign function* in which it gives 3 values (1, 0 and -1) in the Cartesian coordinate system only, but the difference is that the *Position function* gives two values (1 and -1) not only in the Cartesian Coordinate system but in any other Coordinate system. The *Position Function* will by widely used due to its importance. It has many advantages that are discussed in this paper.

Key-words:-Position function, coordinate systems, periodic signal, step function, saturation function, mathematics.

1 Introduction

In mathematics, the *Position Function* is a new and original mathematical function introduced by the author into the mathematical domain, this function determine the position of a point into a certain coordinate system and gives it a value (1) if the point is at the positive part of an axis, or -1 if the point is at the negative part of an axis. For example, in the Cartesian Coordinate system [1] [2] if a point $M\,(x,y)$ has the coordinate ($x=3, y=-2$) therefore the *Position function* $P_x(M(x,y))=1$ because the coordinate (x) of the point is in the positive part of the axis $(x'ox)$, and the *Position function* $P_y(M(x,y))=-1$ because the coordinate (y) of the point is in the negative part of the axis $(y'oy)$. The same thing can be used for other coordinate systems[3] [4] [5], for example in the cylindrical coordinate system [9] which is a three dimensional system using three coordinates as (ρ,θ,z) and suppose that the point M (ρ,θ,z) has the following values M($\rho=2, \theta=-25^0, z=-3$) therefore the *Position function* $P_\rho(M(\rho,\theta,z))=1$ because the coordinate (ρ) of the point is in the positive part of the axis $(o\rho)$, the *Position function* $P_\theta(M(\rho,\theta,z))=-1$ because the coordinate (θ) of the point is in the negative part of the axis $(o\theta)$ between $(2k+1)\pi$ and $(2k+2)\pi$. And finally, the *Position function* $P_z(M(\rho,\theta,z))=-1$ because the coordinate (z) of the point is in the negative part of the axis (oz). This method can be applied in any coordinate system [6] [7] [8]. And one can deduce the importance of this function. In this paper a brief study of the mentioned function is introduced and few examples are treated in order to give an idea about how to use this function and what is its importance.

A definition of the Position Function is presented in the second section, in the third section some examples are treated using the Position function. And finally a conclusion is presented into the fourth section.

2 Definition of the Position Function

The *Position Function* is defined as the following:

$$P_{i_m}(M(i_1,i_2,i_3,\ldots,i_n)) = \begin{cases} +1 \; if\ the\ position\ of\ the\ point\ M(i_1,i_2,i_3,\ldots,i_n) \\ \quad on\ the\ axis\ i_m\ is\ in\ the\ positive\ region \\ \\ -1 \; if\ the\ position\ of\ the\ point\ M(i_1,i_2,i_3,\ldots,i_n) \\ \quad on\ the\ axis\ i_m\ is\ in\ the\ negative\ region \end{cases}$$
(1)

With

• $n \in \mathbb{N}^*$, n should be a real number different from zero in which it is considered as the number of dimensions in a specific coordinate system.
• $\in [1, n]$, is a number between 1 and n. It represents the dimension worked on.
• The positive axis includes 0. Therefore the positive part is considered when the point $M(i_1, i_2, i_3, \dots, i_n)$ is ≥ 0 for a certain dimension
• The negative axis does not include 0. Therefore the negative part is considered when the point $M(i_1, i_2, i_3, \dots, i_n)$ is < 0 for a certain dimension.

This function can be used in any kind of coordinate system as Cartesian, Cylindrical, Spherical, Parabolic Cylindrical, Paraboloidal, Oblate Spheroidal, Prolate Spheroidal, Ellipsoidal, Elliptical Cylindrical, Toroidal, Bispherical, Bipolar Cylindrical

2.1 E ample in the Cartesian coordinate system

Let's consider a three dimensional Cartesian coordinate system with coordinates (x, y, z) and a point M included in this system with $M(x, y, z)$. And let's take values for the point $M(x = -3, y = 4, z = -7)$ in order to get results.

Therefore,

• The Position function of the Point M on the axis (ox) is equal to:

$$P_x(M(x,y,z)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; ox \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; ox \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_x(M(x = -3, y = 4, z = -7)) = -1$

• The Position function of the Point M on the axis (oy) is equal to:

$$P_y(M(x,y,z)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; oy \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; oy \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_y\big(M(x = -3, y = 4, z = -7)\big) = +1$

• The Position function of the Point M on the axis (oz) is equal to:

$$P_z(M(x,y,z)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; oz \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(x,y,z) \\ \quad on \; the \; axis \; oz \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_z\big(M(x = -3, y = 4, z = -7)\big) = -1$

2.2 E ample in the Spherical coordinate system

Let's consider a three dimensional Spherical coordinate system with coordinates (r, θ, φ) and a point M included in this system with $M(r, \theta, \varphi)$. And let's take values for the point $M(r = 3, \theta = 45^0, \varphi = 245^0)$ in order to get results.

Therefore,

• The Position function of the Point M on the axis (or) is equal to:

$$P_r(M(r,\theta,\varphi)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; or \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; or \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_r(M(r = 3, \theta = 45^0, \varphi = 245^0)) = 1$

• The Position function of the Point M on the axis $(o\theta)$ is equal to:

$$P_\theta(M(r,\theta,\varphi)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; o\theta \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; o\theta \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_\theta(M(r = 3, \theta = 45^0, \varphi = 245^0)) = 1$

• The Position function of the Point M on the axis $(o\varphi)$ is equal to:

$$P_\varphi(M(r,\theta,\varphi)) = \begin{cases} +1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; o\varphi \; is \; in \; the \; positive \; region \\ \\ -1 \; if \; the \; position \; of \; the \; point \; M(r,\theta,\varphi) \\ \quad on \; the \; axis \; o\varphi \; is \; in \; the \; negative \; region \end{cases}$$

$\Rightarrow P_\varphi(M(r = 3, \theta = 45^0, \varphi = 245^0)) = -1$

Because the angle is between $(2k + 1)\pi$ and $(2k + 2)\pi$

3 E amples sing the Position Function

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In this section, some examples are presented using the Position function in order to give an idea about the importance of this new function.

E amples in the Cartesian coordinate system

• Let's consider the following expression:

$y(x) = -P_x(x+1)x^2 + 2x - 1 + P_x(x+1)$

This expression can be written as two different equations for $x+1 \geq 0$ and for $x+1 < 0$

Therefore:

$$y(x) = -P_x(x+1)x^2 + 2x - 1 + P_x(x+1) = \begin{cases} -x^2 + 2x & for\ (x+1) \geq 0 \Rightarrow x \geq -1 \\ x^2 + 2x - 2 & for\ (x+1) < 0 \Rightarrow x < -1 \end{cases} \quad (2)$$

The graph of this function is as the following:

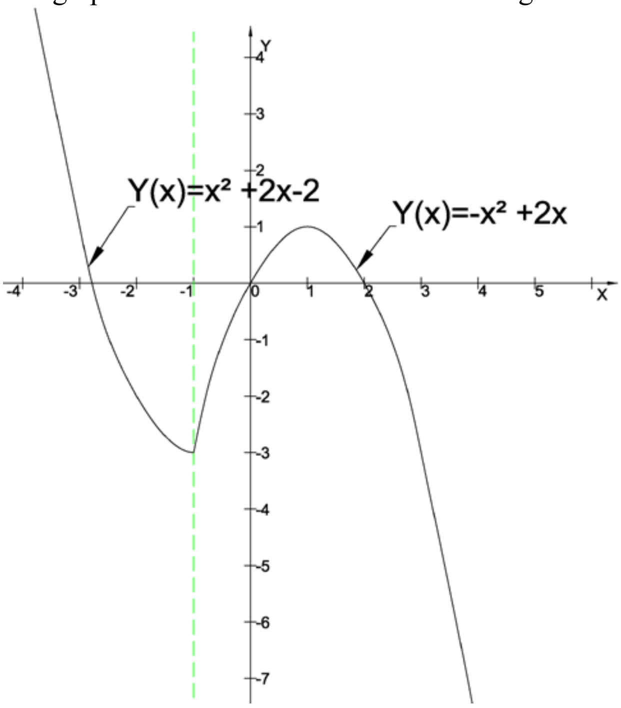

Figure 1: represents the function $y(x) = -P_x(x+1)x^2 + 2x - 1 + P_x(x+1)$ in 2D Cartesian coordinate system

• Let's consider the following expressions:

$$y_1(x) = P_x(x+a) = \begin{cases} +1\ for\ x \geq -a \\ -1 for\ x < -a \end{cases} \quad (3)$$

with $a \geq 0$

$$y_2(x) = P_x(x-b) = \begin{cases} +1\ for\ x \geq b \\ -1 for\ x < b \end{cases} \quad (4)$$

with $b \geq 0$ and $b > a$

Some important expressions are created using the combination of these two equations

1- $y(x) = y_1(x) \cdot y_2(x) =$

$$\begin{cases} +1\ for\ x < -a\ and\ x \geq b \\ -1 for\ x \geq -a\ and\ x < b \end{cases} \quad (5)$$

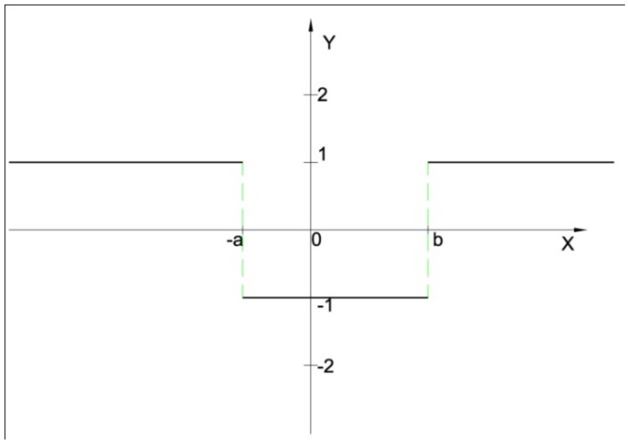

Figure 2: represents the function $y(x) = y_1(x) \cdot y_2(x)$ in 2D Cartesian coordinate system

2- $y(x) = \frac{y_1(x) \cdot y_2(x) + 1}{2} =$

$$\begin{cases} +1\ for\ x < -a\ and\ x \geq b \\ 0\ \ for\ x \geq -a\ and\ x < b \end{cases} \quad (6)$$

This function is called death zone function because in a certain region $[-a, b[$ the value is equal to zero.

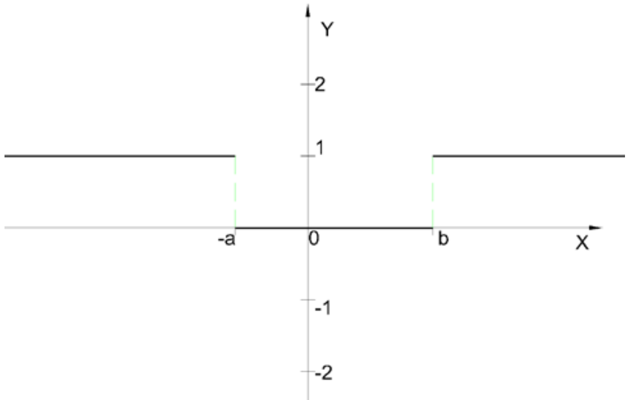

Figure 3: represents the function $y(x) = \frac{y_1(x) \cdot y_2(x) + 1}{2}$ in 2D Cartesian coordinate system

3-

$y(x) = \frac{-y_1(x) \cdot y_2(x) + 1}{2} =$

$$\begin{cases} 0\ for\ x < -a\ and\ x \geq b \\ +1\ \ for\ x \geq -a\ and\ x < b \end{cases} \quad (7)$$

This function is similar to the rectangular function with

$$Rect_{[-a,b]}(x - \frac{(-a+b)}{2}) = \begin{cases} +1\ \ if\ \ x \in [-a, +b] \\ 0\ \ \ \ if\ \ x \notin [-a, +b] \end{cases}$$

The graph of this function is as the following:

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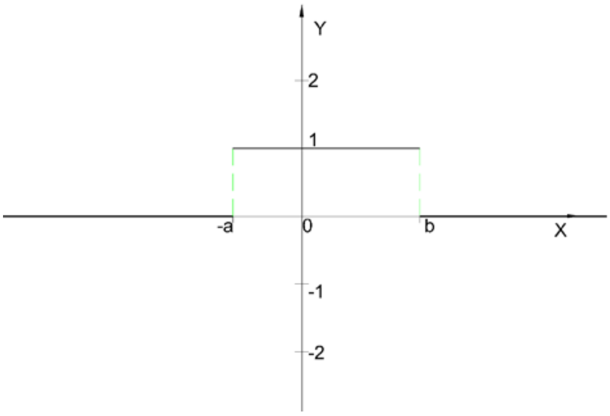

Figure 4: represents the function $y(x) = \frac{-y_1(x)\cdot y_2(x)+1}{2}$ in 2D Cartesian coordinate system

4-

$$y(x) = \frac{-y_1(x)\cdot y_2(x)+1}{2}(x+c) + \frac{y_1(x)\cdot y_2(x)+1}{2}\left(\frac{y_1(x)-1}{2}(a-c) + \frac{y_2(x)+1}{2}(b+c)\right) \quad (8)$$

$$\Longrightarrow y(x) = \begin{cases} b+c \; for \; x \geq b \\ x \;\; for \; x \geq -a \; and \; x < b \\ -a+c \; for \; x < -a \end{cases}$$

The graph of this function is as the following:

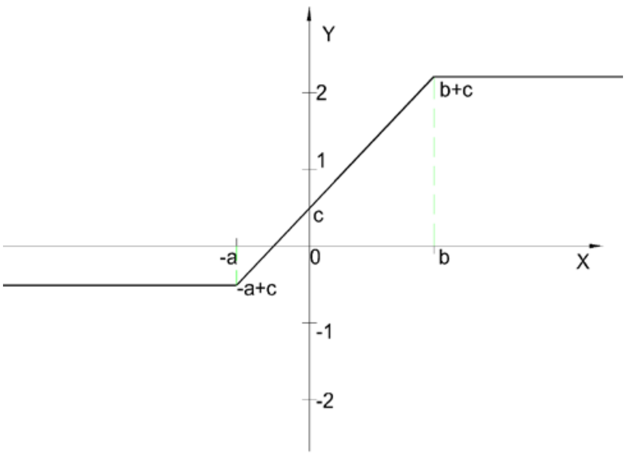

Figure 5: represents the function $y(x)$ in 2D Cartesian coordinate system

This is an original formula introduced by the author and it is the general case of the saturation function which takes the following values

$$Sat(x) = \begin{cases} +1 \;\; if \; x > 1 \\ x \quad if \; x \in [-1,1] \\ -1 \;\; if \; x < -1 \end{cases}$$

5- $y(x) = \frac{y_1(x)+1}{2} = \begin{cases} +1 \;\; if \;\; x \geq -a \\ 0 \quad if \;\; x < -a \end{cases}$ (9)

This function is similar to the step function with

$$U(x+a) = \begin{cases} +1 \;\; if \;\; x \geq -a \\ 0 \quad if \;\; x < \, -a \end{cases}$$

The graph of this function is as the following:

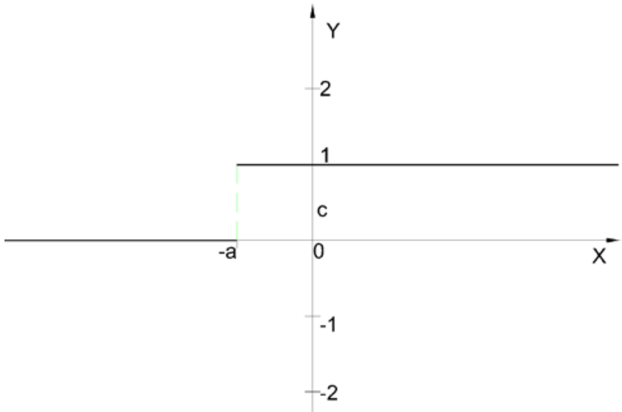

Figure 6: represents the function $y(x) = \frac{y_1(x)+1}{2}$ in 2D Cartesian coordinate system

• Let's consider the following expression:

$$y(x) = P_x(\sin(x)) = \begin{cases} +1 \;\; if \;\; x \in [2k\pi, (2k+1)\pi] \\ -1 \;\; if \;\; x \in](2k+1)\pi, (2k+2)\pi[\end{cases} \quad (10)$$

This equation is similar to the rectangular signal. The graph of this function is as the following:

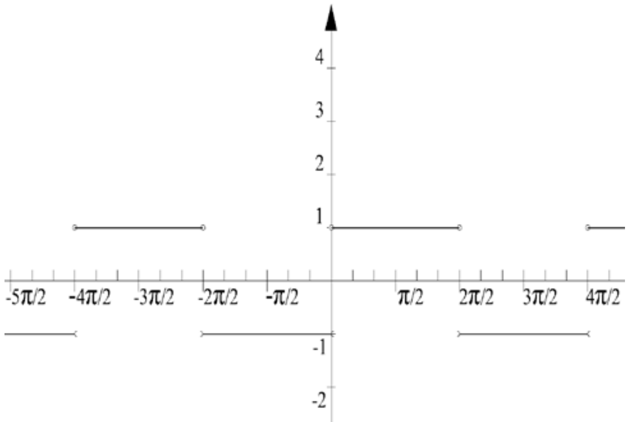

Figure 7: represents the function $y(x) = P_x(\sin(x))$ in 2D Cartesian coordinate system

4 Conclusion

The *Position Function* is a new and original mathematical function introduced by the author into the mathematical domain, the main target of introducing this function is to give two values for a specific point in a specific coordinate system as Cartesian, Cylindrical, Spherical, Parabolic Cylindrical, Ellipsoidal, Elliptical Cylindrical etc these two values are (1 if the Point is in the positive part of the coordinate system), and (-1 if the point is in the negative part of the coordinate system) as indicated in the section 2. Some important examples are developed in order to emphasize the importance of this function into the mathematical domain and its applications. One can deduce many important formulae from certain expressions using the *Position Function* as the rectangular function, Step function, Death Zone function, Saturation function and many others important functions. Moreover the *Position Function* is used in any kind of coordinate system not like others that are used in a specific coordinate system as the previous cited functions.

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