Seismic Wave in Magnetoelastic Irregular Anisotropic Layer

AMARES CHATTOPADHYAY, SANJEEV A. SAHU
Department of Applied Mathematics
Indian School of Mines, Dhanbad-826004
INDIA
amares.c@gmail.com, maths.sngv@gmail.com

Abstract: This paper aims to study the propagation of horizontally polarized shear wave (SH wave) in a magnetoelastic anisotropic and irregular layer. The irregular layer is taken as self-reinforced which is sandwiched between two isotropic elastic half-spaces. Irregularity in layer is considered at the lower interface. The perturbation method is used to find the dispersion equation. Effects of size of irregularity and different values of magnetoelastic coupling parameter on dispersion curves have been studied. Variation of phase velocity with wave number has been presented by graphs.

Keywords: Irregular boundary, Dispersion equation, Perturbation, Shear wave, Anisotropic layer, Magnetoelastic.

1 Introduction
Seismology is the scientific discipline concerned with the study of earthquakes which tells about the structure of the Earth and Physics of earthquakes. Choice of the irregular self-reinforced layer in this study is motivated by the real earth situation. Remarkable works have been done to study the seismic behaviour at different margins of the Earth. The idea of introducing a continuous reinforcement at every point of an elastic solid was given by Belfield et al [1]. Propagation of seismic waves in different media has been studied earlier by many authors. Recently, the propagation of SH waves in an irregular monoclinic crustal layer has been studied by Chattopadhyay et al [2]. In this paper we have discussed the propagation of SH waves in an internal magnetoelastic self-reinforced stratum, with rectangular irregularity in the lower interface. We have used the perturbation approach indicated by Erigen and Samuels [3] and Willis [4] integral formula. Results may be useful for seismologist and engineers. This study may provide some valuable information about the selection of construction material.

2 Formulation of the Problem
The equation of interface between the layer and lower half space is defined as

\[ z = \varepsilon h(x) \]  

where

\[ h(x) = \begin{cases} 
0 & \text{for } x \leq -\frac{s}{2}, \ x \geq \frac{s}{2} \\
 f(x) & \text{for } -\frac{s}{2} \leq x \leq \frac{s}{2} \end{cases} \]

where \( \varepsilon = \frac{H'}{s} \) and \( \varepsilon \ll 1 \).

Let we call the three mediums as \( M_1 \) (upper half-space), \( M_2 \) (irregular layer) and \( M_3 \) (lower half-space). Also, we consider \( \mu_r, \rho_r, u_r \) \((r = 1,2,3)\) as the rigidities, densities and displacements of the upper half space, sandwiched layer and lower half space respectively. The equation of motion for \( M_1 \) and \( M_3 \) are given in eq. (2) and eq. (3).
\[ \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} = 1 \frac{\partial^2 v_1}{\partial t^2} \]  

(2) 

\[ \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_3}{\partial z^2} = 1 \frac{\partial^2 v_3}{\partial t^2} \]  

(3) 

3 Boundary conditions

(i) The stresses and displacements are continuous at the interface of \( M_1 \) and \( M_2 \),

\[ \mu_1 \frac{\partial v_1}{\partial z} = \frac{R}{2} \frac{\partial v_2}{\partial x} + P \frac{\partial v_2}{\partial z} \text{ at } z = -H. \] 

(7)

\[ v_1 = v_2 \text{ at } z = -H. \] 

(8)

(ii) The stresses and displacements are continuous at the interface of \( M_2 \) and \( M_3 \),

\[ \left( R - \frac{Q e^h}{\partial x} \right) \frac{\partial v_2}{\partial z} + \left( P - \frac{R}{2} e^h \right) \frac{\partial v_3}{\partial z} = \mu_3 \left( \frac{\partial v_2}{\partial z} - e^h \frac{\partial v_3}{\partial z} \right) \text{ at } z = e^h(x) \]

(9)

\[ v_2 = v_3 \text{ at } z = e^h(x). \] 

(10)

4 Solution of the problem

Let us assume \( v_i = V_i(z, x) e^{i\omega t} \) \((i = 1, 2, 3)\).

So, eqs. (2), (3) and (6) reduces to

\[ \frac{\partial^2 V_1}{\partial z^2} + \frac{\partial^2 V_1}{\partial x^2} + \frac{\omega^2}{\beta_1^2} V_1 = 0, \] 

(11)

\[ \frac{\partial^2 V_3}{\partial z^2} + \frac{\partial^2 V_3}{\partial x^2} + \frac{\omega^2}{\beta_3^2} V_3 = 0, \] 

(12)

and

\[ \rho \frac{\partial^2 V_2}{\partial z^2} + R \frac{\partial^2 V_2}{\partial z \partial x} + Q \frac{\partial^2 V_2}{\partial x^2} + \rho_2 \omega^2 V_2 = 0. \] 

(13)

With the help of Fourier transform, we get the solutions of above equations and displacements in \( M_1 \), \( M_2 \) and \( M_3 \) as

\[ V_1(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A e^{p z} e^{-i\omega x} d\eta, \] 

(14)

\[ V_2(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\frac{4\pi}{\eta}} (B\cos pz + D\sin pz) e^{-i\omega x} d\eta \] 

(15)

\[ V_3(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( E e^{-pz} + \frac{2}{p_3^2} e^{p_3 z} e^{-p_3 d} \right) e^{-i\omega x} d\eta \] 

(16)

where the second term in the integrand of \( V_3 \) is introduced due to the effect of source at \( S \) in \( M_3 \).
We set the following approximations due to small value of \( \varepsilon \)
\[ B \equiv B_0 + B_1 \varepsilon, \quad D \equiv D_1 + D_2 \varepsilon, \quad E \equiv E_0 + E_1 \varepsilon. \]
Also, the following approximations can be accepted
\[ e^{\pm \varepsilon h} \approx 1 \pm \varepsilon h, \quad \cos \varepsilon h \approx 1, \quad \sin \varepsilon h \approx \varepsilon h. \]

Now, using the boundary conditions (i), (ii) and (iv), and applying the Fourier transform, we get
\[ \left( B_0 - E_0 - \frac{2}{P_3} e^{-p r d} \right) + \varepsilon (B_1 - E_1) = \varepsilon R_1(k) \quad (17) \]
where
\[ R_1(k) = \frac{1}{2\pi} \int \frac{d}{L} B_0 - p D_0 - p_1 E_1 + 2e^{p r d} \left[ \frac{i}{\pi} \int_{-\infty}^{\infty} \tilde{h}(\lambda) d\lambda \right] \]

Similarly, boundary condition (iii) gives
\[ P \left( \frac{a}{2} B_0 - p D_0 + \varepsilon \left( \frac{a}{2} B_1 - p D_1 \right) \right) \left[ 1 + \frac{R}{2} \right] (B_0 + B_1 \varepsilon) - \mu_s (p, E_0 - 2e^{p r d} + \varepsilon p, E_1) = \varepsilon R_2(k) \quad (18) \]
where
\[ R_2(k) = \frac{1}{2\pi} \int \frac{d}{L} P \left( \frac{a}{2} B_0 - p D_0 - p_1 E_1 + 2e^{p r d} \left[ \frac{i}{\pi} \int_{-\infty}^{\infty} \tilde{h}(\lambda) d\lambda \right] \right) \]

Equating the absolute term (terms not containing \( \varepsilon \)) and the coefficient of \( \varepsilon \), we get the following eight equations
\[ B_0 \left[ -i \pi R \cos pH \frac{a}{2} P \cos pH + \pi p \sin pH \right] e^{p r d} + D_0 \left[ -i \pi R \sin pH \frac{a}{2} P \sin pH + \pi p \cos pH \right] e^{p r d} = Ap \mu_s e^{p r d}, \]
\[ B_1 \left[ -i \pi R \cos pH \frac{a}{2} P \cos pH + \pi p \sin pH \right] e^{p r d} = Ap \mu_s e^{p r d}, \]
\[ B_0 \left[ -i \pi R \sin pH \frac{a}{2} P \sin pH + \pi p \cos pH \right] e^{p r d} = Ap \mu_s e^{p r d}, \]
\[ (B_0 \cos pH - D_0 \sin pH) e^{p r d} = Ap \mu_s e^{p r d}, \]
\[ (B_0 \cos pH - D_0 \sin pH) e^{p r d} = Ap \mu_s e^{p r d}, \]
\[ B_0 - E_0 - \frac{2}{P_3} e^{-p r d} = 0, \quad B_1 - E_1 = R_1(k), \]
\[ B_0 \left( \frac{a}{2} P + i \pi R \right) (pP) D_0 = \mu_s (p, E_0 - 2e^{-p r d}) = 0, \]
\[ B_1 \left( \frac{a}{2} P + i \pi R \right) (pP) D_1 = \mu_s \left( (\mu, p, p, E_0 - 2e^{-p r d}) = 0, \right) \]

Solving the above eight equations, we get the values of \( B_0, D_0, E_0, B_1, D_1, E_1, A, A, E_2, E_3, E_4, E_5 \)
and \( G(k) = 4p^2 P^2 \tan pH - 4pp, p, P \)
\[ + 2ikaRP \tan pH + a^2 P^2 \tan pH + 2\alpha p, p, P \tan pH \]
\[ - k^2 R^2 \tan pH + 2ikp, p, P \tan pH \]
\[ - 2ikp, p, P \tan pH - 2p, p, p, P \tan pH \]
\[ - 4p, p, p, p, p, p, p, p, \tan pH \]

which gives the displacement in the magnetoelastic self-reinforced layer as
\[ V_z = \frac{1}{2\pi} \int \frac{d}{\lambda} \left[ 8\mu_s e^{p r d} e^{-p r d} \left[ \frac{i}{\pi} \int_{-\infty}^{\infty} \tilde{h}(\lambda) d\lambda \right] \right] \]

Further on simplification, we get
\[ \psi(k - \lambda) = \frac{2\mu_s}{\lambda} \int \psi(k) + \psi(k + \lambda) \left\{ \frac{1}{\lambda} \sin \left( \frac{\lambda z}{2} \right) \right\} d\lambda \quad (19) \]

where
\[ \psi(k - \lambda) = \left[ \left( B_2 + B_3 + B_4 + B_5 + 2 \right) e^{-p r d} \left( \frac{1}{p} G(k) \right) \right] \]

Finally, we set
\[ B_2 = -2a^2 p, p, P \tan pH - 2a^2 p, P^2 - 8p, p, P^2 - 8p, p, p, P \tan pH + 2p, P G(k), \]
\[ B_3 = -4ika p, p, P \tan pH - 8p, p, P^2 - 8p, p, p, P \tan pH + 4p, p, P^2 \tan pH, \]
\[ B_4 = 8kpp, p, P, P \tan pH + 2ika p, P \tan pH + a^2 p, P \tan pH - 4ika p, P, P \tan pH \]
\[ B_5 = -4apa p, p, p, P \tan pH + 8p, p, p, P \tan pH - 4apa p, P, P \tan pH - 8apa p, p, P \tan pH, \]
\[ B_6 = -2kpp, p, P \tan pH + 8kpp, p, P \tan pH + 8kpp, p, P \tan pH \]
\( B_s = -2i\lambda k^2 p,\mu R \tan pH - 2a\lambda k\mu, p, P \tan pH + 4a\lambda kp,\mu P \tan pH, \)
\( B_3 = -4\lambda k^2 P^2 \tan pH - 2ia\lambda k^2 R \tan pH - 4\lambda kp,\mu P + \lambda R(k), \)
\( B_3 = -\lambda k R^2 \tan pH - a\lambda k^2 P \tan pH - a^2 k p, R^2 \tan pH - 4\lambda kp,\mu p, P, \)
\( B_{3a} = -4ia, p, r, p, R \tan pH - 2ia, p, R p, R \tan pH + 2k, p, R^2, \)
\( B_{3b} = -4a^2, p, R^2, p, R \tan pH - 2a^2, p, R p, R \tan pH + 4a^2, p, R^2 \tan pH, \)
\( B_{3c} = 2a\lambda kp,\mu p, T \tan pH + 2i\lambda k p, p, R \tan pH - 4\lambda kp,\mu p,\mu \tan pH + k p, R^2. \)

Here the argument of \( \psi(k - \lambda) \) is because of our consideration, \( \eta = \lambda = k. \)

Following the asymptotic formula of Willis [4] and neglecting the terms containing \( 2/s \) and higher powers of \( 2/s \) for large \( s \), we get [5]

\[
\int_{-\infty}^{\infty} [\psi(k - \lambda) + \psi(k + \lambda)] \frac{1}{\lambda} \sin \left( \frac{\lambda x}{2} \right) d\lambda \approx \frac{\pi}{2} \psi(k) = \pi \psi(k)
\]

(21)

Hence, using eq. (20) on eq. (21) gives

\[
\mu_s p, R_1 - R_2 = 2s, \mu_s \psi(k) = 2\mu_s \frac{H^2}{\epsilon} \psi(k).
\]

(22)

Therefore, in view of eq. (22) the displacement in the magnetoelastic self-reinforced layer is

\[
V_z = -\frac{1}{2\pi} \int_{-\infty}^{\infty} 8\mu_s e^{-qd} e^{\frac{q}{2}}
\]

\[
\times \left\{ (ikR \tan pH + aP \tan pH + 2P + 2p,\mu \tan pH) \cos p, z
\]

\[
+ (ikR + aP - 2P \tan pH + 2p,\mu \sin p, z) \right\} e^{-qd} dk.
\]

(23)

Since the value of this integral depends entirely on the contribution of the poles of the integrand, hence the dispersion equation for SH waves is given by

\[
G(k) \left[ 1 - \frac{H^2}{2} \psi(k)e^{qd} \right] = 0.
\]

(24)

eq. (24) may be rewritten as

\[
D_2 + D_3 + D_4 - \frac{H^2}{2} (D_6 + D_7 + D_8) = 0.
\]

(25)

Let us assume that the wave is propagating along the surface with \( c \) as the common wave velocity, then we can set the following

\[
p = kP, p_1 = kP, p_2 = kP \text{ and } a = k
\]

where

\[
P = \frac{\left( \frac{c}{\beta^2} \right)^2}{\left( 1 + a^2 \left( \frac{\mu_s}{\mu_T} - 1 \right) + \epsilon_S \sin^2 \phi \right) + \frac{Q}{P} + \frac{R^2}{4P^2}}.
\]

(26)

In the view of above setting, the real part of eq. (25) gives the dispersion relation as

\[
\tan \left( \frac{\left( \frac{c}{\beta^2} \right)^2}{\left( 1 + a^2 \left( \frac{\mu_s}{\mu_T} - 1 \right) + \epsilon_S \sin^2 \phi \right) + \frac{Q}{P} + \frac{R^2}{4P^2}} \right) kH = \frac{J}{L_s}
\]

(26)

\[\epsilon_S = \frac{\mu_T H_0^2}{\mu_T} \] is the magnetoelastic coupling parameter and

\[J_s = (8\mu \varphi, \mu, P + 8\mu \varphi, \mu, P + 8H, k \varphi, \varphi, \varphi, \varphi, \mu_s, P - 8\mu, \varphi, \varphi, \varphi, \mu_s, P, \varphi, \varphi, \mu_s, P) \]

\[ \times \left( 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \mu_s, P \right) \]

\[ \times \left( 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \mu_s, P \right) \]

\[ \times \left( 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \mu_s, P \right) \]

\[ \times \left( 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \mu_s, P \right) \]

\[ \times \left( 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \varphi, \mu_s, P + 8\mu, \varphi, \varphi, \varphi, \mu_s, P \right) \]

(26)

Some particular cases may be deduced considering different combinations (viz. regular, isotropic layer with and without magnetic field).

5 Numerical examples

For the case of an irregular magnetoelastic self-reinforced layer between two isotropic half-spaces
we have used the following data
(i) For upper isotropic half space, [6]
\[ \mu_i = 7.10 \times 10^{10} \text{N/m}^2, \rho_i = 3.321 \text{Kg/m}^3. \]
(ii) For magnetoelastic self-reinforced layer, [7]
\[ \mu_f = 7.07 \times 10^{10} \text{N/m}^2, \mu_r = 3.5 \times 10^9 \text{N/m}^2, \rho_2 = 1.600 \text{Kg/m}^3. \]
(iii) For lower isotropic half space, [6]
\[ \mu_i = 6.77 \times 10^{10} \text{N/m}^2, \rho_i = 3.323 \text{Kg/m}^3. \]
Moreover, following data are also used [8]
\[ a_1 = 0, 0.00316227; \epsilon_H = 0.0, 0.4, 0.8; \phi = 10^\circ. \]

Fig. 2: Dimensionless phase velocity against dimensionless wave number for \( \epsilon_H = 0.0 \) in the presence of reinforcement.

Fig. 3: Dimensionless phase velocity against dimensionless wave number for \( \epsilon_H = 0.0 \) in the absence of reinforcement.

Fig. 4: Dimensionless phase velocity against dimensionless wave number for \( \epsilon_H = 0.8 \) in the presence of reinforcement.

Fig. 5: Dimensionless phase velocity against dimensionless wave number for \( \epsilon_H = 0.8 \) in the absence of reinforcement.

6 Conclusions
The effect of irregularity, magnetic field and self-reinforcement on the propagation of SH wave has been studied. The dispersion equation has been obtained in closed form. We can conclude by presented graphs that the irregular boundary, presence of reinforcement and magnetic field in the medium affects the velocity of SH wave significantly. In particular,

(i) Increment in size of irregularity, decreases the phase velocity of SH wave.
(ii) Presence of reinforcement and magnetic field in the medium increases the phase velocity of SH wave.

(iii) Dispersion equation agrees with the classical Love wave equation for isotropic regular layer over the isotropic half-space.

(iii) This study has possible applications in the field of civil engineering, geophysics and seismology.

7 Acknowledgements

The authors convey their sincere thanks to Department of Science and Technology, New Delhi (India) for their financial support through DST Project No. SR/S4/MS: 436/07 Dated 29.05.2008, Project title: “Wave propagation in anisotropic media”.

References:


