Abstract: The paper focuses on practices of business valuation, in which habitually and most frequently only point estimates of inputs are regarded. It shows that under many common situations this process may be misleading and incorrect. It emphasizes that the pure deterministic analysis relying on point estimates ignores valuable information related to an uncertainty of our estimates. As a solution this paper suggests that the model should be based on probability distributions and the Monte Carlo Simulation, which enables incorporating the risk and uncertainty into the business value estimate. The simulation approach also offers relatively easy solutions to common problem areas, including the integration of expert opinions, nonrecurring events and dependencies between model variables.

Key-Words: Business Valuation; Risk Analysis; Monte Carlo Simulations; Capital Budgeting

1 Introduction

Company valuation methods build on the present value of the future free cash flows, that we expect the company to generate:

\[ PV_{FCF} = \sum_{t=1}^{T} \frac{FCF_t}{(1 + r)^t}, \]

where \( PV_{FCF} \) is the present value of free cash flows, \( FCF_t \) is the free cash flow generated by the company in year \( t \), \( r \) is the required return and \( T \) is the lifetime of the company in years. Since all these parameters are subject to risk, it is not always clear how to estimate their values correctly [5]. This generally accepted valuation approach is based on neoclassical microeconomic principles of an asset valuation under risk, and therefore the expected values of input variables should be considered:

\[ PV_{FCF} = \sum_{t=1}^{T} E(FCF_t) \frac{1 + E(r)}{1 + E(r)}^t. \]

It is important to acknowledge that the structure of inputs for the \( E(FCF) \) in partial years will differ. We can expect important future events like changes in the capital structure, changes in project portfolio or even M&A. Outcome of such events is uncertain and traditional models are very limited in capturing the potential variability. Outputs of deterministic models can be considered accurate strictly under the assumptions that management had already made all decisions remaining for the rest of the company life and all relevant uncertainties and potential scenarios have been quantified via the simple probabilistic calculus.

Generally all the estimates are uncertain numbers with occurrence rather on an interval with varying density of expectations than on a few discrete data points. Deterministic models working with a single point estimate are usually using only one number out of such an interval thus ignoring important and valuable information about the uncertainty. This number can be the mean, the most likely or any other statistically justifiable value. Limiting ourselves only to single points however makes us lose information not only about the variance (degree of uncertainty or risk), but also about the shape that might not be symmetrical or unimodal [7].

Furthermore the portfolio theory introduced by Harry Markowitz [4] and William Sharpe [8] implies that when two assets have the same average value, the market will place greater value on the one with less risk. Two distinct portfolio effects are diversification and statistical dependence. Savage [6] describes that accounting principles are improperly applying these phenomena thus creating numerous inconsistencies. For example a portfolio of homogenous random events is perceived as an addition of single events without the statistical context. It is therefore sometimes impossible to book the expected value of such a portfolio.

Since in reality an occurrence of one factor often influences occurrences of other factors, capturing interdependency among factors within a financial
model is a crucial part of valuation practice. The simplest approach to capture this phenomenon is to use correlations under the assumption that the interdependencies may be modeled as linear. However, in reality non-linear dependencies are much more frequent than linear and thus linearity assumption may generate false outcomes. Non-linearity e.g. implies that the value of a calculation based on average assumptions is not the average value of the calculation. This is technically known as Jensen’s inequality [6].

2 Applying the Simulation Approach

There are several methods for incorporating the uncertainty into the financial model, but due to its simplicity and flexibility, the Monte Carlo simulation is the most popular. This approach calculates numerous scenarios of a model by repeatedly picking values from a user-predefined probability distribution and inserting them in the model. Output of the simulation is a distribution of a monitored variable (e.g. company value), which however should not be understood as a probability distribution, but rather as a distribution of our ignorance or uncertainty concerning the model output.

Replacing uncertain numbers with distributions allows for an integration of a wide range of improvements into financial modeling. Separately analyzing each uncertain variable, approximating its potential occurrence with a corresponding shape of the distribution and observing their joint influence on model outputs provides useful insight into what is beyond the average scenario.

2.1 Quantifying Uncertain Variables

There are essentially two sources of information used to quantify the variables within a risk analysis model. The first is available data and the second is expert opinion [10].

2.1.1 Determining Variability from Data

The observed data may come from a variety of sources: surveys, computer databases, history or research. Before making use of the data, the analyst should be satisfied with their reliability and representativeness. Anomalies in the data should be checked out where possible and proved outliers should be discarded. There are several techniques available to interpret observed data for a variable in order to derive a distribution that realistically models its true variability and our uncertainty about that true variability.

If there is not enough information about the analyzed data set, or for any other reason the assumption about the shape or type of probability distribution cannot be established, non-parametric distribution fitting methods are used. In these cases, for each scenario the Monte Carlo simulation draws randomly from the underlying data set (an empirical distribution).

However, it can also be assumed that the analyzed data come from a known theoretical distribution. A typical example is the normal or log-normal distribution. Their frequent occurrence is likely attributable to the central limit theorem, which predicts that the sum (or product) of a large number of independent random variables, each with finite mean and variance, will be approximately normally (or log-normally) distributed. In these cases parametric distribution fitting methods are used. The empirical distribution of underlying data is then used only to determine the degree of fit to a theoretical (parametric) distribution. Various theoretical distributions can be analyzed to find the one that best fits the observed data. Compared to non-parametric fitting methods mentioned above this approach leads the simulation to abandon the original data set and draw data from the theoretical distribution. This however sometimes means ignoring gained empirical experience.

The shape of the probability distribution is closely related to the uncertainty regarding the estimation of values of its parameters, which is known as Second Order Distribution Fitting. With some simplification, methods for estimating the probability distribution of parameter values can be categorized into the three following groups: classical statistics, Bootstrap method and Bayesian statistics. All three methods are very useful, but require more effort and their applicability varies according to circumstances [2, 10].

2.1.2 Modeling Expert Opinions

Ways of capturing uncertainties between models vary and there are situations where there are no good data available to analyze. Then expert opinions are often more suitable. Again, the deterministic approach requires inserting only a single value, which could be the mean or median of all collected estimates. When applying simulations, the potential of covering expert opinions is much larger as it is possible to create a distribution of all estimates. Thus no important information regarding collected estimates, including the uncertainty of them being correct, would be ignored. Each expert can also be assigned a weight to his or her estimate in order to distinguish between the quality of various expert
respondents. Such a weighting implies the probability of the estimate of the expert is correct, which may for example be derived from his or her reputation or an existing track record.

Furthermore it is easier to cooperate with each expert, because his or her subjective uncertainty concerning the estimate can be captured by a distribution. Thinking hard about the factors that could interfere with an expected base case scenario makes an expert consider both an upside and downside potential variability of the situation. Requiring an expert to also define the worst and the best case scenarios allows for an understanding of the range of potential outcomes. Only then is it possible to realize what can be expected if everything goes wrong and vice versa. That is crucial for a business valuation process as the potential variability of partial uncertain variables within the model add up to a total uncertainty about the value of the company.

The most comprehensible approach to this matter is usually by asking an expert his or her pessimistic, realistic and optimistic estimates. The three values can then be used as parameters for a triangular or betaPERT distribution. The two distributions have achieved a great deal of popularity among risk analysts, since they both offer considerable flexibility in their shape, intuitive nature of their defining parameters and speed of use.

However, for both distributions, minimum and maximum values are the absolute boundaries for the estimated variable, which can lead to underestimating the low probability extreme occurrences and their impact on the value of the company. Generally, an analyst will never be able to define the absolute worst case scenario - it is basically beyond any possibilities to identify all the factors influencing the variability and the extent of their influence. Reasoning of this phenomenon is widely discussed in Taleb’s book Black Swan [9].

It is therefore more suitable to perceive the potential variability on some confidence interval. Phrasing the question of the pessimistic and optimistic estimates in a way that it opens the model to occurrences of potential tail events is an important aspect of its robustness. Asking, what is the worst scenario that cannot happen in 99 cases out of 100, can be a good method to start with, because it simply leaves the 1% of tail events undefined. Statistics will effectively fill this gap with probability distributions.

### 2.2 Modeling Dependencies

Often we are dealing with the question of to what extent within the model does the behavior of one variable determine the expected occurrence of others. Our brain is able to work with similar relationships intuitively based on our empirical experience; however their proper implementation to the valuation model may be complicated. This is not just an infamous problem of distinguishing correlation from causation, but also the issue of mathematical interpretation of the inner dynamics within the model.

Working with linear dependencies expressed by correlations is usually the easiest method. However, it is important to keep in mind that their application is correct only if there is a presumption that the dependence is "approximately" linear. It is inappropriate to use correlations otherwise.

#### 2.2.1 Correlations

In financial modeling, two basic types of dependencies are pursued. The first is the already mentioned dependence between model input variables that can be for example product price and the quantity sold. This type not only affects the overall reliability of the estimate (i.e. the total variance of the distribution), but also the mean value of free cash flow - and thus the value of the company. The reason is the general validity of the theorem on the product of the mean values of two random variables $P$ and $Q$:

**Equation 3**

$$E(PQ) = E(P)E(Q) + \text{cov}(P, Q).$$

The second type of dependence, almost always ignored in the models, is the autocorrelation of time series, i.e. correlation between values of a single input variable at different points in time. An example is the quantity sold in successive years, for which it will certainly be the case that

**Equation 4**

$$\rho(Q_{t+1}, Q_t) \neq 0.$$

Generally the mean value of the sum of random variables is the same as the sum of their mean values. Therefore this kind of dependency will not affect the estimated value of the company. However, this is not the case of the reliability of our estimate, since the definition of the variance of company value estimate is as follows:

**Equation 5**

$$D(PV) = \sum_{k=1}^{T} \sum_{i=1}^{T} \text{cov}\left(\frac{FCF_k}{(1+r)^k}, \frac{FCF_i}{(1+r)^i}\right).$$

In reality the time dependence (i.e., different than zero covariance between variables in time) between $FCF_k$ and $FCF_i$ is more frequent than independence.
Moreover, it is usually a direct dependence, which means that the stronger the dependence the lower the reliability of our estimate, as the variance increases. In practice it is impossible to capture the fact in MS Excel, simply because the above formula for calculation of the covariance works at the level of free cash flows and not of its individual input variables.

2.2.2 Modeling Non-linear Dependencies
A flexible way of capturing non-linear dependencies between random variables X and Y (i.e., input variables of the model) is linking their parameters – in our case parameters of probability distributions. During the simulation only the occurrence of the first variable determines the conditions of the other. For example, assuming that both X and Y come from a uniform distribution \( U [a, b] \), then the dependence of random variable Y, can take the form

\[
Y \sim U[f_1(X); f_2(X)],
\]

where \( f \) represents a function (Figure 1). Both coordinates of this chart are obtained by the Monte Carlo simulation, which will always generate a random value of X first. This will be used as an input parameter for the second random variable Y with parameters \( U [cX + d, eX + f] \). Therefore for each scenario, only the occurrence of a random variable X determines the interval at which the coordinates of Y lie. Similarly any kind of distribution or combination of distributions can be used.

Modeling dependencies between random variables is not limited to the same type of stochastic process or the type of probability distribution. In all cases, however, it is necessary to validate the dependence with statistical analysis first (see Figure 1).

\[\text{Equation 6}\]

2.3 Modeling Nonrecurring Events
Business is always exposed to the risk of nonrecurring events that may significantly affect a company’s cash flows. In terms of financial planning these risks have a binary character, because it is expected that a particular event either occurs or not. However, under both scenarios events may unfold differently. Such risks may include competition entering the market, a change in the legislation, a change in the tax code or natural disasters. All of them may have a major impact on business, its processes and cash flows, and therefore their proper integration into the valuation model is essential.

A frequent mistake is ignoring the less likely events that have been identified as risks, but, despite their significant potential impact on business, the probability of occurrence is considered negligible. Another common approach is that even though the model includes both possible outcomes, they are reflected in a single average distribution, which is also inappropriate, since in reality the “average” situation will never occur.

Nonrecurring events are the kind of uncertainty impossible to be covered using deterministic modelling techniques. As an example of such event, consider the launch of a new technology creating its own market segment (e.g. PC tablets). At first there is an uncertainty concerning the revenues generated by the product of the company being valued, but there is also a risk of competition soon entering the market with a competing product, thus taking share on the revenues. Uncertainty of revenues generated by the product in the two scenarios can be represented by a bimodal distribution as shown in Figure 2. Black line distribution shows an incorrect modeling approach where an average value of the two situations has been taken into account.
It is important to note that both distributions have the same mean value. In traditional models it is therefore impossible to distinguish whether a correct modeling technique has been employed.

3 Discount Rate in Company Valuation

Despite the widespread acceptance and use of the DCF approach to business valuation, there is a growing recognition of its important limitations [3]. DCF analyses are based on a static view ignoring flexibility and variability, and therefore tend to misvalue investments with non-linear payoffs. The major criticism is however pointed to the use of a single discount rate to discount free cash flows, regardless of differences in risk or financing. This approach may help understand the value of the company on average, but it fails to reflect the variety of uncertainties behind each factor within the model.

As discussed throughout the paper, a company cannot be valued without a clear understanding of the risk profile of its operating cash flows. Basic approaches in this matter are stress testing or what-if analysis, whose purpose is to estimate how the value of the company reacts to different potential economic scenarios. A crucial question regards the minimum level of free cash flows the company can be expected to generate under the worst possible economic scenario and whether the company will still be able to cover its debt obligations. In reaction to this information, managers may decide to lower some risk exposures or reduce leverage to lower the company’s risk profile. This concept is related to the probability of the company’s default, which is a very important aspect of its value.

Including a risk analysis of future cash flow, however, requires considering a theoretical shift from traditional DCF conventions. Calculations performed in a risk analysis spreadsheet model are usually presented as a distribution of company value because the cash flows are also expressed as distributions rather than their expected values. Theoretically, this is however incorrect. Since a value of the company presents what the company is worth at the moment of valuation, it can have no uncertainty as there can be only one such number. The problem is that the risk has been double counted by first discounting at the risk-adjusted discounted rate and then showing the company value as a distribution.

The theoretically correct method for calculating the value of the company under these circumstances is to discount the cash flow distributions at the risk free rate. Such distribution is, however, difficult to interpret and incomparable with outputs of other models. Since there is no framework to cover this issue, it is recommended by practitioners to apply risk adjusted discount rate to produce a distribution of company value [11]. The mean value of the company will be more precise than the one calculated by the DCF approach, because it incorporates asymmetrical distributions, correlations and other phenomena described in this paper.

Bancel and Tierny [1] suggest dividing the expected free cash flow of a company into two parts: a relatively certain, or low risk, component; and an uncertain, or risky, component. While the risky component should be funded entirely by economic capital, the low risk component can be financed by debt, since there is a marginal probability of company’s inability to finance it (see Figure 3).

It is further indicated that there could be more than two levels of cash flows according to their levels of risk. It would be easier to estimate discount rate for each of the cash flow groups since it derives from their costs of financing. This would motivate management and analysts to pay closer attention to the risk profile of company cash flows. As a result, their valuation should be more reliable than the one calculated by the DCF approach relying heavily on average cash flows.

4 Conclusion

Company valuation based on point estimates leads to many inaccuracies. Its shortcomings can be conquered by applying simulation approach which clarifies many important questions. Due to incorporating asymmetrical distributions and dependencies between model variables, the value of the company is more precise than the one calculated.
by the DCF approach. Simulation also provides a transparent interpretation of an uncertainty about this value and corresponding input variables, which is a significant improvement over a single value output of DCF model which lacks the understanding of where this value may be located on the real distribution of all possible scenarios.

Furthermore it is possible to determine the minimum level of free cash flows the company is expected to generate under the worst possible economic scenario. This information should reveal whether the company will still be able to cover its debt obligations and help management to decide if any adjustments in the capital structure of the company should be made. The urgency of which would be clearly readable from the risk analysis. The distribution of the value of company may also provide a hint on how the capital structure should be adjusted from the risk perspective.

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