A Colored Petri Net Model for the France Paris Metro System

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Abstract: - This paper describes how colored Petri nets can be used for modeling the Paris underground Metro system. This is one of the most complex metro systems in the world. In this paper, the metro system is analyzed and described. The method of analysis is outlined. A working model for three of the major lines was constructed using CPNs. The results and other issues that have to be tackled are explained.

Key-Words: - Colored Petri Nets, Modeling and Simulation, Systems Analysis, Transport Systems

1 Introduction
An underground metro organization system is a collection of trains and rails that exist at different levels. The system needs to satisfy certain conditions like timings, stopovers, sequential ordering of journeys, min – max waiting time. Normally mathematical models and other graph based methods can be used to model a metro system. Most of these methods are possibly non visual. Petri nets have extensive use for discrete event modeling [4]-[5] apart from other uses in communications and software expression. CPN allows for the proper modeling of temporal problems

2 Motivation
The motivation in this work is to show the usefulness of CPNs in modeling transport systems [1]-[3],[6]-[7], like an underground metro system. Esthetically driven perception of a metro network both from its geographical layout or schematic plan or diagram can be used to create or design a Petri net model layout. Obviously the schematic diagram presents a complex topological layout of the entire metro system along with stops. This is not necessarily drawn to scale. This layout serves as the initial inspirational factor for creating a Petri net model.

3 Problem Definition
The Paris Metro System or Metro’dé Paris was selected for this work for various reasons such as its large size, complexity and distribution. The Paris Metro System is an important symbol of the Paris city and it has an elaborate architectural system. There are about 14 main lines that use different levels underground. There are over 300 stops which can also be considered to be stations. A large number of stops overlap other lines to facilitate transfer.

This metro system is considered to be one of the densest and most complex metro systems in the world. Lines are numbered 1..14. These are identified on the appropriate travel maps by their various colors and names. Lines are bi-directional, i.e. travel takes place in two directions for each line. Most places in Paris are covered by a metro station and some are within close walking distances of 400m. The distribution of the lines create overlap, intersection and meeting points at very important locations. This allows travelers to shift quickly from one line to another. It can be estimated that the trips between stations take from 60 secs to about 2 mins or more. Normally travelers wait for a time of 2 mins or a bit more to board the metro train in most stations.

The aim of this work is to develop a comprehensive working model that represents / parallels characteristics of the system in the real world, as closely as possible. The model must present the actual topological features of the metro system. It should be suitable for execution and analysis. Simulation of the model needs to be reactive in the sense that any change in state ‘outside’ can also be performed inside the model. The idea is to create a CPN model that is fully functional and concise, but simultaneously represents and models the major characteristics of the Paris Metro System.

Some important assumptions are required in order to construct the CPN model. These assumptions are made both about i) the real situation and ii) the actual CPN.
4 Proposed Solution

The proposed solution is to construct a CPN model that closely reflects the real functioning of the Paris Metro System. This can be achieved by examining the actual map of the metro system and constructing a CPN from this graphical layout.

The actual map of the Paris-Metro/RER system is a symbolic graph or symbolical graphical representation of the entire metro networks and its connectivity. This map is composed of a linked set of nodes and edges, but the actual direction is not actually indicated. Each line has its own set of nodes and edges that are represented in the form shown in fig. 2. This is an actual undirected simple finite path graph of the general form \( G = (E,V) \) where \( E = (e_1, e_2, e_3, \ldots, e_n) \) represent the connections from one station to another in a given metro line and \( V = (v_1, v_2, v_3, \ldots, v_n) \). Normally there are no repeated edges in this graph and sequence of vertices such that from the starting node or vertex, following the sequence of vertices in succession to the termination node or vertex, there is always exactly only one edge, joining any two vertices. i.e. this is the path between the vertices. The length of a path is the number of edges in that path.

In the map the directionality is not properly represented so this needs to be transformed or changed before the map can actually be used to implement a Colored Petri net model. In practical terms each metro line can be represented as a digraph. The incoming and outgoing edges to the vertices represent the incoming and outgoing lines from a particular station which is given as a vertex.

The map simple path graph is transformed into something like a Markov Chain graph. This is because from each station, in the metro, it is possible to move out to other stations in a chain like fashion. This is analogous to the Markov chain where it is possible to transit from one state to another in a chain like manner and the next state only depends on the current state only and not on past states.

This is a closer representation of what is actually happening in the metro. It shows that an actual location or station, that services 1 line only, must have at least one entry point and one exit point. Each line normally has at least two metro trains traveling in opposite directions. One from the station and the other to the station, this could be happening simultaneously.

Normal place transition nets or timed Petri nets can be used for modeling however they are not as expressive as CPNs [8]-[10]. CPNs can be used to model different timing issues and even the execution.

Transition firing is an atomic action which has to satisfy certain conditions like token availability. Each token has a unique identity and direction in which it is allowed to travel.

Token firing must occur in predefined temporal order to actually model what is happening in real life.

By using CPNs it is possible to describe, in the minutest detail the different locations of a particular metro route and even its overlapping stations with other metro routes. The metro train is given a unique identity that prevents it from going onto other routes when it is not supposed to do this. We have used the unique modeling features and capabilities of CPN’s. Special places and their physical drawing size can be used to reflect the actual importance of the station. i.e. a large hub can be drawn as a large place when compared to smaller stations. These facts are indicated below: i) Place type is small- single metro line, ii) Medium- Mixed line capacity (i.e. can overlap more than one metro line). This would symbolize 2 or 3 different metro lines converging at the same location. iii) Large- Mix central point or hub, i.e. where more than 3 metro lines meet, cross or converge. Places represent locations or Metro station stops.

The steps for the solution are outlined in fig. 1.

Fig. 1 Proposed Solution Outline
5 Implementation

For implementing and illustrating the solution, three main lines from the Paris-Metro System were selected. The lines are 1,3,13 and these have been considered in detail. These lines overlap each other at certain stations. The graphical layout of these lines along with the stations/stops is given in fig. 2. The more stations or stops are included the more complex the final CPN model becomes to draw. One important step is to get and comprehend the map model which should be accurate. The undirected graphical topology of the metro system is the starting point for constructing the CPN. A directed graph model can be constructed for better comprehension and visualization of the CPN model to be built. The graph model is a simple modification of fig. 2. The edges shown in fig. 2 are not indicative of direction. The graph model indicates direction.

After the CPN is built more details are necessarily added to make the model executable. E.g. token types have to be added. Here the token types used is that of a 3-tuple integer color set of the form (n1,n2,n3) where n1 represents the line number, n2 represents the direction of the train and n3 is for the train id. The proposed solution is to construct a CPN model that closely reflects the real functioning of the Paris Metro System.
Figure 3. Initial CPN Model for Paris-Metro Lines 1,3,13

Figure 4. CPN Model Execution for Paris-Metro Lines 1,3,13
The lines are separated from each other by using arc inscriptions that restrict or prevent it from accessing other lines. E.g. a token having values (1,0,2) and representing an actual train that can only travel on line 1 in a particular direction because the arc inscriptions would prevent it from travel inside other lines.

Other things like functions, variables, etc, need to be defined and used. This is an incremental process and the Petri net is improved as we go along. The diagram in fig. 3 shows the initial colored Petri net that was constructed prior to execution. Fig. 4 shows the model being executed.

6 Results & Conclusion
The model was executed and tested in detail and the following data was used initially to start of the model. The results shown in table 1 are just a brief summary of the actual results that show the basic execution results obtainable.

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>CHEC</th>
<th>Directi on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point de levalois-becon</td>
<td>Anatole France</td>
<td>OK</td>
<td>Down</td>
</tr>
<tr>
<td>Anatole France</td>
<td>Louise Michel</td>
<td>OK</td>
<td>Down</td>
</tr>
<tr>
<td>Louise Michel</td>
<td>Porte de champer ret</td>
<td>OK</td>
<td>Down</td>
</tr>
<tr>
<td>Porte de champer ret</td>
<td>Perreire</td>
<td>OK</td>
<td>Down</td>
</tr>
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Table 1. Fragment of Route test data – Line 3 Down

This work can definitely be developed further. It has been shown how CPNs can be useful for modeling a discrete complex system such as a complex underground metro system or a train metro system. Results are obtainable and a fully functional model is constructed. This shows the relevance of CPNs for modeling Metro or train traffic networks. More work can be done in this respect. Not all the possible results have been catered for in this work. Many other tests, execution and verification techniques can be used. To improve the model it might be necessary to limit place capacities and this requires additional places and more complexity.

Ideally the model should be constructed to contain the entire Paris Metro system. This is definitely achievable. Unfortunately the larger the real system, the more complex will the resultant CPN be. Decomposition techniques would have to be used to simplify it and reduce detail thus less information will be visible.

References:
[8] CPNTools, CPN Group, Department of Computer Science, University of Aarhus, Denmark http://www.daimi.au.dk/CPnets/.