Abstract: - This paper studies the design of a static output feedback controller of a Synchronous Machine. By employing a well-known Takagi-Sugeno approach, the continuous nonlinear system is first described by Takagi-Sugeno (T-S) models. Next, we develop a technique for designing an output feedback control law which stabilizes the Synchronous Machine. The controller is designed in terms of Linear Matrix Inequalities (LMI) problem.

Motivated by stability results developed for parallel distributed compensation (PDC) controller, the Output PDC (OPDC) controller was studied in this work. Simulation results for synchronous machine demonstrate the OPDC controller’s effectiveness.

Key-Words: - Continuous systems, T-S model, OPDC controller, Quadratic stability, LMI formulation.

1 Introduction

The issue of stability and the synthesis of controllers for nonlinear systems described by continuous-time Takagi-Sugeno models [14] has been considered actively. There has been also an increasing interest in the multiple model approach [17,13] which also uses the T-S systems to modeling. During the last years, many works have been carried out to investigate the stability analysis and the design of state feedback controller of T-S systems. Using a Quadratic Lyapunov function and PDC technique [6,8] sufficient conditions for the stability and stabilisability have been established [5,6,10,13,15,16]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The PDC control is a nonlinear state feedback.

Recently a number of control law have been derived from the PDC controller [4,13,14,15,17]. For example, a static Output PDC (OPDC) [7], which is an output feedback controller PDC, used to stabilise T-S models. The LMI approach is used to develop a static output feedback controller for nonlinear systems described by continuous-time T-S models.

In this paper we study the stabilization of a synchronous machine, using fuzzy Takagi-Sugeno approach. By using Output Parallel Distributed Compensation (OPDC) as a nonlinear static output feedback control law, we give a formula for the fishing effort that allows stabilizing the system around a steady state.

The rest of this paper is organized as follows. In section 2, we present an overview of dynamic Takagi Sugeno systems and OPDC formulation in terms of LMI [21]. Section 3 deals with the description of the continuousmodel structured of Synchronous machine, which is transformed to a Takagi-Sugeno fuzzy model, and controlled by an OPDC law.

The simulation results are shown. Finally, a conclusion is given.

2 T-S Model and its stability
2.1 Model representation

A T-S model [9], [10], is based on the interpolation between several LTI local models as follows in the continuous time case:

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left( A_i x(t) + B_i u(t) \right) \\
y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t)
\]  

(1)

where \( r \) is the number of submodels, \( x \in \mathbb{R}^n \) is the state vector and \( u(t) \in \mathbb{R}^m \) is the input vector, \( y(t) \in \mathbb{R}^q \) is the output vector, \( z(t) \in \mathbb{R}^p \) is the decision variables vector and \( h_i(z(t)) \) is the activation function.
By substituting (4) into (1), the closed-loop fuzzy system can be represented as:

\[
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_i + B_iK_j C)x(t)
\]

(5)

The following theorem deals with sufficient conditions in LMI's form to ensure asymptotic stability of (5).

Theorem 1 [14]: The equilibrium of the continuous fuzzy system described by (5) is asymptotically stable in the large if there exists a \( P > 0 \) such that:

\[
L_i^T P + PL_i \leq 0, \text{for } i = 1, \ldots, r
\]

(6)

\[
R_{ij}^T P + PR_{ij} \leq 0, \text{for } i = 1, \ldots, r \text{ and } i < j
\]

(7)

Where:

\[
L_i = (A_i + B_iK_j C)
\]

(8)

\[
R_{ij} = \frac{(A_i + B_iK_j C)^T + (A_i + B_iK_j C)}{2}
\]

(9)

In the following, sufficient conditions in LMI's form are given to ensure asymptotic stability of (5).

Suppose that there exist matrices \( X > 0 \), \( M \) and \( N \), such that:

\[
X = P^{-1} > 0 \text{ and } CX = MC
\]

By substituting (4) into (1), the closed-loop fuzzy system can be represented as:

\[
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_i + B_iK_j C)x(t)
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\[
X = P^{-1} > 0 \text{ and } CX = MC
\]
3.1 The synchronous Machine without amortisor and its Mathematical Model

The mathematical model of the synchronous machine adopted in this work is obtained on consist to transform all stator quantities from phase a, b and c into equivalent d-q axis new variables.

The equations are derived by assuming that the initial orientation of the q-d synchronously rotating reference frame is such that the d-axis is aligned with stator terminal voltage phase. The details of their above equation and its parameters can be found in [16].

The mathematical model of the synchronous machine with damper was established as the follow:

The state equations:

\[ \dot{x}(t) = f(x(t)) + g(x(t))u(t) \]  \hspace{1cm} (13)

Where:

\[ f(x(t)) = \begin{bmatrix} x_2(t) \\ -a_1 \sin(2x_1(t)) - a_2 x_3(t) \sin(x_1(t)) + a_3 x_1(t) \cos(x_1(t)) + a_4 u_1(t) \end{bmatrix}, \]
\[ g(x(t)) = \begin{bmatrix} 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 d_4 \end{bmatrix} \]

The input vector \( u(t) = \begin{bmatrix} E_{ex}(t) \\ P_m(t) \end{bmatrix} \)

Augmented with an output vector:

\[ y(t) = C \times x(t), \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \]

The entire symbols for nonlinear model used in this paper are given in appendix A.

The synchronous machine model parameters are given in appendix B.

A Takagi-Sugeno fuzzy model of this system is given in the following section.

3.2 Construction of T-S Fuzzy Model

As commented earlier, the number of model rules goes as \( e=2^n \) with \( n \) nonlinear terms [2] and [17].

Note that there are four non linearities in the non linear dynamical model (1): \( \sin(2x_1(t)) = 2 \sin(x_1(t)) \cos(x_1(t)) \) and \( x_3(t) \sin(x_1(t)) \).

Thus \( n = 3 \) indicating \( e=2^3=8 \) rules are required. However, with some compromise the number of rules can be reduced to 2 while maintaining model [1], [17].

First, we can rewrite two of the nonlinear terms in \( \sin(x_1(t)) \) and \( \cos(x_1(t)) \) as:

\[ \sin(x) = \frac{x_0 \sin(x) - x \sin(x_0)}{x_0 - \sin(x_0)}, \quad 1 \]
\[ + \frac{x_0(x - \sin(x))}{x_0 - \sin(x_0)} \cdot \frac{\sin(x_0)}{x_0} \]

And

\[ \cos(x_1(t)) = \frac{\cos(x_1(t)) + \cos(x_1_0)}{1 - \cos(x_1_0)}, \quad 1 \]
\[ + \frac{1 - \cos(x_1(t))}{1 - \cos(x_1_0)} \cdot \cos(x_1_0) \]

The membership functions are bounded in the range:

\[ x_1(t) = [-x_{10} + x_{10}] \text{ for } x_{10} = \theta_{d0} \in \left[ \frac{0}{2} \right] \text{ implying:} \]

\[ |x_{10} \sin(x_{10}) - x_1(t) \sin(x_{10})| + \frac{\cos(x_1(t)) + \cos(x_1_0)}{1 - \cos(x_1_0)} \leq 2.4\% \]

Therefore the transformation on \( \cos(x_1(t)) \) can be eliminated with litle compromise and the fuzzy model order reduced to 2 or 4 rules. Then, the final fuzzy model is described by only two rules.

And the premise vector is defined by

\[ z(t) = [z_1(x_1(t)) \quad z_2(x_3(t))] \text{ with } z_1(x_1(t)) = \theta_d(t) \text{ and } z_2(x_3(t)) = x_3(t) \]

For a premise terms, define \( z_i(t) = x_i(t), i = 1, 2 \).

Next, calculate the minimum and maximum values of \( z_i(t) \) under \( x(t) \in [-a, a], a > 0 \).

They are obtained as follows: \( \max z_i(t) = a, \quad \min z_i(t) = -a \).

From the maximum and minimum values, \( z_i(t) \) can be represented by:

\[ z_i(t) = M^1_i(z_i(t))a + M^2_i(z_i(t))(-a) \]
\[ \text{Where } M^1_i(z_i(t)) + M^2_i(z_i(t)) = 1 \]

Therefore the membership functions can be calculated as:

\[ M^1_i(z_i(t)) = \frac{z_i(t) + a}{2a}; \quad M^2_i(z_i(t)) = \frac{a - z_i(t)}{2a} \]

Finally, the complete fuzzy model is comprised of four rules, the premise variable is:

\[ z_1(t) = \theta_d(t) \text{ and } z_2(t) = x_3(t) \]

with the following membership functions respectively,
The Takagi-Sugeno fuzzy model of the synchronous machine connected to infinite bus system can be rewritten by introducing submodels are described respectively by the four matrices $A_i$, $B_i$, $C_i$, $i = 1, \ldots, 4$. as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2 \cdot a_1 + a_2 \cdot a & b_1 & 0 & 0 & 0 \\ c_1 & 0 & c_2 & c_3 & d_1 \\ 0 & 0 & d_2 & d_3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2 \cdot a_1 - a_2 \cdot a & b_1 & b_2 & 0 & 0 \\ c_1 & 0 & c_2 & c_3 & d_1 \\ 0 & 0 & d_2 & d_3 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ b_1 \cos(\theta_d) & 0 & b_2 & 0 & 0 \\ c_1(\sin(\theta_d)/\theta_d) & 0 & c_2 & c_3 & d_1(\sin(\theta_d)/\theta_d) \\ 0 & 0 & d_2 & d_3 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ b_1 \cos(\theta_d) & 0 & b_2 & 0 & 0 \\ c_1(\sin(\theta_d)/\theta_d) & 0 & c_2 & c_3 & d_1(\sin(\theta_d)/\theta_d) \\ 0 & 0 & d_2 & d_3 \end{bmatrix}$$

With:

$$K_1 = \frac{\sin(\theta_{d0})}{\theta_{d0}} [2 \cdot a_2 \cdot \cos(\theta_{d0}) + a_2, a],$$

$$K_2 = \frac{\sin(\theta_{d0})}{\theta_{d0}} [2 \cdot a_2 \cdot \cos(\theta_{d0}) - a_2, a]$$

$$B_1 = \begin{bmatrix} 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & c_4 d_4 \end{bmatrix}^T$$ and $C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$

The T.S fuzzy model exactly represents the nonlinear systems. Notice that this fuzzy model has the common B and C matrix.

In the OPDC design, each control rule is associated with the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights $w_i(z(t))$ in the premise parts. The following output feedback fuzzy controller is constructed via OPDC as follows:

$$u(t) = \sum_{i=1}^{2} h_i(z(t)) K_i y(t)$$

$$h_1(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \quad i = 1, \ldots, r,$$

$$w_i(z(t)) = \prod_{j=1}^{r} M_i^j(z_j(t)), \quad i = 1, \ldots, r$$

Finally, the complete the closed loop model T-S fuzzy (5) is synthesized with the premise variable $z_1(t) = \theta_d(t)$ and $z_2(t) = x_3(t)$

The synthesis of the controller consists of finding the feedback gains of the conclusion parts $K_i$ which guaranteed the asymptotic stability of the output closed loop system.

4 Simulations

To show the effectiveness of the proposed design, a simulation study is performed using simulations tests to a synchronous machine.

Many tests have been performed to prove the goodness of the proposed fuzzy control system. Some results, obtained by means of the SIMULINK program of MATLAB, are reported in what follows.

For these simulations, the model rules are chosen for $\theta_d(t) \in [-30 \quad \theta_{d0}]$ and $x_3 \in [-a \quad a]$.

Assume that the initial conditions $x(t)$:

$$x(0) = (-0.52 \quad 0 \quad 0.36 \quad 0.80 \quad 0.71)^T$$

We presents only the results for the value of $\theta_{d0} = \frac{\pi}{4}$ and $a = 0.82$. Every set of LMI s was solved via the MATLAB LMI toolbox.

Using a software simulator, we have found matrices $X$, $N$, and $M$ that satisfies (10) and (11), and also gains $K_i$ for the output feedback controller (13) that stabilizes the system:

$$K_1 = [243.4088; 29.1002]$$

$$K_2 = [352.6021; 63.3890]$$

$$K_3 = [211.1034; 95.3898]$$

$$K_4 = [197.4842; 68.7478]$$

$$M_1 = [39.9401; 4.7750]$$

$$M_2 = [57.8573; 10.4013]$$

$$M_3 = [34.6392; 15.6522]$$

$$M_4 = [32.4045; 11.2806]$$

The trajectories of the state vector and the system response were illustrated by the above simulation results in the case of Static output quadratic stability.Figs.1 displays overall simulation results of the state vector.
Fig. 1 The trajectories of the state vector: a: $\theta_d$ angular position of rotor (rad), b: $\omega_d$ Rotor angular speed (rad/s), c: $E'_d$ d-axis subtransient F.E.M, d: $E'_q$ q-axis transient F.E.M and e: $E''_q$ q-axis subtransient F.E.M.

The simulations results display the trajectory of the state vector at different values of at startup and those simulations illustrate the effect of the different values of the angle $\theta_d$.

5 Conclusion

This paper presents static output feedback controller for a synchronous machine with damper infinite bus described by T-S models. We have shown that the OPDC controller can be formulated as the solution of LMIs set.

The numerical simulations and experimental results have illustrated the expected performance and indicate that the stability of the OPDC controlled system is very suitable in Synchronous Machine and it leads to an optimization problem.

Appendix

\[
\begin{align*}
    a_1 &= -\frac{\omega_0}{2T_L} \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \\
    a_2 &= -\frac{\omega_0}{T_L} \frac{1}{X_q}; \\
    a_3 &= -\frac{\omega_0}{T_L} \frac{1}{X_d}; \\
    a_4 &= \omega_0 \frac{1}{T_L}; \\
    b_1 &= -\frac{1}{T_q} \left( X_d X_q \right); \\
    b_2 &= \frac{X_q}{T_q} X_d; \\
    c_1 &= -\frac{1}{T_d} \left( X_d X_q - X_a \right) \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \\
    c_2 &= -\frac{1}{T_d} \left( X_d X_q - X_a \right) \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \\
    c_3 &= -\frac{1}{T_d} \left( X_d X_q - X_a \right) \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \\
    c_4 &= -\frac{1}{T_d} \left( X_d X_q - X_a \right) \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \\
    d_1 &= X_d - X_a; \\
    d_2 &= X_d - X_a; \\
    d_3 &= X_d - X_a; \\
    d_4 &= X_d - X_a.
\end{align*}
\]
TABLE I
Machine Synchronous data (capacity power 200VA)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$</td>
<td>$d$-axis magnetic reactance</td>
<td>1.10</td>
</tr>
<tr>
<td>$X'_d$</td>
<td>$d$-axis transient reactance</td>
<td>0.50</td>
</tr>
<tr>
<td>$X_{d''}$</td>
<td>$d$-axis subtransient reactance</td>
<td>0.35</td>
</tr>
<tr>
<td>$X_q$</td>
<td>$q$-axis magnetic reactance</td>
<td>1.10</td>
</tr>
<tr>
<td>$X_{d''}$</td>
<td>$q$-axis subtransient reactance</td>
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<tr>
<td>$X_a$</td>
<td>field leakage reactance</td>
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<td>$T_L$</td>
<td>magnetic dipole moment</td>
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</tr>
<tr>
<td>$T'_d$</td>
<td>$d$-axis transient open circuit time constant</td>
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</tr>
<tr>
<td>$T_{d''}$</td>
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<tr>
<td>$T_{q''}$</td>
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</tr>
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<td>$\omega_0$</td>
<td>synchronous rotor angular</td>
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</tbody>
</table>

References:


