Intrinsic Coordinatability of Agent-Based Systems

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Abstract: Game theory provides a mathematical framework for multiple agent decision making scenarios where the actions of all agents result in consequences for each individual. In scenarios for which it is important for the agents to coordinate to achieve some group-level goal, however, game theory is limited since it does not accommodate a notion of group-level rationality. Conditional game theory is an extension of classical game theory that provides a framework within which to characterize social influence relationships that lead to a social model of the group and, hence, to well-defined notions of both group and individually rational behavior. Building on Shannon information theory, we use the concepts of entropy and mutual information to develop a mathematical concept of the intrinsic ability of the members of an agent-based system to coordinate their behavior.

Key–Words: Conditional game theory, Coordination, Multiagent systems, Shannon information theory

1 Introduction

Agent-based modeling is a powerful tool for characterizing the interaction of multiple autonomous decision makers whose collective actions result in group-level outcomes that define the consequences to each agent. Noncooperative game theory is a widely recognized framework within which to characterize such behavior [6, 17, 11, 12, 14].

A finite strategic (normal form) single-stage noncooperative game consists of a group of \( n \geq 2 \) autonomous agents, denoted \( \mathcal{X} = \{X_1, \ldots, X_n\} \), with each \( X_i, i = 1, \ldots, n \), possessing a finite set \( \mathcal{A}_i \) of actions from which it may instantiate one. The joint action space is the Cartesian product of the individual action spaces, denoted \( \mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \). An action profile is an array of actions \( a = (a_1, \ldots, a_n) \in \mathcal{A} \), such that \( X_i \) may take action \( a_i \in \mathcal{A}_i \). These profiles constitute the outcomes of the game. Each player receives a specific consequence, or payoff, as a result of the outcome. The various payoffs to each player constitute its preference ordering over the outcomes, which is typically expressed in terms of utilities, denoted \( u_{X_i}: \mathcal{A} \rightarrow \mathbb{R} \), that define the benefits (either ordinarily or cardinally) to the players. Classical game theory assumes that each agent comes to the game with a completely and immutably defined utility. Such ex ante utilities are categorical; that is, they are unconditional, and are not subject to modification once contact with other players has occurred.

A game is played by each player choosing its action according to some notion of rational behavior. By far the most widely used notion of rationality is that each player will make a choice that corresponds to its own best interest – the doctrine of individual rationality. Applying this doctrine results in solution concepts such as Nash equilibrium: an action profile that results in a payoff for each player such that, if the player were to change its action, its payoff would diminish.

Game theory has been applied to the subject matter of general economic theory with great success, especially in a context of competitive and market-driven decision making. But once applications expand beyond such scenarios, the limitations of game theory quickly become apparent, particularly when socially sophisticated concepts such as cooperation, compromise, negotiation, and altruism are relevant. Various ways have been devised to modify game theory in such situations. One possible approach is to redefine the utilities in a way that permits an agent to incorporate the interests of others into its utilities, such as (partially) substituting the payoffs of others for its own payoffs or by incorporating parameters to account for such social attributes as fairness and reciprocity into its utility structure [5]. Such approaches, while achieving some success, are still limited conceptually since, at the end of the day, reliance on the mathematical structure of categorical utilities and the logical structure of individual rationality severely lim-

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\(^1\)For many scenarios, a multi-stage game can be described by a series of single-stage games, particularly when the key issue of concern is coordination.
its the ability of the group to engage in meaningful coordination.

According to the Oxford English Dictionary, to coordinate (co- [together] + ordinate [arrange]) is “to place or arrange (things) in proper position relative to each other and to the system of which they form parts; to bring into proper combined order as parts of a whole” [10]. In the context of an agent-based system, coordination means that behavior of the individuals (the parts) must be reconciled with behavior of the group (the whole).

Although the terms are sometimes conflated, coordination is not the same as cooperation. Agents may cooperate when their individual interests just happen to coincide once their payoffs are juxtaposed, but coordination implies a social relationship that involves simultaneous consideration of the behavior of the group and its members. In fact, even antagonists must coordinate (athletic contests and military engagements both require coordination).

As Arrow’s impossibility theorem [1] attests, however, it is not generally possible to define a preference ordering for a group by aggregating individual categorical preferences without violating a set of arguably reasonable and desirable properties. Consequently, classical game theory has proceeded by making assumptions about individual preferences only and then using those preferences to deduce information about the choices (but not the values) of a group.

The purpose of this paper is two-fold: First, we introduce an extension to classical game theory, termed conditional game theory, that permits the explicit modeling of social relationships by means of conditional utilities, and leads to new solution concepts that enable the agents to extend their sphere of interest beyond the self. This extension leads to an emergent notion of a group-level preference ordering as well as individual preference orderings. Second, we apply this theory to define a formal notion of coordinatibility that measures the intrinsic ability of a group to coordinate their actions.

2 Conditional Game Theory

2.1 Social Influence

For multiple agent decision scenarios that involve sophisticated social relationships, it is important to model the social influence that agents have on each other. To introduce this concept, consider the following scenario: $X_1$ and $X_2$ are to purchase an automobile under the following division of labor: $X_1$ will choose the color, either red ($r$) or green ($g$), and $X_2$ will choose the model, either a convertible ($c$) or a sedan ($s$). The corresponding joint action space is $\mathcal{A} = \{(r, c), (r, s), (g, c), (g, s)\}$. Thus, for any profile $(a_1, a_2) \in \mathcal{A}$, a categorical utility for $X_1$ is of the form $u_{X_1}(a_1, a_2)$, and the expression $u_{X_1}(r, c) > u_{X_1}(g, c)$ means that $X_1$ prefers a red convertible to a green convertible, with a corresponding categorical utility for $X_2$.

Suppose, however, that $X_1$ does not possess a categorical utility over the joint action space. Instead, let $X_1$ reason a follows: if $X_2$ were to most prefer a green sedan, then $X_1$ would prefer the green sedan to the green convertible. Such a statement is a hypothetical proposition. $X_1$ does not need to know for certain that $X_2$ does indeed most prefer a green convertible. That statement is merely the antecedent of a hypothetical proposition whose consequent is $X_1$’s preference ordering. Symbolically, we may express this relationship as

$$u_{X_1|X_2}(g, s|g, s) > u_{X_1|X_2}(g, c|g, s),$$

where the argument to the left of the conditioning symbol “|” is the profile under consideration by $X_1$ and the argument on the right is a hypothetical profile for $X_2$ (this syntax is similar to the syntax used to define conditional probability). This relationship does not commit $X_1$ actually to prefer a green sedan to a green convertible. In fact, $X_1$ may also possess the following conditional preference ordering:

$$u_{X_1|X_2}(g, c|g, c) > u_{X_1|X_2}(g, s|g, c).$$

These two expressions indicate a willingness for $X_1$ to conform its preferences to the preferences of $X_2$, but this willingness need not apply to all situations. For example, it may also be true that

$$u_{X_1|X_2}(r, s|r, c) > u_{X_1|X_2}(r, c|r, c),$$

that is, $X_1$ is not willing to conform to $X_2$’s preferences if $X_2$ were to most prefer a red convertible.

This utility structure permits the modeling of social relationships that may or may not go beyond self-interest. One explanation for $X_1$’s conditional preferences is altruism: a willingness to defer to $X_2$’s wishes, thereby expanding beyond self-interest. Another possible explanation is that $X_1$ has a well-defined notion of aesthetics, but does not really care what $X_2$’s preferences are – its only concern is that the car they purchase meets its personal aesthetic criteria. Other explanations are certainly possible. The critical issue is that $X_1$’s preferences can be influenced by $X_2$’s preferences. We shall term this social influence, which is in contrast with notions of influence employed by classical game theory, where it is assumed that each player’s payoff is influenced by the actions of the other player, but not by the other player’s preferences (at least ostensibly).
2.2 Conditional Games

We now formalize the notion of a conditional game, as developed by [15, 16].

**Definition 1** For an agent-based system $\mathcal{X} = \{X_1, \ldots, X_n\}$, let $\text{pa}(X_i) = \{X_{i_1}, \ldots, X_{i_{p_i}}\}$ denote the subset of $\mathcal{X}$ that socially influences $X_i$. The subset $\text{pa}(X_i)$ is termed the parent set of $X_i$. For each $X_{i_j} \in \text{pa}(X_i)$, $j = 1, \ldots, p_i$, let $a_{i_j}$ denote an action profile that $X_i$ hypothesizes as the outcome most preferred by $X_{i_j}$. The profile $a_{i_j}$ is called a conjecture for $X_{i_j}$. The collection $\{a_{i_1}, \ldots, a_{i_{p_i}}\}$ of conjectures is a joint conjecture for $\text{pa}(X_i)$.

For each joint conjecture $\{a_{i_1}, \ldots, a_{i_{p_i}}\}$ for $\text{pa}(X_i)$, the conditional utility for $X_i$ is a mapping $u_{X_i|\text{pa}(X_i)}(\{a_{i_1}, \ldots, a_{i_{p_i}}\}; \mathcal{A} \rightarrow \mathbb{R}$.

**Definition 2** A conditional game comprises a) a set of players $\mathcal{X} = \{X_1, \ldots, X_n\}$ where $n \geq 2$, b) a set of outcomes $\mathcal{A} = A_1 \times \cdots \times A_n$, where $A_i$ is the action set for $X_i$, $i = 1, \ldots, n$, and c) a set of conditional utilities $\{u_{X_i|\text{pa}(X_i)}; \mathcal{A} \rightarrow \mathbb{R}\}$, where $\text{pa}(X_i) = \{X_{i_1}, \ldots, X_{i_{p_i}}\}$ is the parent set for $X_i$. The function $u_{X_i|\text{pa}(X_i)}(a_{i_1}, \ldots, a_{i_{p_i}})$ is the utility that $X_i$ ascribes to action profile $a_i$, given the conjectures $a_{i_k}, k = 1, \ldots, p_i$. If $p_i = 0$, then $X_i$ possesses a categorical utility, denoted $u_{X_i}$. A conditional game reverts to a classical game if all utilities are categorical.

2.3 Concordance

As the conditional preferences propagate through the system via the influence linkages, social bonds are created among the agents. The end result of this propagation is an aggregation function that combines all of the individual conditional utilities to form a group-level utility termed a concordant utility, which provides a complete social model of the group. To guide our search for an appropriate aggregation mechanism, we impose the following constraints:

**Endogeneity.** A concordant utility for a group must be determined by the social interactions of its members.

**Acyclicity.** No influence cycles may occur in the social influence relationships. This principle means that, if $X_i$ influences $X_j$, then $X_j$ does not influence $X_i$. Although this constraint restricts the generality of the influence relationships, it nevertheless permits the modeling of many social systems, such as hierarchical systems. Notice that if all members possess categorical utilities, then the system is trivially acyclic.

**Exchangeability.** If the social relationships can be framed in more than one way using exactly the same information, then all framing should yield the same concordant utility. Essentially this means that the social bonds are invariant to the way they are expressed.

**Monotonicity.** If a subgroup prefers one outcome to another and the complementary subgroup prefers is indifferent with respect to the two outcomes, then the group as a whole must not prefer the latter outcome to the former one.

**Normalization.** With out loss of generality, we may assume that all utilities are nonnegative and sum to unity.

It is shown in [15, 16] that complying with these principles ensures that the utilities possess the same syntax as probability mass functions, and aggregating them is achieved by applying the chain rule of probability. In other words, the aggregation of conditional utilities is mathematically equivalent to computing the joint probability of a family of discrete random vectors by multiplying their conditional probability mass functions. Thus, the concordant utility is of the form

$$U_{X_1 \cdots X_n}(a_1, \ldots, a_n) = \prod_{i=1}^{n} u_{X_i|\text{pa}(X_i)}(a_{i_1}, \ldots, a_{i_{p_i}}).$$

For situations where there are common interests among the stakeholders, concordance could be interpreted as harmony, or the similarity of interests. Concordance, however, need not correspond to harmony or cooperation. For example, with an athletic contest, the opposing players do not cooperate in the sense of pursuing a common objective; rather, success in playing the game depends on their opposition to each other. In other words, there is a preference, from the group perspective, for the players to have disputes regarding their desired behavior, and diametrically opposed conjectures would, from a group perspective, have a low degree of controversy. Thus, even antagonists can behave concordantly.

This structure permits a group to be expressed according to the syntax of a Bayesian network that is, as a directed acyclic graph (DAG) whose vertices are agents and whose edges are mass functions that define the influence linkages. Such a network, termed a utility network, serves as a praxeological analog to Bayesian networks [4]. Consequently, all of the mathematical concepts of probability theory (e.g., independence, conditioning, marginalization, and Bayes rule) can be applied (albeit with praxeological, rather than...
As an example, consider the DAG displayed in Figure 1. The corresponding concordant utility is

\[ U_{X_1X_2X_3}(a_1, a_2, a_3) = u_{X_1}(a_1)u_{X_2|X_1}(a_2|a_1)u_{X_3|X_1X_2}(a_3|a_1, a_2). \]

![Figure 1: The DAG for a three-agent group.](image)

### 2.4 Group and Individual Welfare

Let us now define ex post emergent preference orderings for the group and its constituent members. The concordant utility does not directly serve as the basis for taking action, since it is a function of multiple profiles (conjectures), and only one profile can actually be implemented. However, since each agent can control only its own actions, what is of interest is the utility of all agents making conjectures over their own action spaces.

**Definition 3** Consider the concordant utility \( U_{X_1 \ldots X_n}(a_1, \ldots, a_n) \). Let \( a_{ij} \) denote the \( j \)th element of \( a_i \); that is, \( a_i = (a_{i1}, \ldots, a_{in}) \) is \( X_i \)'s conjecture profile. Next, form the action profile \( (a_{11}, \ldots, a_{nn}) \) by taking the \( i \)th element of each \( X_i \)'s conjecture profile, \( i = 1, \ldots, n \). Now let us sum the concordant utility over all elements of each \( a_i \) except the \( ii \)-th elements to form the group welfare function \( v_{X_1 \ldots X_n} \) for \( \{X_1, \ldots, X_n\} \), yielding

\[ v_{X_1 \ldots X_n}(a_{11}, \ldots, a_{nn}) = \sum_{a_{11}} \ldots \sum_{a_{nn}} U_{X_1 \ldots X_n}(a_1, \ldots, a_n), \]

where \( \sum_{a_{ii}} \) means the sum is taken over all \( a_{ii} \) except \( a_{ii} \). The individual welfare function \( v_{X_i} \) of \( X_i \) is the \( i \)-th marginal of \( v_{X_1 \ldots X_n} \), that is,

\[ v_{X_i}(a_i) = \sum_{a_{ii}} v_{X_1 \ldots X_n}(a_1, \ldots, a_n). \]

These welfare functions can be used to define solution concepts that simultaneously account for both group and individual interests.

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2Epistemology involves the classification of propositions on the basis of knowledge and belief and praxeology involves the classification of propositions on the basis of effectiveness and efficiency.

### 3 Coordination

#### 3.1 Extrinsic Coordination

Despite its inability to accommodate a concept of group-level rationality, classical game theory is often used to model behavior decision scenarios where notions of coordination are relevant. One of the early treatments of coordination was undertaken by Schelling [13] who introduced the concept of tacit coordination: “in tacit coordination . . . one is trying to guess what the other will guess one’s self to guess the other to guess, and so on ad infinitum . . . in the pure-coordination game, the player’s objective is to make contact with the other player through some imaginative process of introspection, of searching for shared clues.” (pp. 92, 93, 96)

Bicchieri [2] also treats coordination from the perspective of game theory, but argues that rational choice theory alone is not enough to effect meaningful coordination. It must be augmented with a theory of belief formation – that is, a theory of learning. If players are permitted to play the same game many times, they may gain insight regarding the social dispositions of the other players, and may be able to predict their behavior, establish their own reputations, and thereby gain the trust of others to their mutual advantage. As players repeatedly interact in a society, they may learn to recognize behavioral patterns and settle on stable ones. Coordination, therefore, is viewed as the end result of social evolution. Malone and Crowston [9], provide a taxonomy of coordination consisting of four elements: goals, activities, actors, and interdependencies. They offer the following operational definition: “Coordination is managing interdependencies among activities” [9, p. 50]. Cooper has introduced the concept of strategic complementarity: “Put in simple words, this condition implies that higher actions by other players provide an incentive for the remaining player to take higher action as well.” [3, p. 19] Decisions are strategic complements if switching to a choice with a higher potential payoff by one player would open up the opportunity for other players to increase their payoffs as well.

The operational concepts of tacit coordination, belief formation, action interdependence, and strategic complementarity all rely on rationality assumptions. But rationality is not part of the mathematical structure of a game. A game is defined by the players, their feasible actions, and their preference orderings; notions of rationality are external to the game description. Consequently, the above concepts of coordination are extrinsic; they are not part of the game’s mathematical structure.
3.2 Intrinsic Coordination

We now ask: Is it possible to define an intrinsic concept of coordination that is a function of, and only of, the mathematical structure of a game and does not rely on notions of rational behavior? At first glance, it may appear that this question is meaningless. Once the payoffs of all players are juxtaposed to form a payoff array, opportunities for cooperation, compromise, exploitation, competition, benevolence, malevolence, and so forth, are exposed. Since these are all social attributes, the argument goes, they cannot be separated from rationality. Thus, to speak of coordination without appealing to notions of rational behavior is impossible. That argument, however, depends upon a critical assumption; namely, that each player comes to the game with its preferences immutably defined. Once social bonds are created as the influence defined by conditional utilities propagates through a system, the structure of the resulting social model will possess an intrinsic ability to coordinate that is dependent on the structure of the utilities and independent of the notions of rationality that may be used to define a solution. It is this social structure that we wish to exploit.

Since the group welfare functions possess the mathematical syntax of probability mass functions, we may apply the mathematical operations and concepts from probability theory, but we must be careful to interpret them in praxeological, rather than classical epistemological, terms, such as marginalization, independence, and Bayes rule. In particular, one of the concepts associated with Shannon information theory is the notion of entropy. In an epistemological context, entropy is a measure of the randomness associated with a random variable. As is well known, entropy is maximized with a uniform distribution, since all outcomes are equally likely. Similarly, a utility function where all outcomes have the same benefit can be viewed as a situation of maximum difficulty in making a good decision. Thus, entropy, in a praxeological context, may be interpreted as the difficulty, or hardness, associated with the choice.

A more operational interpretation of entropy in an epistemological context is that it is a measure of the intrinsic ability to guess wrong. To illustrate, if most of the probability mass is focused on a single event (low entropy), then it is unlikely that any other event will be realized, and guessing that event will likely be correct. On the other hand, if all outcomes are equally likely (high entropy), then the intrinsic ability to make a mistake is high. Reinterpreting these concepts in a praxeological environment, if an agent places most of its utility on a single outcome (low hardness), then making that choice that results in that outcome is easy; but if all outcomes have the same utility (high hardness), then it difficult to settle on an acceptable choice.

The definition of hardness (entropy) for $X_i$ with individual welfare function $v_{X_i}$ is

$$H(X_i) = - \sum a_i v_{X_i}(a_i) \log v_{X_i}(a_i)$$

and the joint hardness of the group $\{X_1, \ldots, X_n\}$ is

$$H(X_1, \ldots, X_n) = - \sum v_{X_1 \ldots X_n}(a_1, \ldots, a_n) \log v_{X_1 \ldots X_n}(a_1, \ldots, a_n).$$

Essentially, hardness has to do with the opportunity cost (the utility of the outcome with the next-highest utility). If, for example, all outcomes were of equal utility, then the opportunity costs are the highest, and the hardness would be greatest, just as the entropy of a uniformly distributed random variable has maximum entropy. Shannon information theory uses the notion of entropy to define mutual information, which is the difference between the sum of the entropies of individual random variables and the joint entropy of the group. This difference, also called the Kullback-Leibler dispersion, provides a measure of the amount of information that random variables contain about each other. In the praxeological domain, this dispersion provides a measure of the amount of social interest shared by the players, and is given by

$$I(X_1, \ldots, X_n) = \sum_i H(X_i) - H(X_1, \ldots, X_n).$$

If all utilities are categorical, then they share no social interest, and if there is a one-to-one relationship between them, they share maximum social interest.

As established by [8, 7], a more convenient dispersion measure is obtained by forming the difference between joint hardness and mutual information and the dividing this difference by the joint hardness, yielding the relative dispersion measure

$$\mathcal{D}(X_1, \ldots, X_n) = \frac{1}{n-1} \left[ 1 - \frac{I(X_1, \ldots, X_n)}{H(X_1, \ldots, X_n)} \right].$$

It has been shown by [8, 7] that the relative dispersion measure is non-negative, symmetric, and, when $n = 2$, satisfies the triangle inequality (and hence is a true metric). The coordination capacity is then given by

$$C(X_1, \ldots, X_n) = 1 - \mathcal{D}(X_1, \ldots, X_n).$$

The coordination capacity is a measure of how intrinsically suited, or fit, the agents are to work harmoniously, irrespective of their notions of rationality.
If $C(X_1, \ldots, X_n) = 0$, as it would be if all utilities were categorical, the system has no intrinsic ability to coordinate, although it might coordinate extrinsically. Conversely, if $C(X_1, \ldots, X_n) \approx 1$, the agents are tightly coupled socially, and the intrinsic capacity to coordinate is high. One way to think of this quantity is as a measure of the ecological compatibility of the group that indicates how well a group is able to function in its environment.

4 Conclusion

The ability of an agent-based system to coordinate in the true etymological sense is a function of its social structure as defined by the influence relationships that exist among the agents. Conditional game theory is an extension of classical game theory that allows such relationships to be explicitly modeled, resulting in the emergence or notions of rational behavior for both the group and its individual constituents. The mathematical structure of Shannon information theory is exploited to define a coordination index that measures the ability of the system to coordinate.

References:


