Detecting Signals in a Non-stationary Environment Modeled by a TVAR Process, from Data Corrupted by an Additive White Noise†

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Abstract: In this paper, a method to detect unknown signals in a non-stationary environment is proposed. In addition, due to the sensor, the data are corrupted by an additive measurement stationary zero-mean white noise. Our approach, which can be useful in a wide range of situations such as the analysis of the object passing by, anomaly detection and digital communications, operates in three steps. Firstly, the non-stationary environment is assumed to be modeled by a time-varying autoregressive (TVAR) process. Secondly, the TVAR parameters and both the variances of the additive measurement white noise and the driving process are estimated by an evolutive method based on an errors-in-variables (EIV) approach. Thirdly, signal detection consists in studying the normalized prediction-error process of the TVAR model. Simulation results point out the relevance of the approach.

Key-Words: Signal detection, non-stationary noise, time-varying autoregressive model, parameter estimation, evolutive method, errors-in-variable approach, prediction-error process.

1 Introduction

Signal detection is one of the most important problems in the signal processing area. It plays a key role in various applications such as radar processing, medical applications, telecommunications, etc. In these fields, the random environment which is usually due to various physical situations may lead to problems in terms of estimation and detection. Thus, when the situation and the condition change, the statistical properties of the environment also change. For example under the sea, the communication is greatly influenced by the conditions of a tidal wave. If the intensity of the tidal wave increases, then the intensity (variance) of the random disturbance also increases. Consequently, in order to reflect the situation, the environmental noise should be modeled as a non-stationary process.

However, detecting the signals in a non-stationary environment falls into the most difficult class of problems, as stated in [1]. Haykin and Bhattacharya [2], [3] proposed a method named the modular learning strategy which includes three fundamental blocks, namely a time-frequency analysis, feature extraction and pattern classification. In addition, Haykin and Thomson [1] proposed an adaptive detector based on learning to detect a target signal embedded in non-stationary background noise. One of the authors also proposed a method to detect signals in a non-stationary noise based on the so-called stationarization and the stationarity test [4], [5].

In this paper, our purpose is to detect an unknown non-random and locally existing signal in a non-stationary environment, when the data are also assumed to be corrupted by an additive measurement white noise. To our knowledge, few papers deal with this issue. Thus, let \( y(k) \) be a scalar observation at time \( k \) given by:

\[
y(k) = s(k) + b(k) + x(k)
\]

where \( b(k) \) is a stationary zero-mean white Gaussian process induced by the sensor with variance \( \sigma_b^2 \) and \( x(k) \) is a non-stationary random environment. In this paper, \( x(k) \) is assumed to be modeled by the time-varying autoregressive (TVAR) process defined as follows:

\[
x(k) = - \sum_{n=1}^{p} a_n (k - n) x(k - n) + u(k)
\]

where \( p \) denotes the model order, the driving process \( u(k) \) is a zero-mean white noise with variance \( \sigma_u^2 \) and the set \( \{a_n (k - n)\}_{n=1,...,p} \) consists of the TVAR parameters.

It should be noted that the TVAR models are very popular and have been used in a wide range of
applications, from radar processing to model the clutter [6] to biomedical applications [7].
Our proposed method operates in three steps as follows:
(i) Given the environmental noise model (2), the TVAR parameters and the variances of both the driving process and the additive noise are unknown and hence must be estimated. For the last 30 years, the TVAR parameter estimation issue from noise-free observations has been mainly addressed by Grenier, by using least squares approaches. Here, to take into account the influence of the sensor, the data are also assumed to be corrupted by the additive measurement noise \(b(k)\). Therefore, the TVAR parameter estimation from noisy observations must be considered. We recently addressed this issue in [8]-[10] where we gave a state of the art on that topic. In [10], we proposed an evolutive method based on an errors-in-variables (EIV) approach and pointed out its relevance by means of a comparative study with other methods. Indeed, our method has the advantage of estimating both the TVAR parameters and the variances of the additive measurement white noise and the driving process.
(ii) The data are filtered by using a filter whose time-varying finite impulse response is defined by the estimates of TVAR parameters. When the data consist of only the non-stationary environment and the additive noise, the filter output corresponds to \(u(k)\) and a time varying moving average (TVMA) process generated by \(b(k)\).
(iii) Using the variances of the \(u(k)\) and \(b(k)\) estimated during step (i), the filter outputs normalized to obtain a non-stationary zero-mean process with unit variance. However, if the data also include a signal, this property is no longer satisfied. Therefore, testing the instantaneous variance of the normalized filter output makes it possible to detect the presence of the signal.
The remainder of the paper is organized as follows: in section 2, our method is presented in details. Simulation results are then given and point out the relevance of the approach.

2 Signal Detection Using Prediction-Error Process of TVAR Prediction

2.1 Estimation of the TVAR Parameters
First, we investigate the problem of the TVAR parameter estimation. Let us consider the data that only consist of the environment noise and the measurement noise. This assumption holds as the duration of the signal \(s(k)\) in equ. (1) is very brief.
Thus, one has:
\[
y(k) = x(k) + b(k)
\] (3)
Given (3), the TVAR parameters have to be estimated from the noisy observations \(y(k)\). Among the approaches that can be considered, we suggest using the one we proposed in [10] and which is based on an errors-in-variables (EIV) algorithm. Let us briefly recall this evolutive method, wherethe TVAR parameters are assumed to be expressed by using a function basis, (as proposed by Grenier [11]):
\[
a_n(k) = \sum_{j=0}^{m} \beta_{nj} f_j(k),
\] (4)
where \(\{f_j(\cdot)\}_{j=0,1,...,m}\) are the basis functions and \(\{\beta_{nj}\}_{j=1,...,m}\) are the corresponding weights.
Therefore, the TVAR parameter estimation issue consists in estimating the weights from the data. This method is hence a deterministic regression method.
By introducing \(A(k) = [f_0(k) \quad \cdots \quad f_m(k)]^T \alpha(k), (A, \alpha) = (Y, y), (X, x), \text{ or } (B, b)\), from (3), one has:
\[
X(k) = Y(k) - B(k).
\] (5)
Then, let us consider the weight vector \(\theta_0 = [\beta_{10} \quad \beta_{11} \quad \cdots \quad \beta_{1m} \quad \beta_{20} \cdots \beta_{pm}]^T\).
Using (4), one has:
\[
P \sum_{n=1}^{p} a_n(k-n)x(k-n) = [X^T(k-1) \quad \cdots \quad X^T(k-p)] \theta_0
\] (6)
Given (6), (2) can be rewritten as follows:
\[
[x(k) - u(k) \quad X^T(k-1) \quad \cdots \quad X^T(k-p)] \cdot \theta^p = 0
\] (7)
where \(\theta^p = [1 \quad \theta_0^T]^T\).
By premultiplying inequ.(7) by
\[
\begin{bmatrix}
x(k) - u(k) \\
x(k-1) \\
\vdots \\
x(k-p)
\end{bmatrix}
\]
and taking the expectation, we have:
\[
E \left[ \begin{bmatrix}
x(k) - u(k) \\
x(k-1) \\
\vdots \\
x(k-p)
\end{bmatrix} \begin{bmatrix}
x(k) - u(k) \\
x(k-1) \\
\vdots \\
x(k-p)
\end{bmatrix}^T \right]
\] (8)
Substituting eqs. (3) and (5) into (8), one has

\[ (R_Y^p - R_B^p)\theta^p = 0 \]  

where \( R_Y^p \) is defined by the unknown variances \( \sigma_1^2 \) and \( \sigma_2^2 \). So, the formulation of an EIV estimation problem using the Frisch scheme consists in determining, only on the basis of the noisy observations, the set of noise variances that satisfies the semi-definite positiveness condition:

\[ R_Y^p - C^p(\alpha, \beta) = 0 \]  

\[ \overrightarrow{R_Y}^p - C^p(\alpha, \beta) = \begin{bmatrix} \alpha & 0 & \ldots & 0 \\ 0 & \beta F(1) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \beta F(p) \end{bmatrix} \geq 0. \]  

To determine this set, let us consider the set of candidates \( [\alpha_1, \beta_1] \) so that \( \alpha_1^2 + \beta_1^2 = 1 \) and \( \alpha_1 \geq \beta_1 \). The 2-tuple \( [\lambda\alpha, \lambda\beta] = [\alpha_1, \lambda\beta] = [\alpha_1, \beta_1] \) makes \( \overrightarrow{R_Y}^p - C^p(\alpha, \beta) \) semi-definite positive provided that \( \lambda \) is the largest eigenvalue of \( \overrightarrow{R_Y}^p - C^p(\alpha, \beta) \). This hence leads to an infinite set of solutions defining a first curve \( S_p(\alpha_1, \beta_1) \). Then, one can iterate the same process by using another model order \( q \) higher than \( p \). This leads to a second curve \( S_q(\alpha_1, \beta_1) \). Therefore, in theory, the variances to be found belong to both curves.

In practice, the expectation is replaced by the temporal mean over a sliding window. As a consequence, there is no longer an intersection between both curves.

On the one hand, for \( S_p(\alpha_1, \beta_1) \), one has:

\[ \left( \overrightarrow{R_Y}^p - C^p(\alpha_1, \lambda_1^{-1}, \beta_1, \lambda_1^{-1}) \right)\theta^p = \theta^p \]

where \( \lambda_1^{-1} \) is the inverse of the largest eigenvalue of \( \overrightarrow{R_Y}^p - C^p(\alpha_1, \beta_1) \).

On the other hand, for \( S_q(\alpha_1, \beta_1) \), one has:

\[ \left( \overrightarrow{R_Y}^q - C^q(\alpha_1, \lambda_q^{-1}, \beta_1, \lambda_q^{-1}) \right)\theta^q = \theta^q \]

where \( \lambda_q^{-1} \) is the inverse of the largest eigenvalue of \( \overrightarrow{R_Y}^q - C^q(\alpha_1, \beta_1) \).

So, we suggest estimating the noise variance by minimizing the following criterion:

\[ J(\alpha_1, \beta_1) = \left\| \left( \overrightarrow{R_Y}^p - C^p(\alpha_1, \lambda_1^{-1}, \beta_1, \lambda_1^{-1}) \right)\theta^p \right\|_2^2 \]

where \( \|\cdot\|_2 \) denotes the Frobenius norm.

The variances \( \sigma_1^2 \) and \( \sigma_2^2 \) are also obtained by using estimated values of \( \alpha \) and \( \beta \).

### 2.2 Detecting Signals Using Normalized Prediction-error Process

The data \( d(k) \) are filtered by using the so-called inverse filter, whose finite impulse response is defined by the estimates of the TVAR parameters \( \hat{a}_n(k - n) \). Thus, one has:

\[ d(k) = y(k) + \sum_{n=1}^{p} \hat{a}_n(k - n)y(k - n) \]

When there is no signal, substituting (3) into (12), one has:

\[ d(k) = x(k) + \sum_{n=1}^{p} \hat{a}_n(k - n)x(k - n) \]

or equivalently:

\[ d(k) = \hat{u}(k) + b(k) + \sum_{n=1}^{p} \hat{a}_n(k - n)b(k - n) \]

The process \( d(k) \) can hence be regarded by the sum of two terms: a time-varying moving average (TVMA) process and \( \hat{u}(k) \) having the same statistical properties as \( u(k) \).
Since $b(k)$ and $\tilde{u}(k)$ are uncorrelated white noise processes, $d(k)$ is a zero-mean process and its variance is time-varying and is equal to $\sigma_d^2(k) = \sigma_u^2 + \sigma_b^2(1 + \sum_{n=1}^{p} a_n^2(k - n))$.

This process $\hat{d}(k)$ is also non-stationary, zero-mean but with unit variance. If a signal exists, it may disturb this property. So in this paper, the instantaneous variance of the noisy TVMA process $\hat{d}^2(k)$ is estimated and plays the role of a signal detector.

3 Simulations

3.1. Simulation protocols

In this section, we suggest carrying out simulation studies. The noisy observations are generated by using equs. (1) and (2), where the variance of $u(k)$ is equal to $\sigma_u^2 = 1$. The order of the TVAR-model and the size of the basis are set to $p = 2$ and $m = 1$, respectively. Basis functions are given by:

$$f_0(k) = 1 \quad \text{and} \quad f_1(k) = \sin \left( \frac{2\pi k}{N} \right),$$

with $N$ the number of samples.

Signal $s(k)$ consists of 3 components of a square pulse signal. Each component exists during 10 samples and has the magnitude set to 30, as depicted in figure 1.

Firstly, a synthetic TVAR process $x(k)$ is generated by (2) and (4) given the weights $\beta_{10} = -1.55$, $\beta_{11} = 0.4$, $\beta_{20} = 0.98$, and $\beta_{21} = 0.02$. The variance of the stationary additive measurement noise $b(k)$ is set to $\sigma_b^2 = 9$. The signal-to-noise ratio (SNR) is hence equal to -4 dB. In this paper, since the environment is assumed to be non-stationary, the SNR is defined as the ratio between the power of the pulse signal and the maximum of the variance of the sum $b(k) + x(k)$; note that, for each time $k$, the variance is estimated by using the window of length equal to 500 and centered around $k$.

Given figure 1 (at the bottom), it can be seen that the process $\hat{d}^2(k)$ has large values when each component of the signal appears.

Figure 2 shows the estimates of the TVAR parameters $\hat{a}_n(k)$ $(n = 1, 2)$.

3.2. Simulation results

In this subsection, the performance of the signal detection is evaluated. More particularly, let us study the ability of the detector from the probabilistic point of view. According to the previous subsection, the process $\hat{d}^2(k)$ can be a useful indicator for the signal detection. Actually, the signal detection is done by introducing the threshold $\rho$. According to the conventional signal detection problem [12], the threshold is chosen by the following process. First, the probability of the false alarm $P_F$ is defined by the practitioner. Then, given the false alarm $P_F$ and the threshold $\rho$, one has:
where \( p(X) \) is the probability density function (pdf) of the process \( d^2(k) \) in the signal-free case. As the pdfs of the environment \( b(k) \) and \( x(k) \) are assumed to be Gaussian, the pdf of the process \( d(k) \) is also Gaussian and the pdf of the squared process \( d^2(k) \) is the chi-square distribution with 1 degree-of-freedom defined as follows:

$$ p(X) = \frac{1}{\sqrt{2\pi}X} e^{-\frac{x}{2}}. \quad (16) $$

Figure 3 shows the probability of detection for the thresholds \( \rho \) equivalent to the probabilities of the false alarm \( P_F = 0.01, 0.05, \) and 0.1 for various values of the SNR, obtained by one hundred Monte Carlo simulations. In these simulations, the probability of detection is obtained by counting the successes corresponding to any value higher than the threshold \( \rho \) when all components of signal appear.

Given figure 3, the larger the probability of the false alarm is, the larger the probability of detection is. This result is reasonable from the theoretical point of view. In addition, the higher the SNR is, better the detection performance are.

When the SNR is higher than -10 dB, the approach is rather reliable.

4 Conclusions

The approach we propose is based on an a priori model of the additive environmental noise. In that case, the key issue is the estimation is the model parameters when the data are corrupted by an additive measurement noise.

References:


