Overview On The Modeling And Digital Linearization Of Power Amplifiers

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Abstract: In modern transmitting devices the digital predistortion technique is widely used for linearizing the power amplifier. There are many algorithms realizing predistortion technique with a different complexity and a stability of results. The several methods are described and compared. The orthogonal polynomials with memory were has proved to be simple enough with suitable results, so it was chosen for modeling a power amplifier and for the predistortion.

Key–Words: Digital predistortion, linearization, power amplifier, Volterra, orthogonal, polynomial, dynamic deviation reduction.

1 Introduction

Using power amplifier (PA) operating near saturation point became inevitable part of modern mobile communication systems. The ideal PA is perfectly linear until certain saturation output power. In practice the PAs are quite non-linear devices exhibiting also memory effects. To achieve maximal efficiency, the operating point of the PA needs to be set close to the saturation point. In such region, the nonlinear effects are very strong. Thus a certain compromise has to be achieved between efficiency and the linearity of the PA. Still nonlinearities in PA generate amplitude and phase distortion, causing spectral spreading [1] in adjacent channels, inter-symbol interference or deformations of constellation diagram.

The demands for increasing the transmitter speeds usually are solved by widening the bandwidths. Due to the large bandwidth of transmitted signals (i.e. 100 MHz for LTE), the memory effects of PAs will increase. There exist several techniques to remove unwanted distortion utilizing predistortion (PD) methods. Comparison of output spectrum for different configurations can be seen on the Fig.1. There is a system with and without predistorter and power spectrum of the input sequence. In the system without predistorter there is a distortion around useful bandwidth, on the contrary a system with PD compensates the distortion. A predistorter is a functional block that precedes a nonlinear device such as PA. The signal of the input passes the predistorter whose characteristic is an inverse of PA. In the last twenty years, many articles on digital predistortion have been already published. One of the most used techniques is indirect learning architecture. Such solution involves using postdistorter for modeling a PA (as an inverse function to predistorter) and than using estimated coefficients for a predistorter. It was shown [9], that certain equivalence between predistortion function and its inverse function postdistortion holds even for such non-linear systems.

There are several methods used in digital predistorters from the most general form known as Volterra series and its derivatives to the polynomials (which can be considered as special case of Volterra series). As for model-based approaches, the orthogonal polynomial model and the dynamic distortion reduction based on simplified Volterra series are a common choice due to its simple complexity, stability of solution and ease of implementation [2].

Figure 1: Power spectrum density of the input \( \hat{x}(t) \) of the system, the output without predistortion \( \hat{y}(t) \).

In this paper, we first report some generalities on baseband predistortion systems. Volterra series, a polynomial series, an orthogonal polynomials with
memory and dynamic deviation Reduction algorithm are discussed. Then, in Section V we discuss the estimation algorithms. In the section VI we present some results of models and linearization. Finally we conclude in Section VII.

2 Volterra Series

The Volterra series is a model for non-linear behaviour similar to the Taylor series. It differs from the Taylor series in its ability to capture memory effects. The Volterra series can be used to approximate the response of a non-linear system to a given input if the output of this system depends strictly on the input at that particular time.

\[
y(t) = \sum_{k=1}^{K} \sum_{q=0}^{Q} a_{kq} x(t-q)x(t-q)^{k-1}.
\] (2)

Despite the simplicity of the definition the stability and condition when performing the inversion of the ill-conditioned matrix, the method is not very stable due the ill-conditioned matrix.

3 Polynomial series with memory

Polynomial series are widely used for modelling the non-linearities. The presented series also compensates the memory effects by sliding the window of delayed copy.

\[
y(t) = k_0 + \sum_{n=1}^{\infty} \int_{-\infty}^{t} k_n(t_1, t_2, \ldots, t_n) x(t-t_2) \ldots x(t-t_n) dt_1 dt_2 \ldots dt_n,
\] (1)

where \( k_n \) is called the Volterra series kernel and can be regarded as a higher-order impulse response of the system.

4 Orthogonal Polynomials with memory

Power amplifiers can be successfully modeled by using memory polynomial models. The memory polynomial models are a special case of Volterra series. The memory polynomials might be perceived as a compromise between memoryless models and the full Volterra series. Let us denote by \( \hat{x}(t) \) the complex input of a nonlinear system and by \( \hat{y}(t) \) the corresponding complex output. Then the polynomial model can be modeled as

\[
\hat{y}(t) = \sum_{k=1}^{K} \sum_{q=0}^{Q} b_{kq} |\hat{x}(t-q)|^{k-1} \hat{x}(t-q) + \hat{e}(t),
\] (3)

where \( K \) is the level of polynomial order and \( Q \) is the depth of memory. Thus the number of coefficients is \( K(Q + 1) \). The time dependent error function \( \hat{e}(t) \) needs to be minimized. If \( \hat{x}(t) \) is uniformly distributed in the range of set \( < 0, 1 > \) then model can be expressed as

\[
\hat{y}(t) = \sum_{k=1}^{K} \sum_{q=0}^{Q} b_{kq} (\sum_{l=1}^{L} (-1)^{l+k} \cdot \frac{(k+l)!}{(l-1)!(l+1)!(k-l)!} |\hat{x}(t-q)|^{k-l} \hat{x}(t-q) + \hat{e}(t).
\] (4)

4.0.1 Dynamic Deviation Reduction

To overcome the complexity of the general Volterra series, an effective model-order reduction method, called dynamic deviation reduction (DDR) [5, 6, 7] was presented. This is based on the fact that the effects of dynamics tend to fade with increasing order in many real PAs, so that the high-order dynamics can be removed in the model, leading to a significant simplification in model complexity. The 1st-order dynamic truncation of the DDR-based baseband Volterra model in the discrete time can be written as

\[
y(t) = \sum_{k=1}^{K} \sum_{q=0}^{Q} \sum_{l=0}^{L} g_{2k+1,1}(i) |x(n-i)|^{2k} x(n-i) + \sum_{k=1}^{K} \sum_{q=0}^{Q} g_{2k+1,2}(i) |x(n-i)|^{2(k-1)} x^*(n-i),
\]

(5)

where \( x(n) \) and \( y(n) \) are the complex envelopes of the input and output of the PA, respectively, and \( g_{2k+1,j} \) is the complex Volterra kernel of the system. Symbol \((.)^*\) stand for the complex conjugate operation and \( |.| \) returns the magnitude. The 2nd-order DDR model can be written and simplified as

\[
y(t) = \sum_{k=1}^{K} \sum_{q=0}^{Q} g_{2k+1,1}(i) |x(n)|^{2k} x(n-i) + \sum_{k=1}^{K} \sum_{q=0}^{Q} g_{2k+1,2}(i) |x(n)|^{2(k-1)} x^2(n-i) + \sum_{k=1}^{K} \sum_{q=0}^{Q} g_{2k+1,3}(i) |x(n)|^{2(k-1)} x(n)|x(n-i)|^2 + \sum_{k=1}^{K} \sum_{q=0}^{Q} g_{2k+1,4}(i) |x(n)|^{2(k-1)} x^*(n) x^2(n-i),
\]

(6)

where \( x(n) \) and \( y(n) \) are the complex envelopes of the input and output of the PA, respectively, and \( g_{2k+1,j} \) is the complex Volterra kernel of the system. Symbol \((.)^*\) denotes the complex conjugate operation and \( |.| \) returns the magnitude.

5 Estimation algorithms

The solution can be obtained by least-squares (LS) by arranging the inputs and outputs into the matrices. To obtain more stable solution (lower condition number) the orthogonal polynomials can be used.[2] Then we can form linear equation

\[
\hat{y} = \hat{b} \hat{\Psi},
\] (7)

where \( \hat{\Psi} \) is a matrix constructed as in [2]. Then the least-squares solution (LS) is

\[
\hat{b} = (\hat{\Psi}^H \hat{\Psi})^{-1} \hat{\Psi}^H \hat{y} = \hat{\Psi}^+ \hat{y},
\] (8)
where \((\cdot)^H\) denotes a complex conjugate transpose and \(\hat{\Psi}\) denotes Moore - Penrose pseudoinverse.

In many applications, adaptive estimation is much more efficiently performed on the sample by sample basis. Such approach minimizes the storage requirements. Usually stochastic gradient algorithms like least mean square algorithm (LMS) is used. But such algorithms are particularly efficient, because the system contains small nonlinearity, which yield in to the ill conditioned covariance matrix. The most popular algorithm is recursive least squares algorithm (RLS). Such algorithm updates the inverse covariance matrix each new sample and forms Kalman-like coefficient update [10]. The RLS is very computationally intensive. another approach is to use damped newton algorithm (DNA), which has comparable performance as RLS but is less complex. It is estimated as

\[
\hat{x} = \Psi b_p. \tag{9}
\]

The error vector between inputs is

\[
e = x - \hat{x}, \tag{10}
\]

ten the update sample-by-sample is performed as

\[
b_{p+1} = b_p + \mu (\Psi^H \Psi)^{-1} \Psi^H e. \tag{11}
\]

6 Simulations and results

The different algorithms have been tested on a model of a class AB PA working at 1.455 GHz described in [8]. Previously mentioned power amplifier was built with a Motorola model MRFC1818 GaAs MESFET. First the models of PA were extracted using orthogonal polynomials with polynomial level \(K = 10\) and memory depth \(Q = 3\) (totally 40 coefficients). The normalized mean square error for modeled PA was \(-30.08\)dB. Than another method known as the dynamic deviation reduction of second order (DDR2) [5], [6] and [7] was tested. The normalized mean square error using DDR2 for modeled PA was \(-30.82\)dB. Because of non-significant differences in results, the orthogonal polynomial model with memory was preferred due to simpler complexity. For simulating the system the baseband input an OFDM signal with QPSK symbols on data sub-carriers was used. For predistorter and postdistorter the polynomial level of nonlinearity was set to \(K = 17\) and the memory depth \(Q = 1\) was used for all the simulations.

There are compared different methods using predistorter in the table (1). The Volterra series are not used because of their complexity. Polynomials with memory are also not used because its instability, which makes it impractical for real implementation. It is obvious, that the DDR first order has a good performance and low complexity.

The level of power ratio in adjacent channels (ACPR) is compared for different configuration. The right and left channel ACPRs are defined by

\[
ACPR_R = 10 \log \left( \frac{\int_{-B/2}^{B/2} P(\hat{g}(t)) dt}{\int_{-3B/2}^{3B/2} P(\hat{g}(t)) dt} \right), \tag{12}
\]

\[
ACPR_L = 10 \log \left( \frac{\int_{-B/2}^{B/2} P(\hat{g}(t)) dt}{\int_{-3B/2}^{3B/2} P(\hat{g}(t)) dt} \right),
\]

where \(B\) represents the bandwidth of the signal and \(P(.)\) is power spectral density. The obtained results are summarized in Table 1. To quantify the deformation of constellation at the output of the PA an error vector magnitude (EVM) was used [3]. In general EVM is a measure of how far the points are from the ideal locations. The computation does not take into account the transmission channel noise and distortions but is focused on the PA distortions. An error vector magnitude (EVM) parameter defined by

\[
EVM = \sqrt{\min_{\alpha, \beta} \sum_k |S_s(k) - (S_t(k) - \beta)/\alpha|^2 / \sum_k |S_s(k)|^2}, \tag{13}
\]

\(S_s(k)\) represents the original source symbols and \(S_t(k)\) the actually transmitted symbols (output from the PA). The parameters \(\alpha\) and \(\beta\) are optimized in order to compensate for a rotation and an offset of the constellation [3].

![Diagram](image)

Figure 2: Power spectrum density of the input and the output without PD and with PD using orthogonal polynomials.
7 Conclusion

In the article there is presented a many algorithms improving a performance of the power amplifiers. A performance is measured by normalized mean square error between the output of power amplifier and the input. First different models of PA are estimated from real measured PA. Extracted models are compared and evaluated. Then the dynamic deviation reduction model of the first order is chosen as a compromise with respect to the complexity and precision. The amplifier has exhibited different output power in adjacent channels. The normalized mean square error was $-30.3dB$ setting $K = 13$ and $Q = 1$ (20 coefficients). The presented methods has proved to be powerful and low complexity. If the readers have any questions, please do not hesitate to contact the authors.

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