

Allometric Models for Predicting Tree Diameter at Breast Height

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Abstract: Mathematical relations that use easily measured variables to predict difficult-to-measure variables are also important in forest inventory. The aim of this study is to develop models which can include variables derived from remote sensing and from previous forest inventory data that would be suitable for estimating tree metrics, needed to calculate a single tree volume. In this paper allometric relations to predict diameter at breast height (DBH) was developed, using different types of data (qualitative and quantitative). Totally seven different linear models were developed, using data collected in Latvia, Jelgava district. General linear model that predicts DBH includes a tree height, effective crown area, soil type and age factors. It showed strongest relationship between predicted and measured DBH ($R^2 = 0,872$). Summary results show that the models predict DBH reasonably well and factors included in all models are significant.

Key-Words: Allometric models; forest inventory; diameter at breast height; remote sensing.

1 Introduction

Foresters and ecologists have long observed regularities in tree growing processes where every aspect has its own role. Canopy leaf surface area, tree height, DBH, volume and structural properties are the main tree characteristics that affect each other.

Tree allometry establishes quantitative relations between some key characteristic dimensions of trees (usually fairly easy to measure) and other properties (often more difficult to assess). Allometric relationships for estimating tree-level and stand-level parameters are very important for managing any forest resources [1, 11, 13, 16]. Allometry varies between tree species [9] and within a stand as trees adapt to the intra and interspecific competition [8], also characteristics such as stand density, stand silvicultural history, genetic factors of tree seed, tree position in a stand, site fertility, height above sea level, distance from sea, mineral soil and stand development class [10, 12, 19] affect variables. These variables are usually measured in the field but can also be predicted by regression models [18]. Direct measurements of forest structure are taken on intensively sampled, relatively small field plots, and these data are used to create allometric models that predict forest parameters from easily measured tree attributes.

There have been numerous studies of this approach. DBH prediction models have been studied by using field measured trees [17] or aerial

photographs [7]. More recent efforts have focused on measuring individual tree heights using airborne laser scanning (ALS) data [14, 18] or crown widths from high resolution aerial [6] and satellite imagery [5], then using the modeled DBH to estimate single tree volume, stand volume, total-tree biomass and carbon.

Tree-level stem volume is usually calculated by using volume models or equations in forest inventory applications. Diameter at breast height (DBH) is commonly used as predictors of stem volume [3] and other tree metrics in a wide variety of allometric equations. In view of the complexity caused by irregularities and diversity of forest stands photogrammetry and airborne LIDAR-based single tree remote sensing (STRS) methods require necessity to adopt the semi-automatic approach [8] and use auxiliary information [15] to make STRS solvable. Most often, this auxiliary information is obtained through allometry.

The goal was to find models that use variables derived from remote sensing and previous forest inventory data that would be suitable for estimating tree metrics needed to calculate single tree and stand volume. In the paper different models for predicting tree DBH were researched.

2 Problem Formulation

2.1 Site description

The study site was a forest (12700 ha) in the middle of Latvia in Jelgava District (56°39' N, 23°47' E).

Totally 350 sample plots (0.045ha) were established during summer 2010. The area consists of mixed coniferous and deciduous forest with different age, high density, complex structure, various components, composition and soil conditions. Represented species are Scots pine (*Pinus sylvestris* L.), Norway spruce (*Picea abies* (L.) H.Karst), silver birch (*Betula p ndula* Roth), black alder (*Alnus glutinos* L.), and European aspen (*Populus tr mula* L.)

2.2 Field measurements

All trees with a diameter at breast height DBH of more than 5 cm were measured and for each tree coordinates, species, height, DBH, crown width and length was recorded. Altogether there were measures of 6154 trees in the data. The mean characteristics of all trees are presented in Table 1.

Table 1. Mean characteristics of the study material.

Species	Characteristic	DBH, cm	Tree height, m	Tree crown width, m	Trees, n
Scots pine	Mean	26.8	23.3	5.3	1617
	Min	5	2.5	0.95	
	Max	83.8	37.1	12.37	
Norway spruce	Mean	17.3	15.4	4.8	2365
	Min	5	2.2	0.5	
	Max	77.8	39.3	13.5	
Silver birch	Mean	17.8	19.1	5.4	1057
	Min	5	4.9	1.1	
	Max	54.6	39.9	18.93	
Black alder	Mean	21.9	21.3	6.1	1016
	Min	5.4	4.3	1.2	
	Max	58.3	36.8	16.48	
European aspen	Mean	27.35	25.8	6.1	99
	Min	6.5	6.7	1.3	
	Max	53.5	35.8	13.2	

Differentially corrected Global Positioning System measurements were used to determine the position of the center of each plot. Accuracy of the positioning was approximately 1 meter.

Database processing and field data acquisition process model are presented in Fig.1.

The tree crown width was measured by projecting the edges of the crown to the ground and by measuring the length along one axis from edge to edge through the crown centre. The diameters of any two axes at 90 degrees to each other were selected and averaged by using arithmetic mean. Tree locations within a plot were measured using center as the origin for determining tree azimuth and distance to the center. Information about tree age

and soil conditions was taken from year 2009 forest inventory.

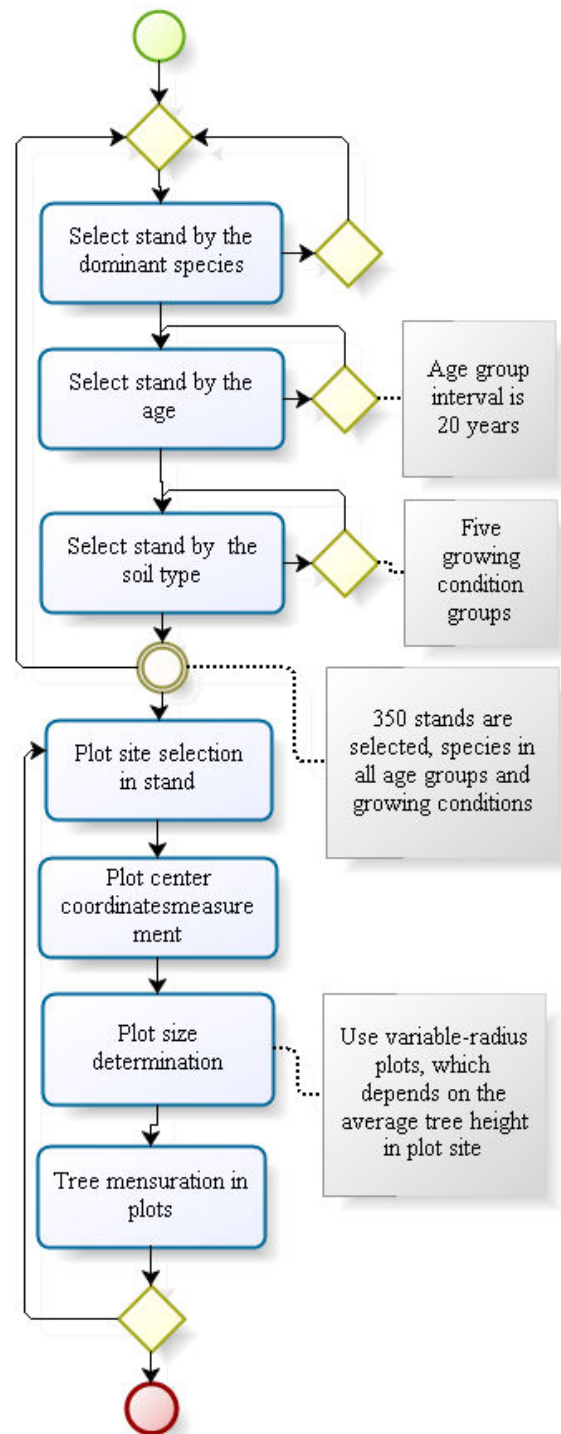


Fig.1 Database processing and field data acquisition.

Effective crown area (the area that does not overlap with another tree crown) for each tree (first and second storey trees equally) was calculated using information about its location within a plot and the width of its crown. The foliage was projected on the ground and generally known area calculation formulas were used for calculations of effective

crown area. Two more reasons for using effective crown area instead of the simple one are as follows - it acquires more sunlight than the rest of its share and in the process of tree identification by using remote sensing data usually only effective leaf area can be detected. Further data analyzes showed that by using it, instead of simple tree crown, considerable increase of the regression model accuracy can be achieved.

2.3 Statistical analysis

The analysis of covariance was used to evaluate if certain factors have an effect on the outcome variable. Multiple linear regressions consider more than one independent variable, and it was used to develop allometric models to predict DBH (cm) of an individual tree in SPSS for Windows using untransformed data (for age, soil type and species) and transformed data (for DBH, tree height, crown width and effective crown area). Power function was used of the form:

$$Y_i = \beta_0 + \beta_1 * X_{i1} + \beta_2 * X_{i2} + \dots + \beta_p * X_{ip} + \epsilon_i \quad (1)$$

where Y_i is the i th observation of the dependent variable (DBH), X_{ij} is i th observation of the j th independent variable (collected data in sample plots and data from previous forest inventory), $j = 1, 2, \dots, p$. The values β_j represent parameters to be estimated, and ϵ_i is the i th independent identically distributed normal error.

The following seven general linear models were developed, by using data collected in Latvia, Jelgava District:

$$\ln(DBH)_i = \beta_0 + \beta_1 * A + \beta_2 * AAT + \beta_3 * S + \epsilon_i \quad (2)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * S + \epsilon_i \quad (3)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * A + \beta_3 * AAT + \beta_4 * S + \epsilon_i \quad (4)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * \ln(VP)_i + \beta_3 * S + \epsilon_i \quad (5)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * \ln(VPE)_i + \beta_3 * S + \epsilon_i \quad (6)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * \ln(VP)_i + \beta_3 * A + \beta_4 * AAT + \beta_5 * S + \epsilon_i \quad (7)$$

$$\ln(DBH)_i = \beta_0 + \beta_1 * \ln(H)_i + \beta_2 * \ln(VPE)_i + \beta_3 * A + \beta_4 * AAT + \beta_5 * S + \epsilon_i \quad (8)$$

Where DBH is the diameter at breast height (cm), H - tree height (m), VP – crown width (m), VPE - effective crown area (m²), A - age (years) and S - species dummy variable, but AAT - soil type dummy variable for the districts.

All models were extended also to include the factors interaction effects. The intercept was tested to determine if they were statistically different from zero ($P < 0.05$). Root mean square error of the estimate (RMSE) and the coefficient of determination (r^2) were used to evaluate goodness of fit.

3 Problem Solution

One of the main tasks was to find out what data is required in order to, as closely as possible, determine DBH. The basis for nearly all models is well-known relationship between the diameter and height of a tree. In Fig.2 are shown relationships between \ln DBH and \ln H in the study area.

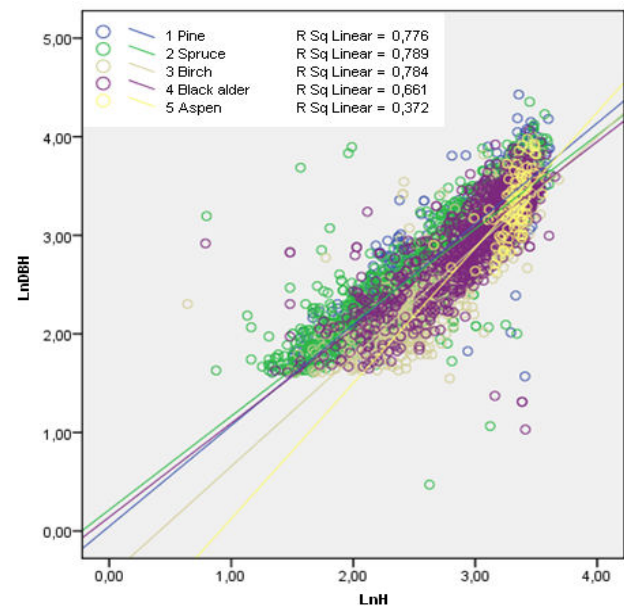


Fig.2 Relationships between \ln DBH and \ln H in the study area of main species.

There is a linear relationship between \ln DBH and \ln H, but depending on the species linear regression growth rate is different.

Model (model no. 2. in Table 2) that uses measurement of a tree height as a predictor variable can be expected to produce a reasonably accurate estimate of DBH ($r^2=0.792$), but often it is not enough.

Overview of statistical indicators of all models is shown in Table 2.

Table 2. Overview of models statistical indicators.

Model, (Factors)	RMSE	F	Sig.	r2
1. (A;AAT;S)	9.2	50.6	0.000	0.409
2. (H;S)	165.7	2602.3	0.000	0.792
3. (H;A;AAT;S)	12.1	218.8	0.000	0.822
4. (H;VP;S)	114.2	2471.7	0.000	0.849
5. (H,VPE,S)	114.4	2500.6	0.000	0.851
6. (H,VP,A,AAT,S)	9.6	237.1	0.000	0.871
7. (H,VPE,A,AAT,S)	9.6	239.2	0.000	0.872

Model (model no.1. in table 2) that includes only information about species, age, soil type and factors interaction effects (Table 3) showed poor results.

Table 3. Overview of model no.1 (A;AAT;S) factor interaction effects.

Model (A;AAT;S)	RMSE	F	Sig.
S	0.4	2.6	0.033
AAT	2.7	14.8	0.000
A	13.2	72.1	0.000
S * A	0.6	3.4	0.007
S * AAT	1.9	10.3	0.000
S * AAT * A	1.8	10.3	0.000
AAT * A	1.3	7.2	0.000

It proved that a tree height is the most important factor in all models.

Models with different combination of factors were tested and the best results were obtained by using one that included 5 factors - tree height, effective crown area, age, tree species and soil type. Using this general linear model, the following results were obtained (Table 4), where all 5 factors-tree height ($p=0.01$), effective crown area ($p=0.000$), age ($p=0.068$), species ($p=0.000$) and soil type ($p=0.000$), as well factors interaction effects, are significant at different level on significance.

This model gave the best performance according to values of statistics used to compare models in the fitting phase. Consequently, this model was accepted.

Results of DBH estimation model accuracy show a strong relationship between predicted and measured DBH (shown in Fig. 3).

Results of this study show that the developed model can be used in LIDAR-based single tree remote sensing methods to predict DBH if information about soil type and age is available.

Table 4. Overview of model no.7 (H,VPE,A,AAT, S) factor interaction effects.

Model (H,VPE,A,AAT,S)	RMSE	F	Sig.
S	0.32	7.96	0.000
AAT	0.14	3.51	0.000
A	0.13	3.32	0.068
LnH	0.41	10.35	0.001
LnVPE	0.61	15.27	0.000
S * AAT	0.15	3.74	0.000
S * AAT	0.07	1.84	0.117
S * LnH	0.07	1.79	0.126
S * LnVPE	0.07	1.82	0.120
AAT * A	0.03	0.79	0.647
AAT * LnH	0.16	4.17	0.000
AAT * LnVPE	0.09	2.23	0.006
S * AAT * A	0.08	2.17	0.001
S * AAT * LnH	0.15	3.86	0.000
S * AAT * LnVPE	0.07	1.95	0.003

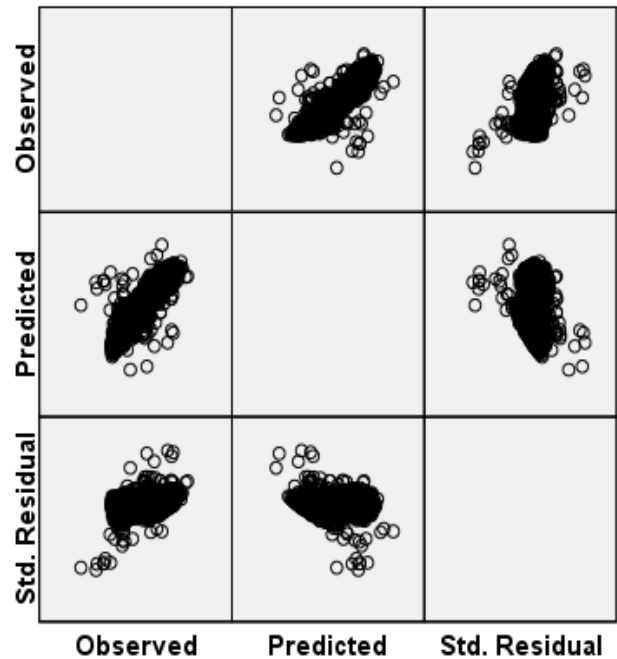


Fig.3 Predicted and measured DBH.

It should be possible to improve DBH estimation accuracy by using more precise measurements of foliage density, foliage mass or crown length. The parameter that could cause most problems when this model is used in practice is tree species, because methods that are capable to determine its value from remote sensing data are still being developed and those that are used in

practice usually are able to distinct three or four species.

A number of studies have also investigated the relationship between DBH and crown dimensions for different tree species and a strong relationship was noted [2, 4].

4 Conclusion

Models that use field or remotely-sensed measurement of a tree height as a predictor variable can be expected to produce a reasonably accurate estimate of DBH ($R^2=0,792$) in Latvian forests, but when the model uses crown dimension measurements and information about age and soil type, the accuracy of DBH increases ($R^2=0,872$).

Interaction effects between factors included in models must be considered, and statistical analysis shows that they are significant at different level on significance.

Simple leaf area calculation must be replaced with one that considers tree concurrency, because it better suits to be used in systems for processing remote sensing data.

It should be possible to improve DBH estimation accuracy if information of tree foliage density, foliage mass or crown length were available. Developed models can be used in LIDAR-based single tree remote sensing methods to predict DBH.

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