INS and magnetic sensor aided carrier phase differential GPS for attitude determination

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Abstract: - This paper presents a fast and reliable method for precise orientation (attitude) determination for a moving vehicle. The method uses the carrier phase measurements of GPS receivers. To solve the so called integer ambiguity problem the additional accelerometer and magnetometer is used. These sensors should be calibrated, the calibration method is also presented. The attitude determination method is tested in real circumstances, during a real flight with a sailplane. Based on the results further corrections are initiated which results more precise measurements.

Key-Words: - Carrier phase, differential GPS, magnetometer, precise navigation

1 Introduction
The outdoor navigation of a moving vehicle is usually in the focus of researches. It has a plenty of possible applications in the field of transportation, agriculture or military.

Increasing the level of autonomy the demand of high precision positioning and attitude determination arises. In outdoor circumstances one of the typical sensors for positioning is the GPS.

As the accuracy of the normal GPS receivers is over a meter, special types of solutions are needed to increase precision. One possible solution is the usage of the measurements of the carrier phase of the GPS signal. As the wavelength of the signal is about 19 cm, the reachable precision is in the centimeter level.

The most difficult problem with the carrier phase GPS is the solution of the integer ambiguity problem. Existing solutions are shown in [1][2][3]. These methods have disadvantages which are described in section 5.

This article presents a novel method which uses additional sensors like magnetometers and accelerometers to solve the integer ambiguity problem.

This paper is structured as follows. Section 2 and 3 show the calibration methods of the magnetometer and the accelerometer. Section 4 introduces the theory of carrier phase based differential GPS. Section 5 presents a solution for integer ambiguity problem. Section 6 describes the validation of the developed methods during a real flight with a sailplane. Finally Section 7 gives a method which can increase the precision of the previous attitude determination methods.

2 Magnetic sensor calibration

The low-cost magnetic sensors consist usually a magneto-resistive Wheatstone-bridge, an analog-digital converter and some kind of communication interface.

In this case the sensors should be calibrated to measure the real magnetic vector. The length of the magnetic vector on a given point of the Earth can be obtained from the World Magnetic Model produced by the U.S. National Geophysical Data Center. Let \( f_M \) denote the strength of the magnetic field in the place of calibration.

2.1 Error model
Let us introduce the error model of the magnetic sensor.

\[
S m_{real} + m_{bias} = m_{meas}
\]  

where \( S \) is a diagonal matrix responsible for the scaling error, \( m_{bias} \) is an additional offset, \( m_{real} \) is the real magnetic vector and \( m_{meas} \) is the measured value. At the end of the calibration process every unknown parameter of this model can be obtained.

2.2 Calibration method
The base of the measurement method is that if the sensor is rotated into different orientations the sensor should measure the same length of the vector. Therefore the vector of the measurement
should be on the surface of a sphere. But because of errors this sphere is distorted into an ellipsoid. Beside this error model this ellipsoid has parallel axes to the axes of the sensor. This quadratic surface can described with the following equation:

$$\begin{align*}
m_{\text{meas}}^T \mathbf{Q} m_{\text{meas}} &= 0
\end{align*}$$

where $\mathbf{Q}$ is in the form of:

$$\mathbf{Q} = \begin{bmatrix} p_1 & p_4 \\ p_2 & p_5 \\ p_3 & p_6 \\ p_4 & p_5 & p_6 & p_7 \end{bmatrix}$$

where $p_i$ are unknown parameters. Equation (2) and (3) is linear in the unknown parameter vector:

$$\begin{align*}
m^T m &= \mathbf{Q} \mathbf{p} = 0
\end{align*}$$

where $m_x, m_y, m_z$ are the x,y,z components of the measured vector respectively. (4) has a trivial solution, therefore to find the proper solution an additional constraint

$$p^T p = 1$$

should be introduced. In this case the best solution is come from the SVD decomposition:

$$\mathbf{USV}^T = \mathbf{M}_n$$

where $\mathbf{M}_n$ has the measurement components of (4) in its rows. The least square error solution for $\mathbf{p}$ is the column of $V^T$ corresponding to the least singular value.

The task is now to find the homogenous transformation $\mathbf{T}_{\text{magn}}$ satisfying

$$\mathbf{T}_{\text{magn}}^T \mathbf{T}_{\text{magn}} = \hat{\mathbf{Q}}$$

where $\hat{\mathbf{Q}}$ describes the same quadratic surface as $\mathbf{Q}$ but its lower right component should be 1. This can be achieved by

$$\hat{\mathbf{Q}} = r\mathbf{Q}$$

where $r$ is

$$r = \begin{bmatrix} p_4 & p_5 & p_6 \\ p_2 & p_5 & p_7 \\ p_3 & p_6 \end{bmatrix}^{-1}$$

With the composition of $\hat{\mathbf{Q}}$ the transformation $\mathbf{T}$ can easily be achieved. In this case the calibrated magnetic vector can be calculated from the measured one by

$$\hat{\mathbf{m}}_{\text{cal}} = \mathbf{f}_M \mathbf{T}_{\text{magn}} \mathbf{m}_{\text{meas}}$$

### 3 Accelerometer calibration

In the methods of this paper the accelerometer will be used to measure the direction of the gravity vector when the vehicle is stationary. The calibration method presented here is enough for this purpose. But this solution neglects some phenomenon, for example the non-perpendicularity of the axes. A more complex calibration of the accelerometer is described in [4].

The calibration method of the accelerometer is very similar to the magnetic sensor’s. In stationary situation the measured acceleration is the gravitational acceleration, which length is 1G. Therefore in different orientations the measured vectors should be on the surface of a sphere.

The error model of the accelerometer is

$$a_{\text{acc}} + a_{\text{bias}} = a_{\text{meas}}$$

Hence (2)-(9) can be applied for this calibration as well. Then the calibrated acceleration is

$$a_{\text{cal}} = \mathbf{T}_{\text{acc}} \mathbf{m}_{\text{meas}}$$

### 4 Carrier phase differential GPS

The casual GPS receivers have the precision about 1-3 meters. For the attitude determination of a mobile vehicle it is not enough. Another possible solution is the usage of the carrier phase measurement of the GPS signal. As the wavelength of this signal is about 19 centimeters, subdecimeter precision can be achieved with this technique.

A carrier phase differential GPS system has at least two receivers. In this case the structure of the carrier signal can be seen in Fig. 1.

![Figure 1. The carrier phase signal of GPS and its measurement](image)

Fig 1. shows the actual state of the carrier signal at time $t$. The two receivers are indexed with $r$ and $s$. The distance between the $i$th satellite and the
receivers are denoted by $\rho_i^r(t)$ and $\rho_i^s(t)$ and called
pseudorange. The actually broadcasted phase of the
signal is $\varphi_i^s(t)$, the integer number of broadcasted
full wavelength between the satellite and the
receivers are $N_i^s(t)$ and $N_i^r(t)$. The received signal
phases are $\varphi_i^r(t)$ and $\varphi_i^s(t)$. $ICP_i^r(t)$ and $ICP_i^s(t)$
are the integer number of received full periods of
the carrier phase signal since the start of the
receiver. $\Phi_{\rho_i^r}(t)$ and $\Phi_{\rho_i^s}(t)$ are the received
wavelengths since switching on the receivers. Every
value of the parameters are measured in wavelength
unit.

From Fig. 1. the following equations can be derived:

\[ \varphi_i^r(t) + N_i^r(t) - \varphi_i^s(t) = \rho_i^r(t) + \mu_i^r(t) \quad (13) \]

\[ \varphi_i^s(t) + N_i^s(t) - \varphi_i^r(t) = \rho_i^s(t) + \mu_i^s(t) \quad (14) \]

where $\mu_i^r(t)$ and $\mu_i^s(t)$ are measurement noises.
These noises can be modeled as follows:

\[ \mu_i^r(t) = I_i^r(t) - T_i^r(t) + c\lambda^{-1}(\delta_i^r(t) - \delta_i^s(t)) \]
\[ + c\lambda^{-1}(D_i^r(t) - D_i^s(t)) + \mu_i(t) \quad (15) \]

where $c$ is the speed of light, $\lambda$ is the wavelength
of the signal, $I_i^r(t)$ and $T_i^r(t)$ are the ionospheric
and tropospheric delay, $\delta_i^r(t)$ and $\delta_i^s(t)$ are the
time drift of the satellite and the receiver, $D_i^r(t)$ and
$D_i^s(t)$ are the hardware delay of the satellite and the
receiver and $\mu_i(t)$ is a zero mean white noise.

4.1 Single differencing

To handle the complex model of the measurement
error one technique is to obtain the so called single
differenced equations. Subtracting (13) from (14)
and using the notations

\[ \Delta \varphi_i^r(t) = \rho_i^r(t) - \rho_i^s(t) \quad (16) \]

\[ \Delta \mu_i^r(t) = \mu_i^r(t) - \mu_i^s(t) \quad (17) \]

the following equation yields:

\[ N_i^r(t) - N_i^s(t) - \varphi_i^r(t) + \varphi_i^s(t) = \]
\[ \Delta \varphi_i^r(t) + \Delta \mu_i^r(t) \quad (18) \]

Let $\beta(t)$ be

\[ \beta(t) = c\lambda^{-1}(\delta_i^s(t) - \delta_i^r(t) + D_i^r(t) - D_i^s(t)) \quad (19) \]

which is a satellite independent value. Assuming that the two receivers are close to each other, the
tropospheric and ionospheric errors of the receivers are
assumed to be the same. Moreover the receiver
independent parts of $\mu_i^r(t)$ and $\mu_i^s(t)$ are exactly the
same, therefore

\[ \Delta \mu_i^r(t) = \beta(t) + \mu_i(t) \quad (20) \]

Examining the value of $\varphi_i^r(t) + N_i^r(t) + ICP_i^r(t)$
and $\varphi_i^s(t) + N_i^s(t) + ICP_i^s(t)$, it can be seen that their
difference is a time independent constant. Let us
introduce

\[ \Delta N_i^s(t) = N_i^s(t) - N_i^r(t) + ICP_i^r(t) - ICP_i^s(t) \quad (21) \]
called single difference integer ambiguity and

\[ \Delta \varphi_i^s(t) = -ICP_i^r(t) + ICP_i^s(t) - \varphi_i^r(t) + \varphi_i^s(t) \quad (22) \]

which is a pure measurement value from the
receivers at time $t$.

Using (18)-(20) in (16) the single differenced
equation can be formed:

\[ \Delta \varphi_i^r(t) = \Delta \rho_i^r(t) + \beta(t) - \Delta N_i^s(t) + \Delta \mu_i^r(t) \quad (23) \]

For the attitude determination the position vector
between the two receivers play an important role.
The connection between this vector and $\Delta \rho_i^r(t)$ can
be seen in Fig. 2.

![Figure 2. Geometry of single differencing](image)

The position vector between the two receivers is
$x(t)$, called baseline. As the pseudoranges are
typically larger than 22000km hence the length of a
typical baseline is less than 30km. Therefore the
direction vectors $e^i_r$ and $e^i_s$ are approximated to be
the same. The connection between the baseline and
$\Delta \rho_i^r(t)$ is

\[ \lambda^{-1} \Delta \rho_i^r(t) = (e_i^r)^T x(t) \quad (24) \]

4.2 Double differencing

The $\beta(t)$ component in (23) is an unknown variable
and depends only on the property of the two
receivers. One of the possible techniques to
eliminate this component is double differencing. It
means that (23) should be described for two
different satellites and they should be subtracted
from each other.

Introducing the following parameters:
the double differenced equation is

\[ \nabla \Delta \phi^{i,j} (t) = \nabla \phi^{i,j} (t) - \Delta \phi^j (t) \]  

where \( \mu^j(t) \) has zero mean. The problem with (25) is that if there are \( m \) satellites in view then \( m-1 \) equations can be formed, but \( m+2 \) unknown parameters are present. Therefore the solution is indefinite.

### 5 Fixing the integer ambiguities

As the solution of (27) is indefinite the casual solutions for real value equations cannot work. In spite of that there are some methods which try to find the proper solution. These methods can be categorized.

The simplest solution is called floating point solution. In this case many of different measurements in different times are collected and using the time independent property of \( \nabla \Delta N^{i,j} \) the solution of the equation system became obvious. The only problem is that the formed equations should be significantly different. This difference comes from the movement of the satellite. Therefore these solutions can work only if there is enough time to wait a significant satellite movement, which is typically some minutes.

The second type of solutions uses the result of the floating point solutions and also the integer property of \( \nabla \Delta N^{i,j} \). The most well-known method called LAMDA [2]. These solutions can give a more reliable result, but have the same disadvantage as the floating point solution.

The third category uses only the integer property of \( \nabla \Delta N^{i,j} \) and calculates it only from one measurement equation. Therefore the solution formed immediately. The disadvantage is the large computation capacity and usually the number of false positive solutions can be high. A typical method of them is Knight’s method [3].

Common in these solutions is that they use only the measurement of GPS. The solution in this paper uses other sensor data to estimate the baseline first. Then the integer property of \( \nabla \Delta N^{i,j} \) is used to make the solution more precise.

### 5.1 INS and magnetic aided approximation

In this paper the carrier phase based differential GPS method is used for attitude determination. For this purpose two GPS antennas are attached to the roof of a moving (aerial, ground etc.) vehicle and the baseline is calculated for each measurement epoch. By this way the direction of the baseline gives information about the orientation of the vehicle. Let the vehicle’s frame be denoted by \( K_B \).

The magnetic sensor and the accelerometer are also attached to the body of the vehicle. Therefore the direction of the baseline vector between the antennas is known in the frame of the vehicle. Let \( R_B \) be the rotation transformation which transforms the baseline to the x axis of \( K_B \).

The accelerometer measures the gravity vector if the vehicle is stationary. Denote this measurement with \( g_d \) in \( K_B \). This vector gives also the z axis of the North-East-Down (NED) coordinate frame.

The magnetometer measures the magnetic north direction in \( K_B \). Let \( m_N \) be this measured vector. The difference between the x axis of NED and the magnetic North direction is described with the declination angle \( \delta \). This angle can be achieved by the World Magnetic Model.

From these measurements an approximation can be given for the baseline in the NED coordinate system:

\[ \hat{m}_E = g_d \times m_N \]  

\[ \hat{m}_N = \hat{m}_E \times g_d \]  

\[ \hat{x}_{NED} = R_B^T [-S_{\delta} C_{\delta} 0 m^T_N 0 0 0 1 g_d^T 0] \]  

where \( || \) is the real length of the baseline. As the GPS measurement data, similarly to satellite positions, are described in Earth-Center-Earth-Fixed (ECEF) coordinate system, the transformation between ECEF and NED should be calculated based on World Geodetic System (WGS84) model. The baseline in ECEF is

\[ \hat{x}_{ECEF} = R_{ECEF,NED} \hat{x}_{NED} \]

### 5.2 Search for the integer ambiguity

To find the proper integer ambiguity, the first task is to bound the space where the proper value is in. This method is based on (24). Assuming error parameters \( \epsilon^i \) for (30)

\[ e^{T} \hat{\epsilon} - \epsilon^i < \lambda^i \Delta \rho^i < e^{T} \hat{\epsilon} + \epsilon^i \]  

Forming the bounds for the double differenced pseudorange it yields

\[ \left[ \left( e^{T} - e^{T}_i \right) \hat{\epsilon} - \epsilon^i - \epsilon^j \right] - 1 \leq \lambda^i \nabla \Delta \rho^{i,j} \leq \left[ \left( e^{T} - e^{T}_i \right) \hat{\epsilon} + \epsilon^i + \epsilon^j \right] + 1 \]

Hence using (27)
\[ \nabla \Delta \phi_{i,j} + \left[ \left( e^{T} - e^{jT} \right) i \right] - e^{i} - e^{j} - 1 \leq \nabla N_{i,j} \leq \nabla \Delta \phi_{i,j} + \left[ \left( e^{T} - e^{jT} \right) i \right] + e^{i} + e^{j} + 1 \] (34)

gives the bounds for the double differenced integer ambiguities.

The task is now to find the optimal integer within the search space. The solution is based on (27). For a given possible integer ambiguity (27) can be solved if the number of the visible satellites is at least four.

Let \( \bar{x} \) be one of the possible solutions for the baseline. The decision about the optimal solution is based on the correspondence to \( \hat{x} \). The simplest way is to find the least square error solution

\[
x_{\text{opt}} = \arg \min \| \hat{x} - \bar{x} \| 
\] (35)

Another weighted least square solution can be found in [5], which can decrease the computational capacity of the method.

### 6 Measurement results

The measurement process was evaluated during a flight with a sailplane. The path of the test flight can be seen in Fig. 3.

![Figure 3. Path of the test flight](image)

The GPS antennas were attached to the roof of the sailplane, by this way the baseline can describe the heading direction of the plane.

Unfortunately one baseline is not enough to reconstruct the full orientation of the plane. Therefore only the North-East direction can be examined. The same problem occurs with the magnetometer.

The measurement from the magnetic sensor can be seen in Fig. 4.

![Figure 4. Magnetic sensor data](image)

Using the measurement method described in section 5, the integer ambiguity can already be fixed from the first measurement epoch. Hence forth the baseline can be calculated for the full measurement period. The baseline was projected to the N-E plane. The N-E angle of the results can be seen in Fig. 5.

![Figure 5. N-E direction from GPS sensors](image)

Validating the result of the carrier phase GPS measurement, the length of the baseline can be calculated. The real length in this case was 2.45 m. This length can be seen in Fig. 6.

![Figure 6. Length of the baseline](image)

### 7 Increasing precision

It can be seen in Fig. 6 that the length of the baseline differs from the ideal value. Sometimes the error value reaches the 10 cm.
This effect can come from the approximations of (13)-(23) and from the measurement noise. Therefore it can be worth the computation capacity to perturb the integer ambiguities a little bit.

Let $\nabla \Delta \vec{N}^{i,j}_k$ be the $k$th component of the double differenced integer ambiguity. As the error in Fig. 6 is less than one wavelength the perturbation can be

$$\nabla \Delta \vec{N}^{i,j}_k = \nabla \Delta \vec{N}^{i,j}_k + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(36)

In the case of $m$ visible satellites there are $3^{m-1}$ possible baseline values which are small enough to handle in real-time.

Examining the possible solutions the smallest length error can be chosen. The corrected results for the full measurement record can be seen in Fig. 7. In this case the error is usually under 1 cm.

8 Conclusion and future work
This paper presented a simple but reliable calibration method for magnetic sensor and accelerometer. An introduction to the carrier phase based differential GPS is also shown. A fast and reliable solution for the integer ambiguity problem is presented with fusion of the calibrated magnetometer and accelerometer measurement. The methods are tested in a real flight with a sailplane. Finally, based on the result a correction method is shown which gives better solution for baseline calculation.

The carrier phase differential GPS method can be extended to a three antennas system, which is able to measure the full orientation of the plane. Using this information the angle of attack and the side slip angle of airplanes can also be calculated.

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